Triples "Φ, e, π" and "π, 4, 6" as Input Data for Modified Koide-Formulas and the Common Grounds of the Results

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1) Abstract:

In this report the common grounds of the results of modified Koide-Formulas are presented, in which the Triples "Φ, e, π" and "π, 4, 6" are set as basis values of various exponents.
The figures of the first Triple are the Quotient of the Golden Ratio Φ, the Euler Figure Figure e and the Circle Figure π. Besides the circle/sphere diameter the figures of the second Triple "π, 4, 6" determine the circle area and the sphere volume.
The exact exponent value, which results by the Equalization of the two modified Koide-Formulas, is close to the figure 0.444, which is also used at an approximation for the mass ratio of the elementar particles Tauon and Electron.
The results of the two Koide-Formulas are close to each other over a relatively wide exponent range.

2) Formulas of the modified Koide-Formulas:

As already presented in the author's report[1] the Koide-Formula[2] connects the masses of the three Leptons, namely Electron, Myon and Tauon and is readable as (verbatim taken from german wikipedia.de-entry "Yoshio Koide"[2]):

$$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 0.66666056 \qquad \text{(m0)}$$

The result is very close to the fraction 2/3. Values for the masses m_e[3], m_μ[4], m_τ[5] are given at the last page. In the Koide-Formula the Triple "m_e, m_μ, m_τ" is applied.

A general Formula of the Koide-Formula can be written as:

$$F_K = (Inp_1 \pm Inp_2 \pm Inp_3 \pm Inp_4 \pm \ ...) / (Inp_1^{Exp} \pm Inp_2^{Exp} \pm Inp_3^{Exp} \pm Inp_4^{Exp} \pm ...)^{1/Exp} \qquad \text{(FK0)}$$

$$F_K = T_N / T_{Da}^{1/Exp} \quad \text{with } T_N = Inp_1 \pm Inp_2 \pm Inp_3 \pm Inp_4 \pm \ ...$$

$$\text{and } T_{Da} = Inp_1^{Exp} \pm Inp_2^{Exp} \pm Inp_3^{Exp} \pm Inp_4^{Exp} \pm ...$$

$$F_K = T_N / T_D \quad \text{with } T_D = (Inp_1^{Exp} \pm Inp_2^{Exp} \pm Inp_3^{Exp} \pm Inp_4^{Exp} \pm ...)^{1/Exp}$$

[At these formulas the downsized letter N stands for Numerator and the downsized letter D stands for Denominator. Term T_N expresses the Numerator and Term T_D the Denominator of a formula F_K.]

The Formula, in which with three positive Input data and only plus-operators are applied, is given by:

$$F_K(x) = (Inp_1 + Inp_2 + Inp_3) / (Inp_1^x + Inp_2^x + Inp_3^x)^{1/x} \qquad \text{(FK)}$$

For the first Triple "Φ, e, π" the above Formula (FK) is written to:

$$F_{K1}(x) = (\Phi + e + \pi) / (\Phi^x + e^x + \pi^x)^{1/x} = 7.477908471 / (\Phi^x + e^x + \pi^x)^{1/x} \qquad \text{(FK1)}$$

For the second Triple "π, 4, 6" the above Formula (FK) is written to:

$$F_{K2}(x) = (\pi + 4 + 6) / (\pi^x + 4^x + 6^x)^{1/x} = 13.14159265 / (\pi^x + 4^x + 6^x)^{1/x} \qquad \text{(FK2)}$$

The x-value, which fulfills the Equation $F_{K1}(x) = F_{K2}(x)$, is determined to $x_W = 0.570365150$.

The equalization of the two Equations $F_{K1}(x)$ and $F_{K2}(x)$ leads to a result close to the figure 0.444:

$$F_{K1}(x_w) = (\Phi + e + \pi) / (\Phi^{x_w} + e^{x_w} + \pi^{x_w})^{1/x_w} = 0.444025902$$
$$F_{K2}(x_w) = (\pi + 4 + 6) / (\pi^{x_w} + 4^{x_w} + 6^{x_w})^{1/x_w} = 0.444025902$$

with $x_w = 0.570365150$

Remarkable relations with exponent x_w:

$$Rel_1 = 0.444 * \Phi / (x_w * \Phi - 0.444) = 1.500213 \quad [\approx 3/2]$$
$$Rel_2 = (0.444 * e - x_w) * \pi = 1.999787$$
$$Rel_1 + Rel_2 = 3.4999996$$
$$Rel_3 = [\pi + 1/(0.444 * x_w)] / \sqrt{\pi} = 4.000320$$

