Quantum Erasure Experiment is not a Miracle

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Abstract

This paper rejects the common notion that ”erasing path information causes interference fringes to reappear”. This paper argues that it is a misnomer to call this experiment a quantum ”erasure”. Instead of ”erasing”, the diagonal polarizers actually filter the photons, and it is this filtering that causes the interference fringes to reappear.

Further, there is no need to introduce anticausality to explain the delay selection experiment. The wavefunction of a photon does not collapse after it comes out of the double slit until it reaches the position of the screen. It is meaningless to ask which slit the photon came through. Therefore, instead of calling this experiment a ”quantum erasure experiment”, it would be more appropriate to call it a ”quantum screening experiment”, and it is this means of screening that restores coherence.

Keywords：Quantum eraser experiment, Double-slit experiment

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1 Introduction

In the original double-slit interference experiment, when an electron is shot from the gun, it is in the state $|\psi_0\rangle$. Upon reaching the double-slit plate, only the wave function that can pass through the two slits remains, while the rest is absorbed by the plate. $|\psi_1\rangle$ or $|\psi_2\rangle$ represents the wave function of the electron being at slit 1 or slit 2, respectively.

$|\psi_0\rangle \rightarrow (|\psi_1\rangle + |\psi_2\rangle)$

Since electrons that do not hit the two slits are absorbed by the plate, the above evolution is not unitary. Also, because the two slits are symmetric, the two quantum states can be superposed with equal weights.

After passing through the slits, the electron continues to fly towards the screen, evolving into a superposition of quantum states at various points. The evolution of the quantum state passing through slit 1 is:

$||\psi_1\rangle \rightarrow \int a(z)\,|\varphi(z)\rangle\,dz$

$|\varphi(z)\rangle$ is the quantum state of the electron at different z coordinates, and $a(z)$ is its weight. Similarly, the evolution of the quantum state passing through slit 2 is:

$||\psi_2\rangle \rightarrow \int b(z)\,|\varphi(z)\rangle\,dz$

$\varphi(z)$ forms a complete orthogonal basis set. There is $\langle\varphi(z')|\varphi(z)\rangle = 0,\ z' \neq z$, which means that the electron cannot appear at two places on the screen at the same time; and $\int |\varphi(z)|^2 dz = |\psi_1|^2 + |\psi_2|^2$, which means that all electrons passing through the two slits fall on the screen.

The number of electrons detected at position z is:

$(a(z)+b(z))^*(a(z)+b(z)) = |a(z)+b(z)|^2 = |a(z)|^2+|b(z)|^2+a(z)b(z)+a(z)b(z)^*$

$a(z)^*b(z) + a(z)b(z)^*$ are called interference terms, which do not exist in classical situations. We must emphasize that double-slit interference is a result of an electron ”interfering with itself,” not due to interference between electrons. Therefore, even if electrons are emitted one by one with a long interval, interference fringes will still appear.

1.1 Einstein's Thought Experiment

At the Fifth Solvay Conference, Einstein proposed a thought experiment: install rollers on the double-slit plate so it can slide freely left and right. When an electron passes through the slit, it will collide with the slit, imparting momentum to the plate in the left or right direction. By observing the momentum acquired by the plate, we can deduce through which slit the electron passed (since the electron gun is positioned between the two slits, if the plate moves to the left, it indicates the electron passed through the left slit). Thus, without disturbing the electron, we can determine its path by observing the movement of the plate.

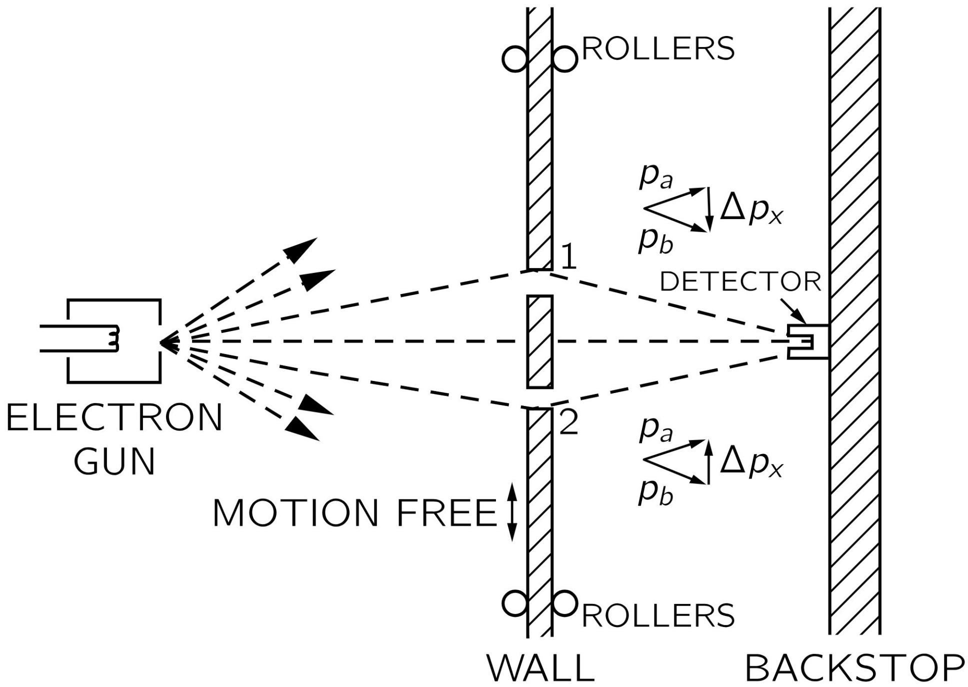


Figure 1: Einstein's Thought Experiment

Bohr replied that to achieve this, one must know the momentum of the electron before it passes through the plate. By measuring the electron's momentum after it passes through the plate, one can calculate how much the plate's momentum has changed. However, remember that according to the uncertainty principle, we cannot know the position of the plate with arbitrary accuracy. If we do not know the exact position of the plate, we cannot precisely state the locations of the two slits. For each electron passing through the slit, the slit will be in a different position, meaning the center of the interference pattern will be in different positions for each electron, causing the interference fringes to disappear.

If we accurately measure the plate's momentum to determine which slit the electron passed through by measuring the recoil momentum, the uncertainty in the plate's x position, according to the uncertainty principle, will cause the observed pattern on the screen to shift up and down by a distance equivalent to that between the maximum and nearest minimum of the interference fringes. This random movement will erase the interference pattern, resulting in no observable interference.

Bohr used the "complementarity principle" to explain wave-particle duality, saying, "The behavior of quantum systems is both particle-like and wave-like, depending on the experimental setup." If a which-path detector is used, the electron behaves as a "particle"; if not used, the electron behaves as a "wave." This notion hints at "observation creating reality." Bohr's physical explanation relies on Heisenberg's uncertainty principle, stating that observing a microscopic object inevitably introduces uncontrollable momentum and energy disturbances. Heisenberg himself used a thought experiment with a microscope to illustrate the momentum-position uncertainty relationship, attributing the uncertainty to the momentum impact of photons on the observed object.

Next, we will use the concept of entanglement to provide another explanation. If the electron passes through slit 1, the plate will shift left, denoted as $|left\rangle$, and if it passes through slit 2, the plate will shift right, denoted as $|right\rangle$. The evolution from the electron gun to the double-slit plate is described by:

$|\Psi\rangle = |\psi_1\rangle \otimes |left\rangle + |\psi_2\rangle \otimes |right\rangle$

The electron's wave function and the information about the plate's movement become entangled. What difference will this make compared to the original double-slit experiment?

The interference pattern on the screen can be described by:

$|a(z)|^2 + |b(z)|^2 + a(z)b(z)\,\langle left|right\rangle + a(z)b(z)\,\langle right|left\rangle$

In the original double-slit experiment, the plate is fixed, so $|left\rangle$ and $|right\rangle$ are indistinguishable, and can be considered the same state, so $\langle right|left\rangle = 1$. However, if $\langle right|left\rangle = 0$, it is akin to using a light source to detect which slit the electron passed through, causing the interference fringes to disappear. Due to the significant uncertainty in measuring the plate's momentum, it's impossible to precisely measure the change in momentum caused by the electron's collision, and measuring the plate won't clearly indicate which slit the electron passed through. Experimentally, $|left\rangle$ and $|right\rangle$ are not orthogonal.

We have discovered that entanglement always leads to decoherence, but experimentally, maximum entanglement is not always achievable. Generally, $0 \leq \langle right|left\rangle \leq 1$. If the inner product of these two states is 0, they are fully distinguishable, and the fringes will completely disappear; if 1, they are indistinguishable, and clear fringes are visible. If the inner product value is in between, the fringes will blur but not disappear.

If I entangle the electron's wave function with something, and the two entangled states are orthogonal, the interference fringes will disappear. In the next experiment, we will find that "measurement" does not always occur during the interaction between macroscopic instruments and microscopic objects.

1.2 Feynman's Thought Experiment

In "The Feynman Lectures on Physics," Feynman used an illumination system as a which-path detector, explaining the disappearance of interference fringes by the uncontrolled impact of photons on electrons. Recent research indicates that the decoherence effect of a which-path detector mainly comes from the entanglement between the detected object and the detector's quantum states, rather than from the momentum-energy impact traditionally believed (Feynman, 1963).

Feynman proposed a thought experiment: place a light source behind the double-slit, allowing us to determine through which slit the electron passed. This light source must be as weak as possible to minimize the scattering effect on the electron's momentum, avoiding the argument that the collision between photons and electrons deflects the electron's trajectory, thereby altering the interference fringes.

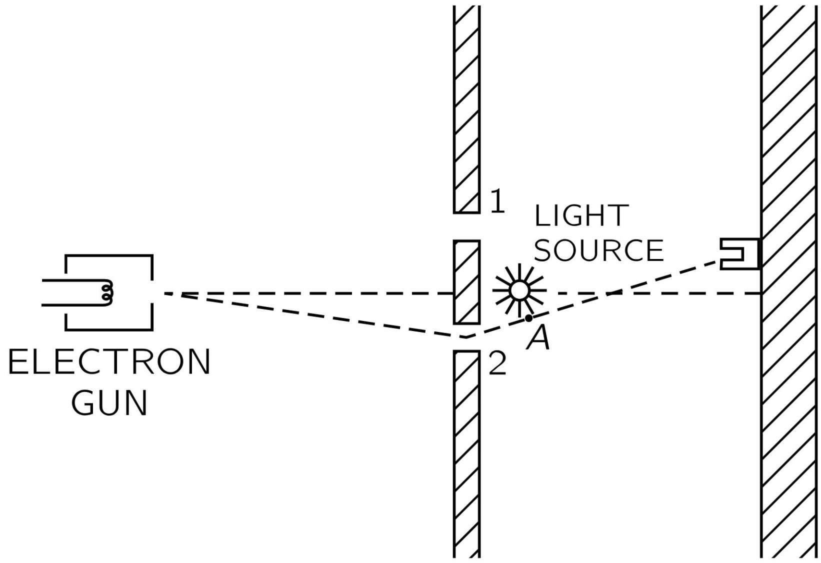


Figure 2: Feynman's Thought Experiment

If we detect the path without disturbing the wave function evolving through the two slits, recording electrons passing through slit 1 as M_1 and those passing through slit 2 as M_2, the evolution from the electron gun to the double-slit plate is described by:

$|\psi_0\rangle \rightarrow |\psi_1\rangle \otimes |M_1\rangle + |\psi_2\rangle \otimes |M_2\rangle$

The electron's wave function becomes entangled with its path information, differing from the original double-slit experiment. After passing through the slits, the evolution towards the screen is:

$|\psi_1\rangle \otimes |M_1\rangle + |\psi_2\rangle \otimes |M_2\rangle \rightarrow |M_1\rangle \otimes \int a(z)\,|\varphi(z)\rangle\,dz + |M_2\rangle \otimes \int b(z)\,|\varphi(z)\rangle\,dz$

Taking the square modulus of the above equation gives the number of electrons detected at position z. Since the two states of the detected information are orthogonal (the electron cannot be observed passing through both slits simultaneously), $\langle M_1|M_2\rangle = \langle M_2|M_1\rangle = 0$, causing the interference terms $a(z)^* b(z)$ and $a(z)b(z)^*$ to disappear. As the path information is entangled with the wave function, and we detect through which slit the electron passed, the interference fringes disappear.

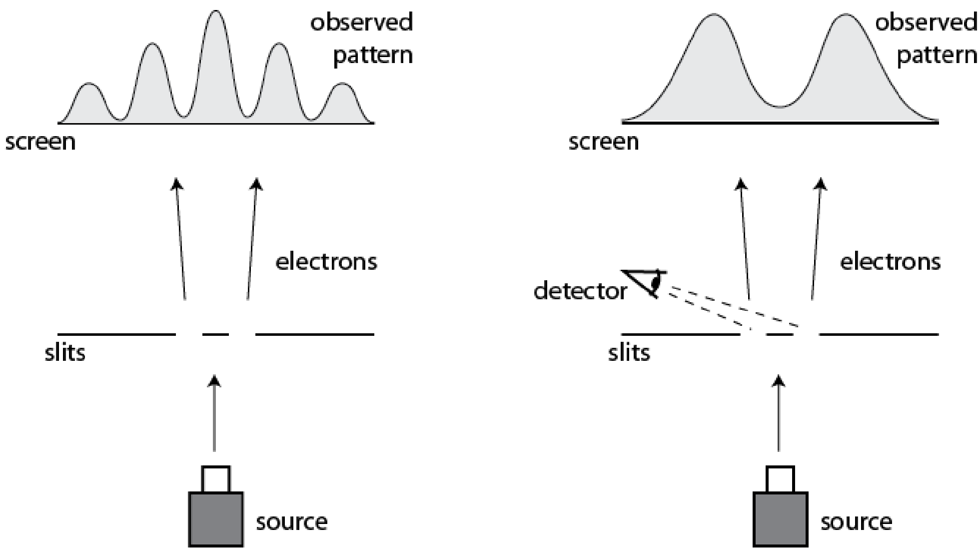


Figure 3: Detecting the hole through which the electrons pass causes the interference fringes to disappear

Previously, we believed that the disappearance of interference fringes in "which-path" experiments was due to the complementarity principle of wave-particle duality. According to Heisenberg's uncertainty principle, it was thought that measurement would cause uncontrollable disturbances to the particle's momentum. However, this is not the only mechanism explaining the disappearance of interference fringes. We can also use quantum entanglement to explain the disappearance of interference fringes.

1.3 Placing a Stern-Gerlach Apparatus in Front of the Double Slits

We place a Stern-Gerlach apparatus in front of the double slits. To avoid interference from Lorentz force, we replace the electron with a neutron. After passing through a non-uniform magnetic field, the neutron beam splits into two beams, and the double-slit plate is positioned so that the two neutron beams pass through the double slits. Will there still be interference fringes? The conclusion is that there will be no interference. But why? What is different in this case compared to the ordinary double-slit interference? Since the neutron's spin is random, it should randomly pass through the two slits.

A neutron passing through slit 1 has spin-up, while a neutron passing through slit 2 has spin-down. The evolution from the neutron source to the double-slit plate is described by:

$|\psi_0\rangle \to |\psi_1\rangle \otimes |u\rangle + |\psi_2\rangle \otimes |d\rangle$

The spatial position of the neutron becomes entangled with its spin state, which is the key difference from previous double-slit interference. The number of electrons detected at position z is:

$|a(z)|^2 + |b(z)|^2 + a(z)^* b(z) \langle u|d \rangle + a(z) b(z)^* \langle d|u \rangle$

Since the spin states are orthogonal, $\langle u|d \rangle = \langle d|u \rangle = 0$, the interference terms $a(z)^* b(z)$ and $a(z) b(z)^*$ disappear. In this experiment, we still do not know which slit a single neutron passes through, but the interference fringes still disappear.

In this experiment, the disappearance of the interference phenomenon is actually due to "measurement," though this measurement is not obvious. After the measurement, neutrons flying upwards are labeled $|u\rangle$, spin-up; and neutrons flying downwards are labeled $|d\rangle$, spin-down. This measurement is relatively "concealed": first, the measurement result or label $|u\rangle$ and $|d\rangle$ is a microscopic state; second, this spin is intrinsic to the neutron, not from photons or other particles or systems. Due to this concealment, we cannot determine through which slit the neutron passes, but the interference phenomenon still disappears. This "measurement" is not the conventional interaction with a macroscopic instrument but rather the "marking" using the neutron's microscopic spin degree of freedom.

From the above examples, we see that entangling with anything (even microscopic) will kill the interference effect. As long as the two states become distinguishable, the interference phenomenon will disappear, and any degree of freedom can be used for " marking." This again highlights the deficiency of the Copenhagen interpretation, where measurement does not require interaction with macroscopic instruments; it can also occur with microscopic spin entanglement.