	TN1 = Φ + e + π = 7.477908471			TN2 = π + 4 + 6 = 13.14159265			ΔFK =	
Exponent x	TD1a	TN1 / TD1a	FK1 = TN1 / TD1	TD2a	TN2 / TD2a	FK2 = TN2 / TD2	= FK2 – FK1	ΔFK / FK2 (%)
0,5703477	5,00579	1,49385	0,444000	6,90451	1,90334	0,444000	-1,02E-08	-2,31E-06
0,5703652	5,00587	1,49383	0,444026	6,90468	1,90329	0,444026	4,71E-12	1,06E-09
0,10	3,27575	2,28281	0,000053	3,46621	3,79134	0,000052	-6,95E-08	-1,32E-01
0,20	3,57970	2,08897	0,012722	4,00775	3,27904	0,012710	-1,17E-05	-9,24E-02
0,30	3,91492	1,91010	0,079074	4,63725	2,83392	0,079028	-4,65E-05	-5,88E-02
0,40	4,28482	1,74521	0,196765	5,36951	2,44745	0,196703	-6,23E-05	-3,17E-02
0,50	4,69319	1,59335	0,339503	6,22194	2,11214	0,339466	-3,68E-05	-1,09E-02
0,60	5,14427	1,45364	0,487799	7,21497	1,82143	0,487817	1,78E-05	3,64E-03
0,70	5,64274	1,32523	0,631281	8,37262	1,56959	0,631356	7,50E-05	1,19E-02
0,80	6,19384	1,20731	0,765297	9,72313	1,35158	0,765403	1,07E-04	1,39E-02
0,90	6,80340	1,09914	0,888237	11,29974	1,16300	0,888325	8,84E-05	9,95E-03
1,00	7,47791	1,00000	1,000000	13,14159	1,00000	1,000000	0,00E+00	0,00E+00
1,10	8,22458	0,90921	1,101181	15,29479	0,85922	1,101007	-1,74E-04	-1,58E-02
1,20	9,05146	0,82616	1,192655	17,81369	0,73772	1,192211	-4,44E-04	-3,72E-02
1,30	9,96751	0,75023	1,275371	20,76237	0,63295	1,274553	-8,18E-04	-6,42E-02
1,40	10,98271	0,68088	1,350256	24,21648	0,54267	1,348955	-1,30E-03	-9,65E-02
1,50	12,10819	0,61759	1,418167	28,26527	0,46494	1,416270	-1,90E-03	-1,34E-01
1,60	13,35633	0,55988	1,479877	33,01419	0,39806	1,477271	-2,61E-03	-1,76E-01
1,70	14,74096	0,50729	1,536074	38,58784	0,34056	1,532646	-3,43E-03	-2,24E-01
1,80	16,27746	0,45940	1,587364	45,13351	0,29117	1,583003	-4,36E-03	-2,75E-01
1,90	17,98300	0,41583	1,634280	52,82539	0,24877	1,628878	-5,40E-03	-3,32E-01
2,00	19,87669	0,37621	1,677290	61,86960	0,21241	1,670742	-6,55E-03	-3,92E-01
2,10	21,97988	0,34022	1,716803	72,51011	0,18124	1,709007	-7,80E-03	-4,56E-01
2,20	24,31633	0,30753	1,753180	85,03581	0,15454	1,744039	-9,14E-03	-5,24E-01
2,30	26,91257	0,27786	1,786736	99,78895	0,13169	1,776159	-1,06E-02	-5,96E-01
2,40	29,79817	0,25095	1,817751	117,17509	0,11215	1,805649	-1,21E-02	-6,70E-01
2,50	33,00610	0,22656	1,846471	137,67505	0,09545	1,832761	-1,37E-02	-7,48E-01
2,60	36,57316	0,20446	1,873112	161,85903	0,08119	1,857719	-1,54E-02	-8,29E-01
2,70	40,54038	0,18446	1,897869	190,40346	0,06902	1,880719	-1,71E-02	-9,12E-01
2,80	44,95354	0,16635	1,920911	224,11110	0,05864	1,901940	-1,90E-02	-9,97E-01
2,90	49,86371	0,14997	1,942393	263,93485	0,04979	1,921539	-2,09E-02	-1,09E+00
3,00	55,32788	0,13516	1,962451	311,00628	0,04226	1,939657	-2,28E-02	-1,18E+00
3,10	61,40964	0,12177	1,981206	366,66942	0,03584	1,956422	-2,48E-02	-1,27E+00
3,20	68,17994	0,10968	1,998768	432,52122	0,03038	1,971949	-2,68E-02	-1,36E+00
3,30	75,71797	0,09876	2,015236	510,45954	0,02574	1,986340	-2,89E-02	-1,45E+00
3,40	84,11211	0,08890	2,030698	602,74054	0,02180	1,999690	-3,10E-02	-1,55E+00
3,50	93,46101	0,08001	2,045233	712,04698	0,01846	2,012081	-3,32E-02	-1,65E+00
3,60	103,87476	0,07199	2,058915	841,56963	0,01562	2,023592	-3,53E-02	-1,75E+00
3,70	115,47629	0,06476	2,071809	995,10439	0,01321	2,034292	-3,75E-02	-1,84E+00
3,80	128,40280	0,05824	2,083973	1177,16808	0,01116	2,044243	-3,97E-02	-1,94E+00
3,90	142,80747	0,05236	2,095463	1393,13649	0,00943	2,053505	-4,20E-02	-2,04E+00
4,00	158,86134	0,04707	2,106326	1649,40909	0,00797	2,062129	-4,42E-02	-2,14E+00