Taking double-slit interference or any double-path interferometer (such as a Mach-Zehnder interferometer) as an example, suppose the electron (or photon) passes through two paths with partial waves $|\psi_1\rangle$ and $|\psi_2\rangle$, and the quantum state of the instrument and environment is $|D\rangle$. If no which-path detector is installed, the total quantum state of the experimental setup is:

$|\Psi_0\rangle = (|\psi_1\rangle + |\psi_2\rangle) \otimes |D\rangle$

This means the instrument and environment treat the two beams of particles (or waves) equally, the total quantum state is expressed as a direct product form, with no entanglement. When a which-path detector is installed, it responds to the particle passing through a particular path, and the total quantum state becomes:

$|\Psi\rangle = |\psi_1\rangle \otimes |D_1\rangle + |\psi_2\rangle \otimes |D_2\rangle$

$|D_1\rangle$ and $|D_2\rangle$ become different, coupling respectively with the two partial waves, thus making them distinguishable. Now let's look at the probability distribution of the particles, which is described by the square modulus of $|\Psi\rangle$:

$\langle\Psi|\Psi\rangle = \langle\psi_1|\psi_1\rangle + \langle\psi_2|\psi_2\rangle + \langle\psi_1|\psi_2\rangle \langle D_1|D_2\rangle + \langle\psi_2|\psi_1\rangle \langle D_2|D_1\rangle$

In the absence of a which-path detector, $|D_1\rangle = |D_2\rangle = |D\rangle$, $\langle D_1|D_2\rangle = \langle D|D\rangle = 1$, hence the interference term is fully retained.

"Two paths can be completely distinguished" means $|D_1\rangle$ and $|D_2\rangle$ are orthogonal, i.e., $\langle D_1|D_2\rangle = 0$, and the interference fringes disappear. Intermediate situations occur when $|D_1\rangle$ and $|D_2\rangle$ are neither completely identical nor completely orthogonal, $0 < |\langle D_1|D_2\rangle| < 1$, resulting in reduced contrast of the interference fringes, indicating partial coherence.

How do the two states $|D_1\rangle$ and $|D_2\rangle$ of the which-path detector become orthogonal? The above discussion may be somewhat abstract. We can summarize the double-path interferometer as shown below. The particle beam is split into two beams by a beam splitter (BS), travels through different paths, and reunites at a beam merger (BM), causing interference.

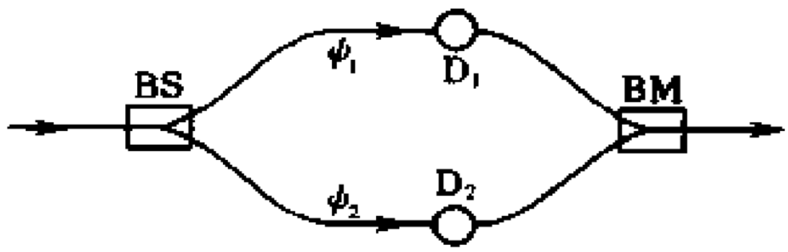


Figure 4: The double-path interferometer

Until the era of Wheeler and Feynman, people still believed that detecting which-path information required the detector to react to the passing particle, thus always causing uncontrollable interference with the particle's motion. This is not necessary; for example, we can use the particle's internal state to mark its path. Suppose a particle has two internal states, spin-up and spin-down ($\uparrow$ and $\downarrow$), which can be the up and down spin states of an electron, or the two polarization states of a photon (vertical and horizontal, V and H), or two energy levels resulting from the hyperfine splitting of an atom. Detecting the internal state of the particle can, in principle, not interfere with the particle's overall motion. It can be seen that the decoherence effect of the which-path detector does not necessarily result from uncontrollable impacts on the particle.

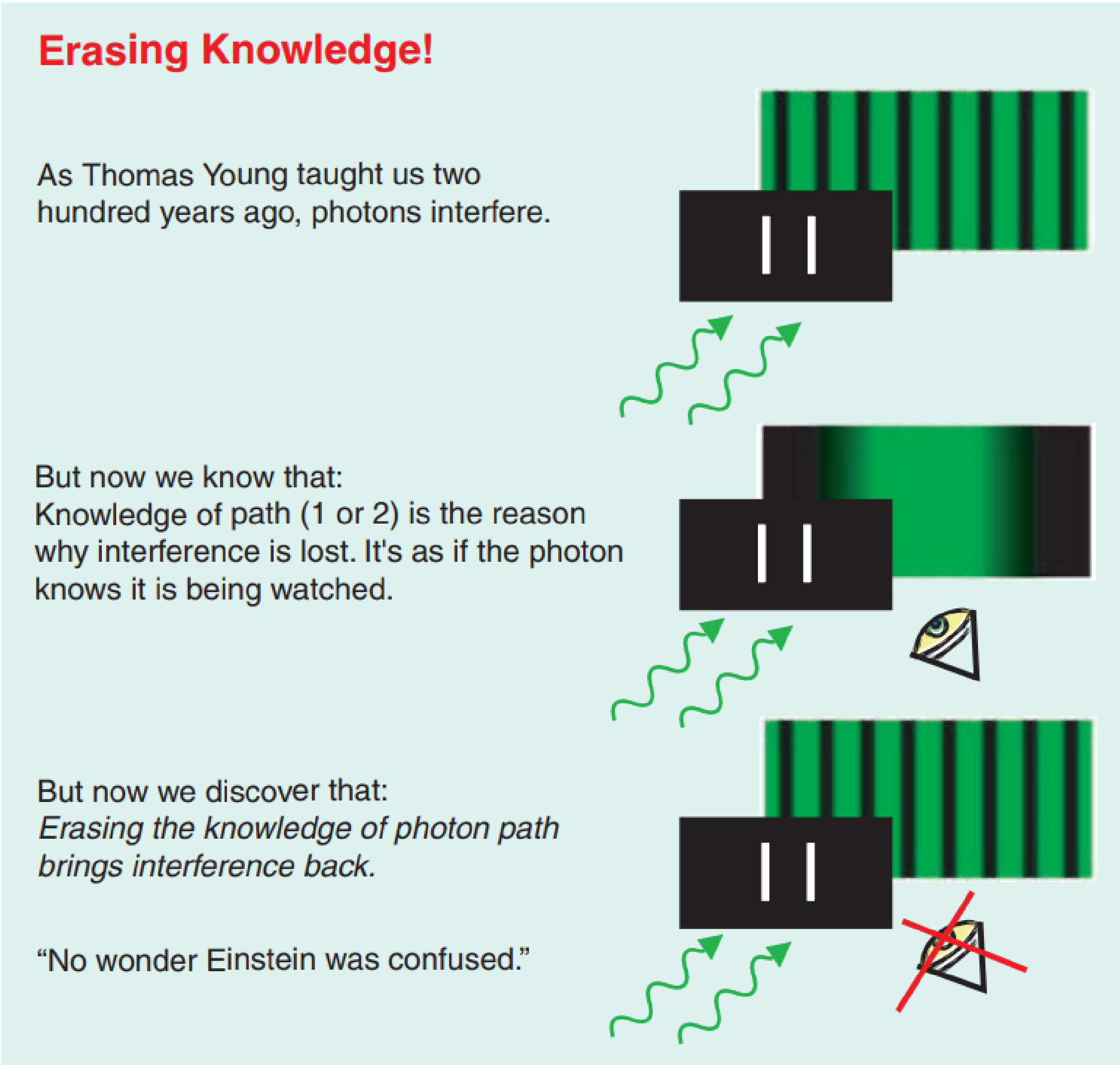


Fig. 1. Schematics for the Young's double-slit experiment. The which-path information wipes out the interference pattern. The interference pattern can be restored by erasing the which-path information.

The quantum eraser experiment discussed in this article uses the vertical and horizontal polarization states of photons to mark path information, causing the interference fringes to disappear. Placing a diagonal polarizer (as a quantum eraser) restores the interference fringes.

This article refutes a common view: ”Erasing path information will lead to the reappearance of interference fringes (Aharonov, 2005; Hillmer, 2007).” The article argues that calling this experiment a quantum ”eraser” is a misunderstanding. Rather than erasing, the diagonal polarizers actually filter the photons, and this filtering restores the interference fringes.

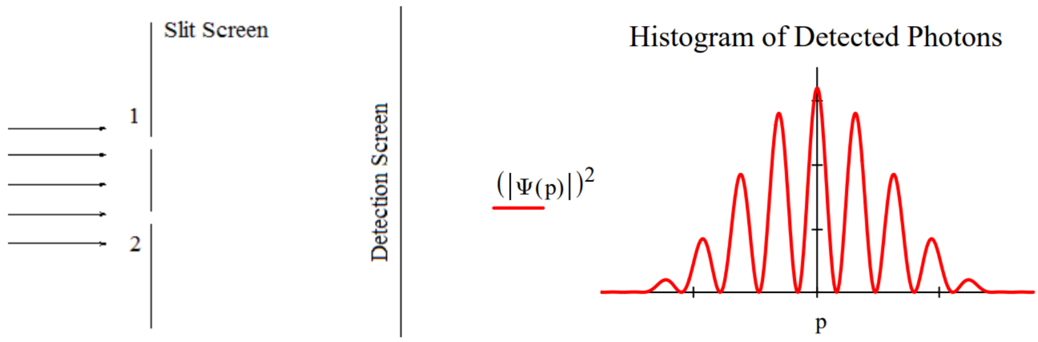
Furthermore, there is no need to introduce retrocausality to explain the delayed choice experiment. The photon’s wave function, after passing through the double slit and before reaching the screen, does not collapse. Asking which slit the photon passed through during this process is meaningless. Therefore, this experiment is more appropriately called a ”quantum filtering experiment” rather than a ”quantum eraser experiment,” as the filtering restores coherence.