Table 1: Comparison of the Results with the Triples "Φ, e, π" and "π, 4, 6"

As one can see at Table 1, the results of the Fomulas $F_{K1}(x)$ and $F_{K2}(x)$ are very close to each other over a wide exponent range x. Up to an exponent x=1.4 the relative absolute deviations $|\Delta F_K|$ of the two formulas are less than 0.1%. From x=1.5 to x=2.8 the relative absolute deviations $|\Delta F_K|$ are less than 1%.

If one compares the ratios of the single figures of each Triple setting the smallest figure (Φ at Triple 1 and π at Triple 2) in the denominator, one gets the following ratios:

Triple 1: $R1_{T1} = \pi / \Phi = 1.94161$ $R2_{T1} = e / \Phi = 1.679991$ [≈ 1.68]

Triple 2: $R1_{T2} = 6 / \pi = 1.90985$ $R2_{T2} = 4 / \pi = 1.273240$

One would not expect, that the deviations ΔF_K (= F_{K2} - F_{K1}) are comparatively small over the just presented exponent range, because the ratio "$R2_{T1}$ / $R2_{T2}$ (≈1.32)" is relative big - that means more than 30% difference related to the ratio $R2_{T2}$ - in comparison to the relative absolute deviations $|\Delta F_K / F_{K2}|$ less than 1% for an exponent x<2.8. The ratio $R1_{T1}/R1_{T2}$ (=1.017) of the bigger ratio values $R1_{T1}$ and $R1_{T2}$ is close to the figure 1, by that these ratios might have the crucial influence on the result ΔF_K (= F_{K2} - F_{K1}).

If one takes the masses of the Elementar Particles Electron, Myon and Tauon in comparison to the masses of the Elementar Particles Electron, Proton[6] and Neutron[7] and uses them as Input Triples in the Equation (FK), one gets the following results depending on various exponents x (see Table 2).

Masses marked with the character * are related to the mass of the Electron!

	m_e^*	m_μ^*	m_τ^*	m_e^*	m_p^*	m_n^*		
	1,00	206,768283	3477,230	1,00	1836,1526734	1838,68366173		
	$T_{N3} = m_e^* + m_\mu^* + m_\tau^* = 3684.9983$			$T_{N4} = m_e^* + m_p^* + m_n^* = 3675.8364$			$\Delta F_K =$	
Exponent x	T_{D3a}	T_{N3} / T_{D3a}	$F_{K3} = T_{N3} / T_{D3}$	T_{D4a}	T_{N4} / T_{D4a}	$F_{K4} = T_{N4} / T_{D4}$	$= F_{K4} - F_{K3}$	$\Delta F_K / F_{K4}$ (%)
0,4050302	3,68500E+01	9,99999E+01	0,500010	4,29887E+01	8,55070E+01	0,340943	-1,59E-01	-4,67E+01
0,5625000	1,19228E+02	3,09071E+01	0,750063	1,38135E+02	2,66105E+01	0,575936	-1,74E-01	-3,02E+01
0,10	4,96439E+00	7,42287E+02	0,000405	5,24083E+00	7,01384E+02	0,000235	-1,70E-04	-7,24E+01
0,20	9,01262E+00	4,08871E+02	0,061970	9,99232E+00	3,67866E+02	0,036900	-2,51E-02	-6,79E+01
0,30	1,74948E+01	2,10634E+02	0,265098	2,00674E+01	1,83174E+02	0,167385	-9,77E-02	-5,84E+01
0,40	3,55283E+01	1,03720E+02	0,489781	4,14309E+01	8,87221E+01	0,332695	-1,57E-01	-4,72E+01
0,50	7,43475E+01	4,95645E+01	0,666661	8,67302E+01	4,23824E+01	0,488669	-1,78E-01	-3,64E+01
0,60	1,58779E+02	2,32083E+01	0,791493	1,82784E+02	2,01103E+01	0,624397	-1,67E-01	-2,68E+01
0,70	3,43973E+02	1,07130E+01	0,876658	3,86457E+02	9,51164E+00	0,740454	-1,36E-01	-1,84E+01
0,80	7,52933E+02	4,89419E+00	0,934311	8,18328E+02	4,49189E+00	0,839841	-9,45E-02	-1,12E+01
0,90	1,66087E+03	2,21872E+00	0,973392	1,73407E+03	2,11977E+00	0,925533	-4,79E-02	-5,17E+00
1,00	3,68500E+03	1,00000E+00	1,000000	3,67584E+03	1,00000E+00	1,000000	0,00E+00	0,00E+00
1,10	8,21221E+03	4,48722E-01	1,018210	7,79318E+03	4,71674E-01	1,065207	4,70E-02	4,41E+00
1,20	1,83631E+04	2,00674E-01	1,030739	1,65236E+04	2,22459E-01	1,122711	9,20E-02	8,19E+00
1,30	4,11670E+04	8,95134E-02	1,039403	3,50359E+04	1,04916E-01	1,173754	1,34E-01	1,14E+01
1,40	9,24705E+04	3,98505E-02	1,045423	7,42894E+04	4,94799E-02	1,219334	1,74E-01	1,43E+01
1,50	2,08020E+05	1,77147E-02	1,049625	1,57523E+05	2,33352E-02	1,260258	2,11E-01	1,67E+01
1,60	4,68487E+05	7,86574E-03	1,052571	3,34014E+05	1,10050E-02	1,297190	2,45E-01	1,89E+01
1,70	1,05600E+06	3,48958E-03	1,054644	7,08246E+05	5,19005E-03	1,330673	2,76E-01	2,07E+01
1,80	2,38184E+06	1,54712E-03	1,056108	1,50177E+06	2,44766E-03	1,361160	3,05E-01	2,24E+01
1,90	5,37496E+06	6,85586E-04	1,057145	3,18438E+06	1,15433E-03	1,389028	3,32E-01	2,39E+01
2,00	1,21339E+07	3,03695E-04	1,057882	6,75222E+06	5,44390E-04	1,414598	3,57E-01	2,52E+01
2,10	2,73998E+07	1,34490E-04	1,058408	1,43175E+07	2,56737E-04	1,438138	3,80E-01	2,64E+01
2,20	6,18851E+07	5,95458E-05	1,058784	3,03590E+07	1,21079E-04	1,459877	4,01E-01	2,75E+01
2,30	1,39796E+08	2,63598E-05	1,059053	6,43737E+07	5,71015E-05	1,480013	4,21E-01	2,84E+01
2,40	3,15832E+08	1,16676E-05	1,059247	1,36499E+08	2,69294E-05	1,498714	4,39E-01	2,93E+01
2,50	7,13605E+08	5,16392E-06	1,059386	2,89435E+08	1,27001E-05	1,516128	4,57E-01	3,01E+01

Table 2: Comparison of the Results won by the Triples "m_e*, m_μ*, m_τ*" and "m_e*, m_p*, m_n*"

The well-known result close to the value 2/3 (result of the Koide-Formula) is won by the exponent x=0.5 for the Triple "m_e*, m_μ*, m_τ*", which is listed at the left side of Table 2. By use of this exponent (x=0.5) the relative absolute deviation $|\Delta F_K|$ of the result to the one of the Triple "m_e*, m_p*, m_n*" is 36.4%.