2 The Quantum Eraser Experiment with Photon Polarization

The quantum eraser experiment with photon polarization is designed in the following four steps:

1. In the traditional double-slit experiment, the photon's wave function passes through both slits simultaneously, forming interference fringes.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|x_1\rangle + |x_2\rangle\right] \qquad \Psi(p) = \langle p|\Psi\rangle = \frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle + \langle p|x_2\rangle\right]$$



To calculate the interference pattern, the equation $|\Psi\rangle$ must be Fourier transformed into momentum space to obtain it (Rioux, 2004).

The specific mathematical derivation is as follows:

The wave function passing through a slit of width δ is:

$$\int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\delta}} e^{-ipx} dx$$

The wave function passing through two slits of width δ located at x_1 and x_2 is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)$$

$$\Psi(p) = \langle p|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\langle p|x_1\rangle + \langle p|x_2\rangle\right)$$

$$\Psi(p) = \frac{1}{\sqrt{2}}\left(\int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\delta}} e^{-ipx} dx + \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\delta}} e^{-ipx} dx\right)$$

The interference pattern is $\left|\Psi(p)\right|^2$, with the function graph shown below:

$$\text{Plot}\left[\text{Abs}\left[\int_{-1-0.2}^{-1+0.2} e^{-i p x}\, dx + \int_{1-0.2}^{1+0.2} e^{-i p x}\, dx\right]^2, \{p, -20, 20\}\right]$$
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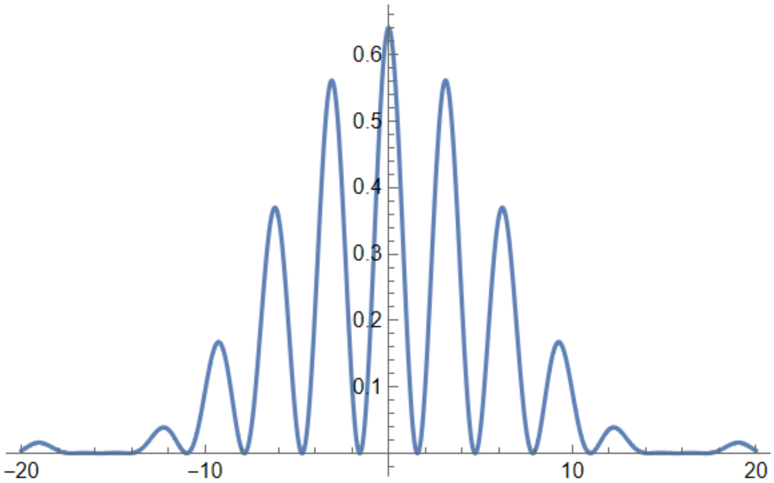
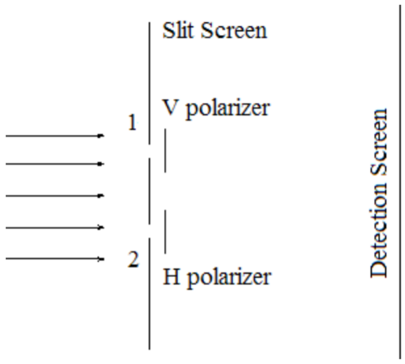
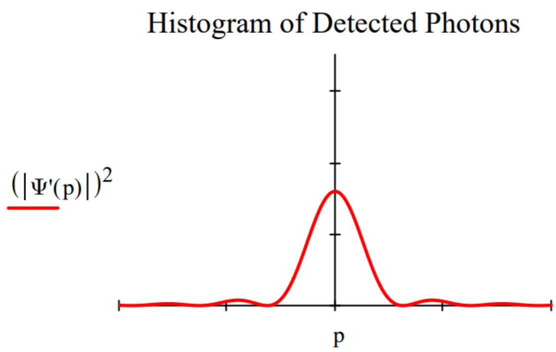


Figure 5: Double slit interference pattern

2. Place horizontal and vertical polarizers behind the double slits, causing the interference fringes to disappear due to the entanglement between the photons from the slits and the path information.

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left[|x_1\rangle|V\rangle + |x_2\rangle|H\rangle\right] \qquad \Psi'(p) = \langle p|\Psi'\rangle = \frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle|V\rangle + \langle p|x_2\rangle|H\rangle\right]$$





The specific mathematical derivation is as follows:

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(|x_1\rangle\,|V\rangle + |x_2\rangle\,|H\rangle\right)$$

$$\langle\Psi'|\Psi'\rangle = \frac{1}{2}\left(\langle x_1|x_1\rangle\,\langle V|V\rangle + \langle x_1|x_2\rangle\,\langle V|H\rangle + \langle x_2|x_1\rangle\,\langle H|V\rangle + \langle x_2|x_2\rangle\,\langle H|H\rangle\right)$$

Since horizontal and vertical polarizers are orthogonal, $\langle V|H\rangle = \langle H|V\rangle = 0$, causing the two cross-interference terms to disappear. Projecting it onto momentum space yields:

$$\langle \Psi(p)'|\Psi(p)'\rangle = \frac{1}{2}\left(\langle x_1|p\rangle \langle p|x_1\rangle + \langle x_2|p\rangle \langle p|x_2\rangle\right)$$

$$\left|\Psi(p)'\right|^2 = \frac{1}{2}\left(\left|\int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\delta}}e^{-ipx}dx\right|^2 + \left|\int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{\delta}}e^{-ipx}dx\right|^2\right)$$

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Plot[(Abs[∫_{1-0.2}^{1+0.2} Exp[-i*p*x] dx])^2 + (Abs[∫_{-1-0.2}^{-1+0.2} Exp[-i*p*x] dx])^2, {p, -20, 20}]
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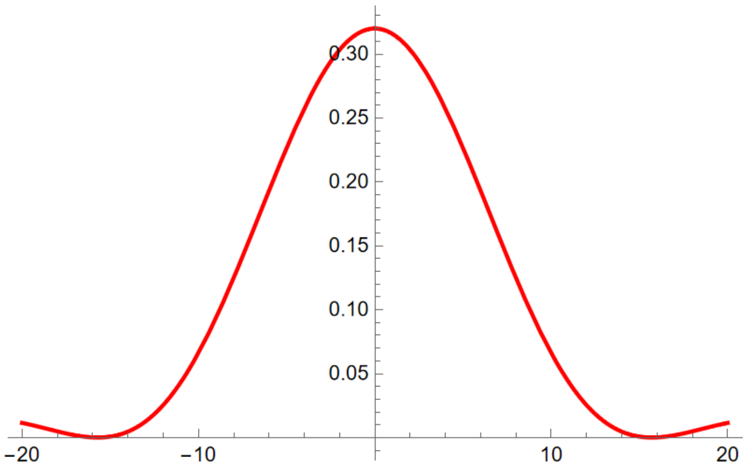

Figure 6: Double-Slit Pattern with Polarizers

Not only do the interference fringes disappear, but the peak light intensity at the center also reduces by half, as some energy fills in the dark fringes.