The Equations applying the mass Triples are:

$$F_{K3}(x) = (m_{e*} + m_{\mu*} + m_{\tau*}) / (m_{e*}^x + m_{\mu*}^x + m_{\tau*}^x)^{1/x} = 3684.9983 / (m_{e*}^x + m_{\mu*}^x + m_{\tau*}^x)^{1/x} \qquad \text{(FK3)}$$

$$F_{K4}(x) = (m_{e*} + m_{p*} + m_{n*}) / (m_{e*}^x + m_{p*}^x + m_{n*}^x)^{1/x} = 3675.8364 / (m_{e*}^x + m_{p*}^x + m_{n*}^x)^{1/x} \qquad \text{(FK4)}$$

The exponent x=0.4050302 (see the first line of Table 2) corresponds to a value, which is dependent on the Golden Ratio Φ by the expression:

$$0.40503017 = (3/4)^{(1.2*\Phi*\Phi)}$$

The exponent x=0.4050302 fulfills quite exactly the result value 0.5 for the Equation F_{K3}.

Remarkable: the ratio T_{N3}/T_{Da3} results to the value 99.99994, which is won by the relation:

$$x = x_3 = 0.4050302: \; T_{N3}/T_{Da3} = (m_{e*} + m_{\mu*} + m_{\tau*}) / (m_{e*}^x + m_{\mu*}^x + m_{\tau*}^x) = 99.99994$$

Further: $x = x_3 = 0.4050302$: $FK_3(x_3) = T_{N3} / T_{D3} = 3684.9983 / 7369.8550 = 0.5000096$

$T_{N3} = m_{e*} + m_{\mu*} + m_{\tau*} = 3684.9983$ [≈ 3685 = 5 * 11 * 67; only primes]

$x = x_3 = 0.4050302$: $T_{D3}/T_{Da3} = (m_{e*}^x + m_{\mu*}^x + m_{\tau*}^x)^{1/x} / (m_{e*}^x + m_{\mu*}^x + m_{\tau*}^x) = 199.996$ [≈ 200]

$x_3 * T_{D3} = 0.4050302 * 7369.8550 = 2985.014$ [≈ 2985 = 3 * 5 * 199; only primes]

$m_{p*}^{x_4} = 1836.15267343^{0.4050302} = 20.9885$ [≈ 21 = 3 * 7; again only primes]

$m_{n*}^{x_4} = 1838.68366173^{0.4050302} = 21.0002$ [≈ 21 = 3 * 7]

$x = x_4 = 0.4050302$: $T_{Da4} = (m_{e*}^x + m_{p*}^x + m_{n*}^x) = 42.9887$ [≈ 43; a prime]

$1.286 * T_{N2} = 1.286 * 13.14159 = 16.9001$ [≈ 0.1 * 13 * 13; prime 13]

Results with the exponent value $x = x_1 = x_2$ (see the first line of Table 1):

wanted x: $F_{K1}(x_1) = T_{N1} / T_{D1} = F_{K2}(x_2) = T_{N2} / T_{D2} = 0.4440000$ ---> $x = x_1 = x_2 = 0.570347681$

$x = x_2 = 0.570347681$: $T_{D2} / F_{K2}(x) = 29.59818 / 0.444 = 66.6626$ [≈ 100 * 2 / 3]

$x = x_2 = 0.570347681$: $x / 1.286 = 0.443505$ [$\approx 0.666^2 = 0.443556$]

$T_{N2}/(1.286*T_{Da2}) = 13.14159/(1.286*6.904506) = 1.48004$ [≈ 148/100 = 4 * 37 / 100; prime 37]

$x = x_1 = 0.570347681$: $(\pi / \Phi)^{0.570347681} = 1.4599993$ [≈ 146 / 100 = 2 * 73 / 100; prime 73]

The figure 1.286 is often used at the upper relations! Isn't it worth for an investigation by experts, which qualities this figure possesses (not only) in connection with the figures Φ, π, 1.44, 6.66?
Please look further below at the approximations for the mass ratio Myon to Electron, at which the figure 1.286 connects the two exponents of basis 0.999, and at the relations at the last page!

Referring to this context some approximations from the report [8] are listed in the following:

Approximation for the Mass Ratio Tauon[5] to Electron[3.1] (=3477.23 ±0.23):

$$m_\tau / m_e \approx MR_{\tau/e} = 0.999^{-2*0.99} * (2*\pi)^{0.999*4.44} = 3477.2429 \qquad (MR\tau/e)$$

Exponent term 0.999*4.44 is equal the term 6.66*0.666.