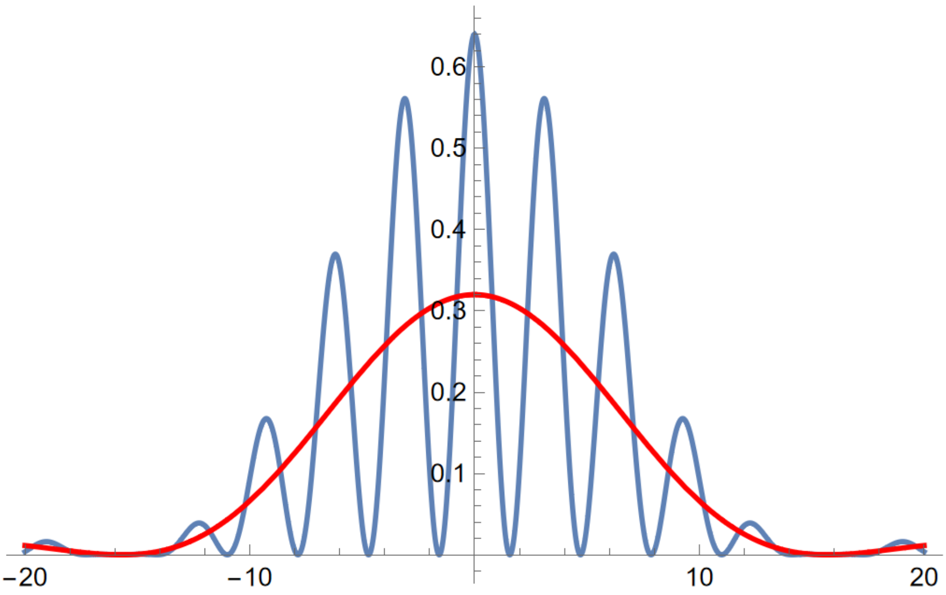

Figure 7: Comparison of Double-Slit Patterns with and without Polarizers

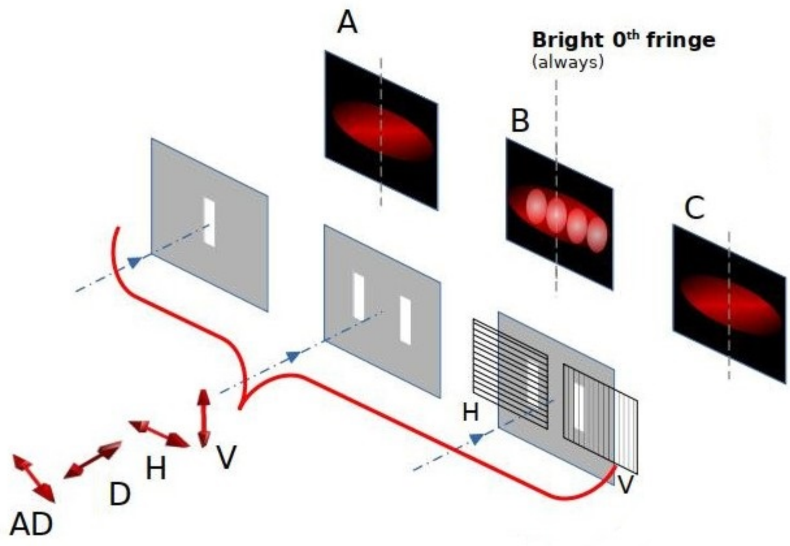

Figure 8: Experimental devices 1 and 2 correspond to B and C in this diagram

3. Next, place a polarizer at a 45° angle (Diagonal) as a "quantum eraser." Since $\langle D|V\rangle = \langle D|H\rangle = \frac{1}{2}$, the interference fringes reappear.

$$\Psi''(p)=\langle D|\Psi'(p)\rangle=\frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle\langle D|V\rangle+\langle p|x_2\rangle\langle D|H\rangle\right]=\frac{1}{2}\left[\langle p|x_1\rangle+\langle p|x_2\rangle\right]$$

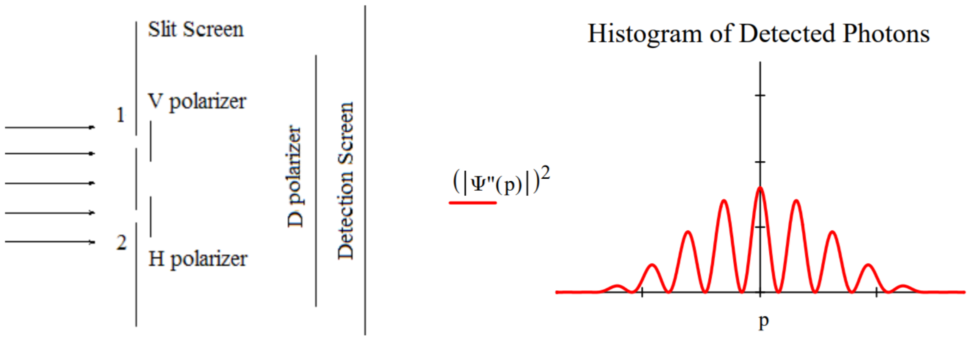

Figure 9: Double slit experimental setup with +45° polarizer in place

The function graph of $|\Psi''(p)|^2$ is half of $|\Psi(p)|^2$.

4. Finally, if a polarizer at a -45° angle (Anti-diagonal) is placed, the interference fringes will shift by $\frac{\pi}{2}$ in phase, with bright and dark fringes inverted.

$$\Psi'''(p)=\langle A|\Psi'(p)\rangle=\frac{1}{\sqrt{2}}\left[\langle p|x_1\rangle\langle A|V\rangle+\langle p|x_2\rangle\langle A|H\rangle\right]=\frac{1}{2}\left[\langle p|x_1\rangle-\langle p|x_2\rangle\right]$$

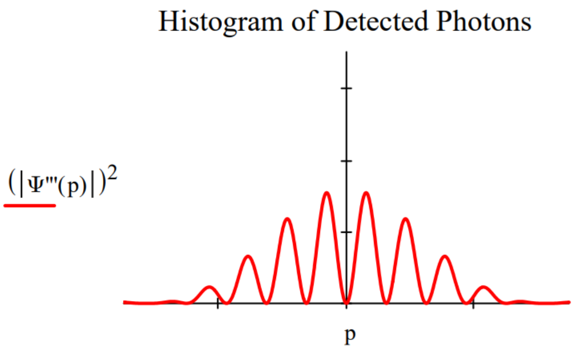

Figure 10: Double-Slit Experimental Setup with -45° Polarizer

The two cases with 45° polarizers can be explained by whether the polarization direction of the incident light is parallel or perpendicular to the 45° polarizer, corresponding to a central maximum or minimum.

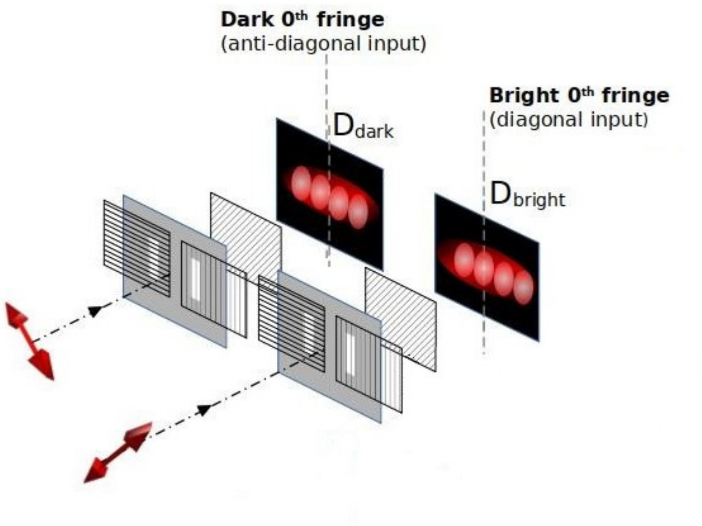

Figure 11: Double-Slit Pattern with ±45° Polarizers

If the upper half of the D polarizer and the lower half of the A polarizer are combined and placed in the light path, the following phenomenon will be observed: the interference fringes corresponding to the D polarizer appear in the upper part of the light spot, while the fringes corresponding to the A polarizer appear in the lower part. The interference pattern looks like mismatched teeth.

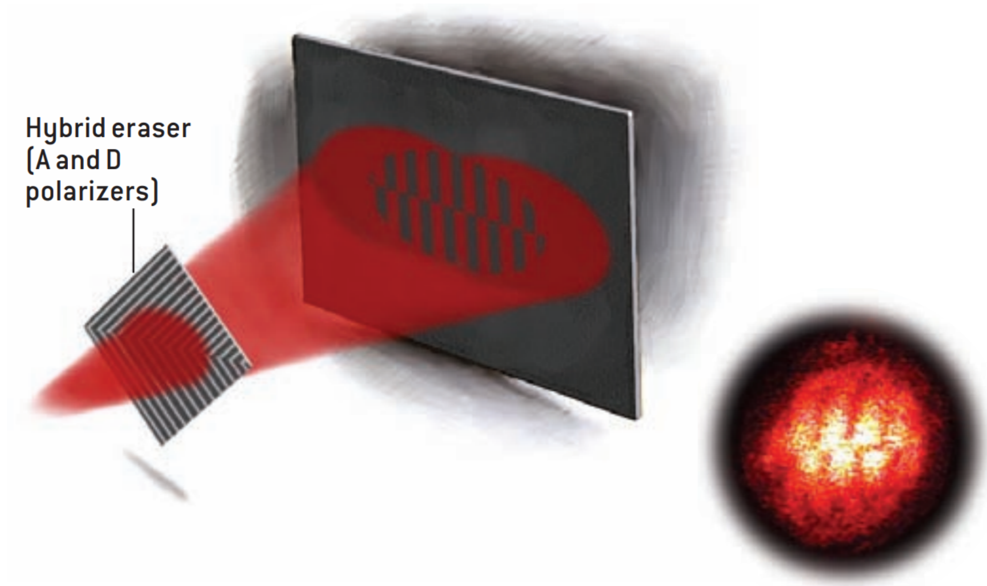

Figure 12: Combining D and A Polarizers

Finally, place the four graphs together for comparison. The blue, red, orange, and green curves represent $|\Psi(p)|^2, |\Psi'(p)|^2, |\Psi''(p)|^2, |\Psi'''(p)|^2$ respectively.

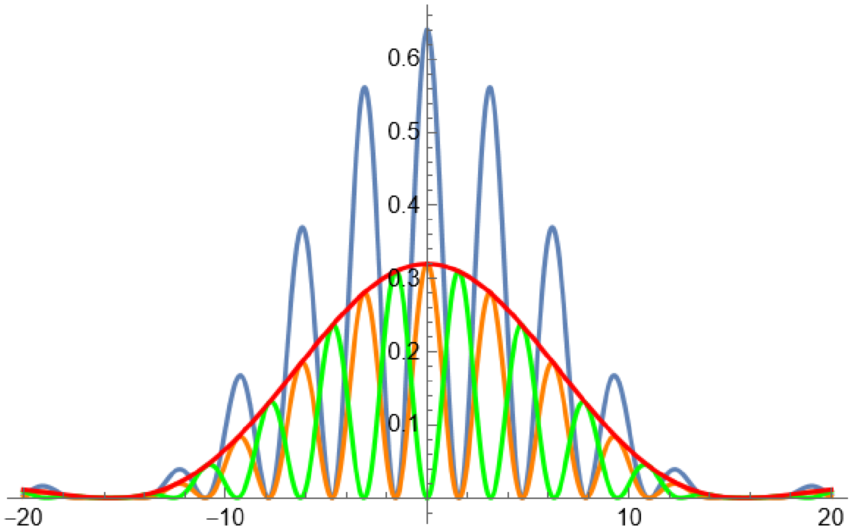

Figure 13: Four Graphs

It is easy to extend and infer that if the eraser polarizer is at an angle between ±45°, the screen will show an interference pattern with reduced contrast, a result of partial coherence.

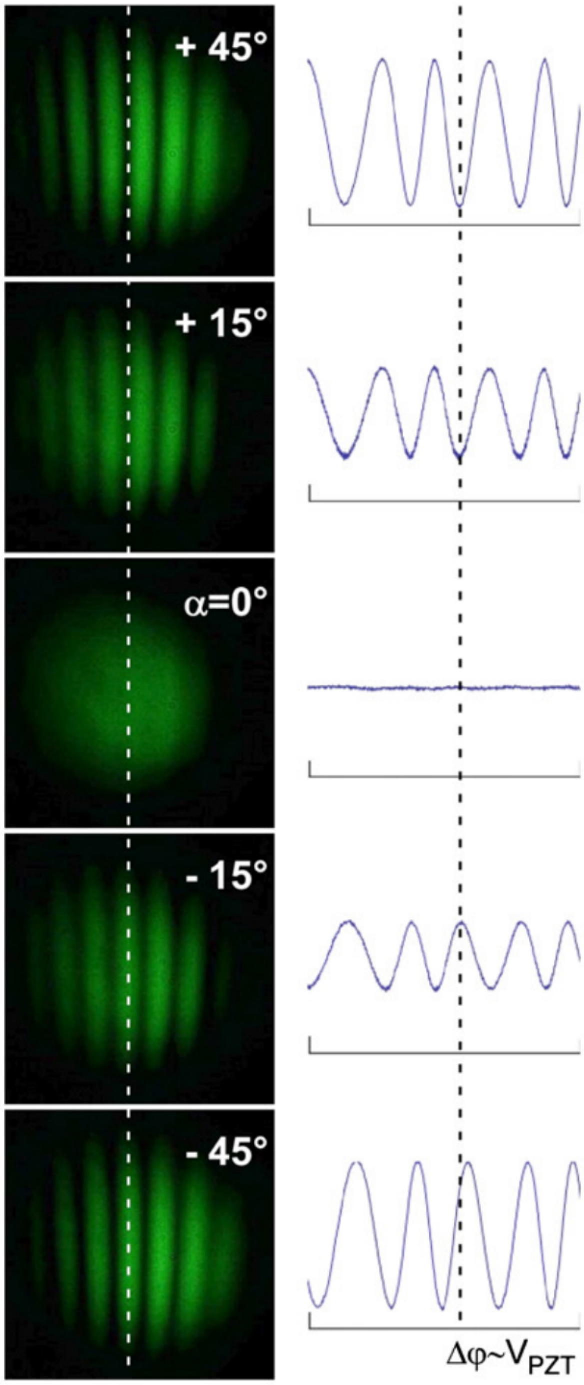

Figure 14: Double-Slit Pattern with Polarizers

5. For any eraser polarizer at an angle between ±45°, a universal solution can be derived.

Place a vertical polarizer at slit 1 and a polarizer at an angle θ relative to the vertical direction at slit 2. The final eraser polarizer is at an angle ϕ relative to the vertical direction.

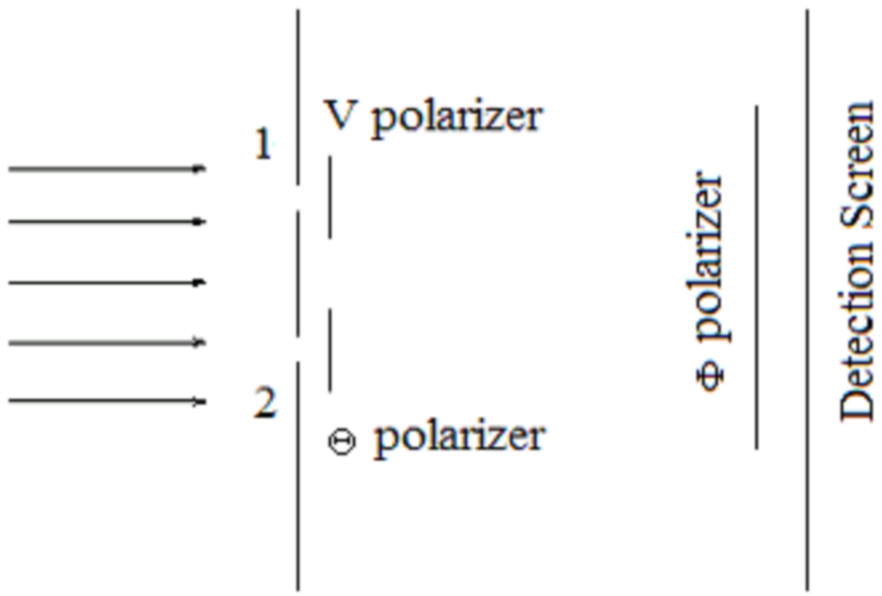

Figure 15: Universal Case

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|x_1\rangle|V\rangle + |x_2\rangle|\theta\rangle\right)$$

$$\Psi(p,\phi) = \langle p,\phi|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\langle p|x_1\rangle\langle\phi|v\rangle + \langle p|x_2\rangle\langle\phi|\theta\rangle\right)$$

$$\Psi(p,\phi) = \langle p,\phi|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\langle p|x_1\rangle\cos\phi + \langle p|x_2\rangle\cos(\theta-\phi)\right)$$

3 Quantum ”Erasure” is a Misleading Term

The usual explanation for the quantum eraser experiment is that knowing through which slit the photon passed will destroy the interference because the photon is no longer in a superposition state of passing through both slits but rather a mixed state of passing through one slit and the other.

This article argues that calling this experiment a quantum ”eraser” is a misleading colloquialism. Polarizers only allow photons parallel to their polarization direction to pass through. Thus, rather than erasing, it is more accurate to say they filter the photons.

A better explanation is that the photon’s state entangles with the polarizer, thus getting ”tagged.” Since V/H two ”tags” are orthogonal, the cross-interference terms in $|\Psi'\rangle$ disappear, leaving the pattern on the screen as the sum of two single-slit diffraction patterns.

When a diagonal polarizer (D) is placed before the detection screen, interference fringes reappear. A common view says this means the diagonal polarizer (D) acts as a ”quantum eraser,” erasing the path information provided by the V/H polarizers, thus restoring coherence. The diagonal polarizer (D) is called a quantum eraser because it seems to restore the interference pattern lost due to the V/H polarizers’ path information.

Placing an anti-diagonal polarizer (A) before the detection screen causes a 180° phase shift in the restored interference pattern (the bright and dark fringes are misplaced).

This phase shift phenomenon cannot be explained by erasing path information. Erasing implies deleting path information, and if the information were deleted, the original interference pattern should be restored without a phase shift. Therefore, whether placing A or D, the same interference fringes should be seen.

From this analysis, it is clear that the diagonal polarizer does not erase information; it only filters out the diagonal component of $|\Psi'\rangle$, allowing half the photons to pass through and showing the original interference pattern $\left|\Psi(p)\right|^2$ at half intensity.

In Figure 13, the red curve is the sum of the orange and green curves, i.e., $\left|\Psi'(p)\right|^2 = \left|\Psi''(p)\right|^2 + \left|\Psi'''(p)\right|^2$, so rather than calling ±45° polarizers erasers, they are actually filters for the photons.

The role of polarizers on photons can be divided into two steps: 1. Perform a projection measurement; 2. Block photons whose polarization angle is not aligned, collapsing those in the other direction and absorbing them.

Thus, polarizers A or D do not perform any ”erasing path information” but filter out photons at the appropriate angle and let them pass through. The correct understanding is that these appropriately angled photons can interfere with each other.

Therefore, this experiment is more appropriately called a ”quantum filtering experiment” than a ”quantum eraser experiment,” as the filtering restores coherence.

4 Explaining the Quantum Eraser Experiment with "Record" Electrons

Assume that for each "traveling" electron passing through the slits, we have a separate "record" electron. This electron pair is entangled such that if the traveling electron passes through the left slit, the record electron is in the spin-up state; if it passes through the right slit, the record electron is in the spin-down state (Carroll, 2019). We get:

$$|\Psi'\rangle = \frac{1}{\sqrt{2}} \left(|x_1\rangle\,|u\rangle + |x_2\rangle\,|d\rangle\right)$$

$$\langle\Psi'|\Psi'\rangle = \frac{1}{2} \left(\langle x_1|x_1\rangle\,\langle u|u\rangle + \langle x_1|x_2\rangle\,\langle u|d\rangle + \langle x_2|x_1\rangle\,\langle d|u\rangle + \langle x_2|x_2\rangle\,\langle d|d\rangle\right)$$

Since horizontal and vertical polarizers are orthogonal, $\langle u|d\rangle = \langle d|u\rangle = 0$, causing the two cross-interference terms to disappear. Projecting it onto momentum space yields:

$$\langle\Psi(p)'|\Psi(p)'\rangle = \frac{1}{2} \left(\langle x_1|p\rangle\,\langle p|x_1\rangle + \langle x_2|p\rangle\,\langle p|x_2\rangle\right)$$

$$\left|\Psi(p)'\right|^2 = \frac{1}{2} \left(\left| \int_{x_1-\frac{\delta}{2}}^{x_1+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\delta}} e^{-ipx} dx \right|^2 + \left| \int_{x_2-\frac{\delta}{2}}^{x_2+\frac{\delta}{2}} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\delta}} e^{-ipx} dx \right|^2 \right)$$

This is the same situation as in Figure 6, where the interference fringes disappear.

$$\texttt{Plot}\left[\left(\texttt{Abs}\left[\int_{1-0.2}^{1+0.2} \texttt{Exp}[-i * p * x]\, dx\right]\right)^2 + \left(\texttt{Abs}\left[\int_{-1-0.2}^{-1+0.2} \texttt{Exp}[-i * p * x]\, dx\right]\right)^2, \{p, -20, 20\}\right.$$

繪圖

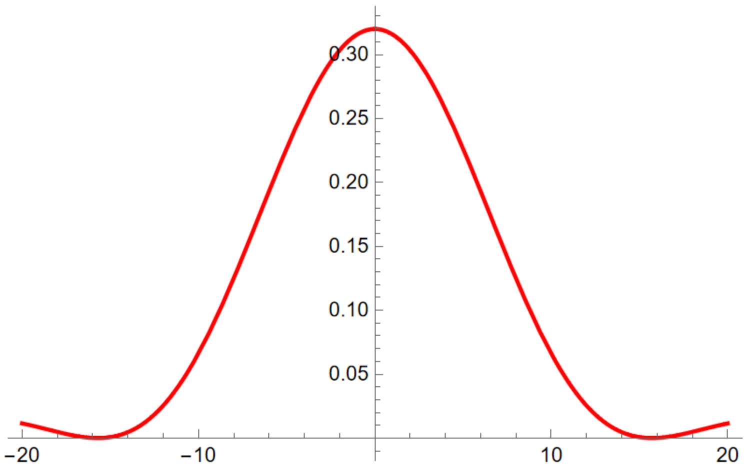

Figure 16: Double-Slit Pattern with Polarizers

Now let us measure the record spin along the horizontal axis. There is a relationship between the horizontal and vertical spin states; we can write it as:

$$|\uparrow\rangle = |\rightarrow\rangle + |\leftarrow\rangle$$

$$|\downarrow\rangle = |\rightarrow\rangle - |\leftarrow\rangle$$

(For simplicity, we omit the various factors of square root two.) So before we make such a measurement, the state is:

$$\begin{aligned}
|\Psi\rangle &= |x_1\rangle \left(|\rightarrow\rangle + |\leftarrow\rangle\right) + |x_2\rangle \left(|\rightarrow\rangle - |\leftarrow\rangle\right) \\
&= |x_1\rangle |\rightarrow\rangle + |x_1\rangle |\leftarrow\rangle + |x_2\rangle |\rightarrow\rangle - |x_2\rangle |\leftarrow\rangle \\
&= \left(|x_1\rangle + |x_2\rangle\right) |\rightarrow\rangle + \left(|x_1\rangle - |x_2\rangle\right) |\leftarrow\rangle
\end{aligned}$$

When we measure the record spin along the vertical direction, the resulting entanglement is between $|\uparrow\rangle$ and $|x_1\rangle$, and $|\downarrow\rangle$ and $|x_2\rangle$. Thus, by making this measurement, we can determine whether the electron passed through one slit or the other. However, when we measure the record spin along the horizontal axis, the situation is different. After each measurement, we again find ourselves in a branch of the wave function where the traveling electron has passed through both slits.

The interference pattern is:

$$|\Psi|^2 = \left|\left(|x_1\rangle + |x_2\rangle\right)\right|^2 \langle\rightarrow | \rightarrow\rangle \qquad +(|x_1\rangle + |x_2\rangle)(|x_1\rangle - |x_2\rangle)\langle\rightarrow | \leftarrow\rangle \qquad + (|x_1\rangle - |x_2\rangle)(|x_1\rangle + |x_2\rangle)\langle\leftarrow$$

If we find that measuring spin to the left gives the traveling electron a negative sign in its wave function contribution, this negative sign results in a central dark fringe in the interference pattern.

$$p1 = \mathrm{Plot}\Big[\Big(\mathrm{Abs}\Big[\int_{1-0.2}^{1+0.2} \mathrm{Exp}[-i * p * x]\, dx - \int_{-1-0.2}^{-1+0.2} \mathrm{Exp}[-i * p * x]\, dx\Big]\Big)^{2}$$

繪圖

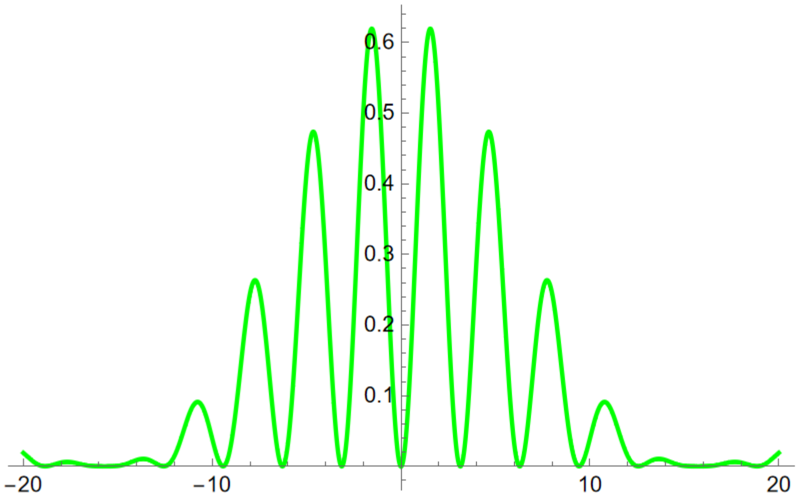

Figure 17: Central Dark Fringe of $\left|(|x_1\rangle - |x_2\rangle)\right|^2$

By choosing this type of measurement, we have erased the information about which slit the electron passed through. Hence, this is called a ”quantum eraser experiment.” This erasure does not affect the overall distribution of fringes on the detection screen. It remains non-interfering.

The interference patterns $\left|(|x_1\rangle + |x_2\rangle)\right|^2$ (blue) and $\left|(|x_1\rangle - |x_2\rangle)\right|^2$ (green) are complementary, and their sum is the red pattern, which is the non-interference case.

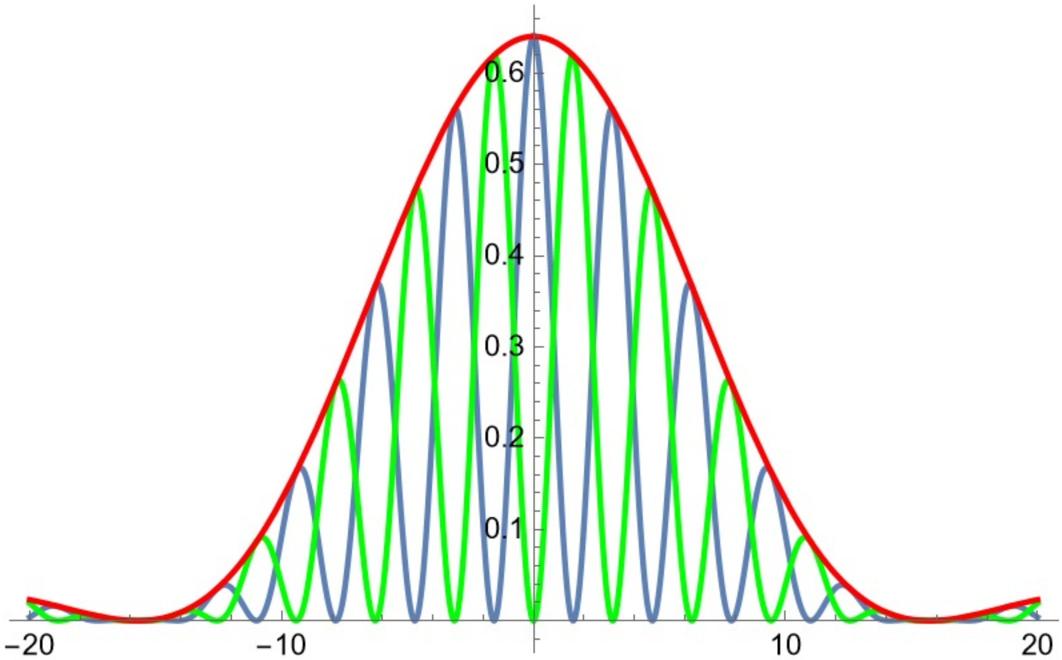

Figure 18: One's Peak Matches the Other's Valley

If we use a computer to separate the flashes on the detection screen into two groups —those associated with the record electron's left spin and those associated with the right spin. What do we see now?

Interestingly, the interference patterns reappear. The traveling electrons associated with the record electrons' left spin form an interference pattern, as do those associated with the right spin. (Remember, we do not see the pattern all at once; it appears gradually as we detect many individual flashes.) But the two interference patterns are slightly offset, so one peak matches the other's valley. Hidden within the seemingly featureless fuzzy distribution is actually an interference pattern.

The result would be as shown in the figure below, where the yellow and blue bright spots are traveling electrons associated with the record electrons' left and right spin, respectively. One peak matches the other's valley, and they are offset from each other. But if we do not filter the traveling electrons and view them together on the screen, there are no interference fringes.

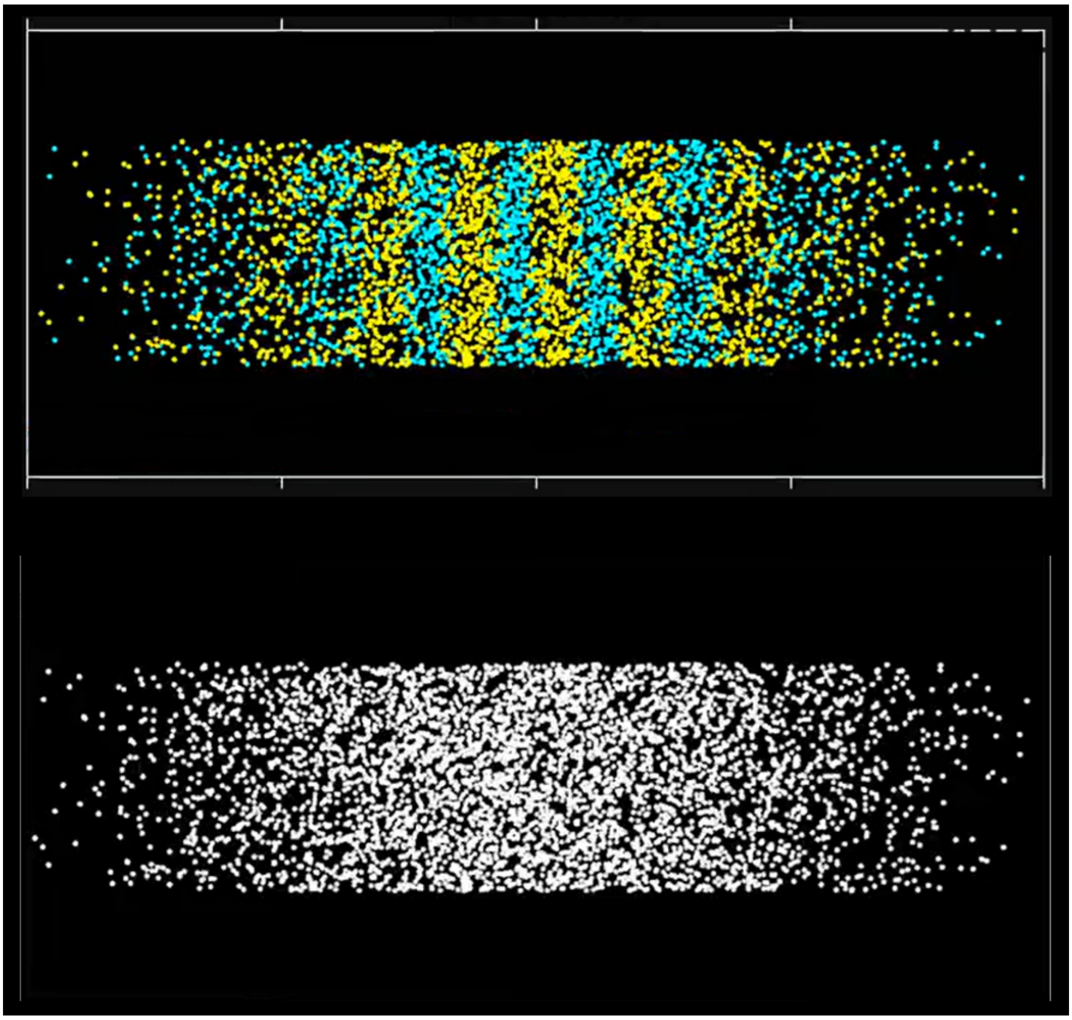

Figure 19: Filtering Two Groups

In retrospect, this is not so surprising. By observing how the quantum state $|\Psi\rangle$ is entangled with the record electrons' left and right spins, we can see that each measurement entangles with the traveling electron passing through both slits, so it certainly can interfere. And the negative sign just offsets one pattern, so combining the two patterns creates a smooth distribution.

The electron does not ”choose” to pass through one slit or the other. Its wave function (and the wave function of anything it is entangled with) always evolves according to the Schrödinger equation. The electron does not choose; its wave function explicitly passes through both slits. By measuring the record electrons along different directions, we can pick out different parts of the entangled wave function, some of which show interference. In fact, nothing is conveying information into the past.

The Copenhagen interpretation considers electrons as ”things with both wave-like and particle-like properties.” If we believe the Copenhagen interpretation, we might think the electron must exhibit wave-like or particle-like behavior when passing through the slits. This leads to the conclusion that the delayed choice experiment must convey information into the past to help the electron decide, ”choosing” which slit to pass through.

All these ideas should be resisted. The electron does not choose to exhibit wave-like or particle-like behavior. But some fundamental quantum mechanics researchers indeed view delayed choice quantum erasure and similar experiments (which have been successfully completed) as evidence of retrocausality in nature—the transmission of information into the past to influence it.

5 Conclusion: Quantum Erasure Demonstrates that ”Information is Physical”

What is quantum erasure? Simply put, quantum erasure is a means to restore coherence by changing the measurement basis.

In the quantum eraser experiment, there is always a process of filtering information. For example, in Figure 13, the orange light intensity is only half of the blue light intensity, indicating that half of the photons are actually filtered out. And these filtered photons are represented by the green light intensity.

The role of the ”quantum eraser” is to restore coherence by filtering information. In Figure 13, the orange and green interference fringes are the results of filtering.

John Preskill proposed a perspective: quantum erasure demonstrates that ”information is physical” (Preskill, 1998). The act of filtering information causes physical phenomena.

Mathematically, rigorously speaking, changing the measurement basis is a means of preparing different ensembles as described by the HJW theorem.

There is no need to introduce retrocausality to explain the delayed choice experiment. The photon's wave function, after passing through the double slit and before reaching the screen, does not collapse. Asking which slit the photon passed through during this process is meaningless. Therefore, this experiment is more appropriately called a ”quantum filtering experiment” than a ”quantum eraser experiment,” as the filtering restores coherence.

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