Approximation for the Mass Ratio Myon[4] to Electron[3.1] is:

$$m_\mu / m_e \approx MR_{\mu/e} = 0.999^{-2/1.14} * (2*\pi)^{2.9} = 0.999^{-2*0.99/(1+0.1*1.286)} * (2*\pi) * (2*\pi)^{1.9} =$$
$$= 206.7682821 \qquad (MR_\mu/e)$$

Remarkable referring the exponents of basis 0.999 at the last two Equations: there are connections, which lead to the figure 0.99 as well to figure 1.9 at the exponents of Equation (MRτ/e) and (MR_μ/e) by use of the figure 1.286:

$0.99 = (1 + 0.1*1.286) / 1.14$ and $1000 * (1 + 0.1*1.286) / (6*99) = 1.9$; $19 * 6 = 114$

$0.666^{1./1.286} = 0.72900865$ [$\approx 0.729 = 0.9*0.9*0.9$];

$9*9*9 / (10*0.666^{1./1.286}) = 137.035411$ [$\approx \alpha^{-1}$]; α^{-1}: Inverse of the Fine Structure Constant[3.3]

3) "Particle Masses related to the Electron Mass" with additional exponents as Input Data

The ratio "(mass of the Myon related to the Electron) with the exponent 2/3: $(m_\mu / m_e)^{2/3}$" leads to the result value 34.966805, which is close to the term "35*0.999 = 34.965".

The ratio "(mass of the Tauon related to the Electron) with the exponent 1/3: $(m_\tau / m_e)^{1/3}$" leads to the result value 15.149948, which is close the figure 15.15 (= 15 * 1.01).
An Approximation of the related Tauon mass ($m_\tau / m_e = 3477.23 \pm 0.23$), which is far within the mass tolerance of the Tauon, is given as follows:

$$m_{Myon\#*} = 15^3 * 0.99^{-3*0.99} = 3477.2607$$

Remarkable: two times the figures 3 and 0.99

The related Input Data (particle masses related to m_e and marked with *) with chosen exponents are:

$$m_{e5*} = m_{e6*} = 1$$
$$m_{\mu 5*} = (m_\mu / m_e)^{2/3} = 34.966805$$
$$m_{\tau 5*} = (m_\tau / m_e)^{1/3} = 15.149948$$
$$m_{p6*} = (m_p / m_e)^{1/2} = 42.850352$$
$$m_{n6*} = (m_n / m_e)^{1/2} = 42.879875$$

The Equations with these related mass Triples are:

$$F_{K5}(x) = (m_{e5*} + m_{\mu 5*} + m_{\tau 5*}) / (m_{e5*}^x + m_{\mu 5*}^x + m_{\tau 5*}^x)^{1/x} = 51.116753 / (m_{e5*}^x + m_{\mu 5*}^x + m_{\tau 5*}^x)^{1/x} \quad \text{(FK5)}$$

$$F_{K6}(x) = (m_{e6*} + m_{p6*} + m_{n6*}) / (m_{e6*}^x + m_{p6*}^x + m_{n5*}^x)^{1/x} = 86.730227 / (m_{e6*}^x + m_{p6*}^x + m_{n6*}^x)^{1/x} \quad \text{(FK6)}$$

The x-value, which fulfills the Equation $F_{K5}(x) = F_{K6}(x)$, is determined to $x_1 = 0.46589815$.

Masses marked with the character * are related to the mass of the Electron!

	me*	mμ*^(2/3)	mτ*^(1/3)	me*	√mp*	√mn*		
	1,00	34,966805	15,149948	1,00	42,8503521	42,87987479		
	TN5 = me*+mμ*^(1/3)+mτ*^(2/3) = 51.116753			TN6 = √me* + √mp* + √mn* = 86.730227				
Exponent x	TD5a	TD5	FK5 = TN5 / TD5	TD6a	TD6	FK6 = TN6 / TD6	TD5 / TD5a	TD6 / TD6a
0,46589815	9,78599E+00	1,33729E+02	0,382241	1,25193E+01	2,26899E+02	0,382241	1,36654E+01	1,81240E+01
0,65650250	1,72695E+01	7,66751E+01	0,666667	2,45782E+01	1,31256E+02	0,660773	4,43991E+00	5,34033E+00
0,66112220	1,75155E+01	7,59936E+01	0,672646	2,49911E+01	1,30095E+02	0,666667	4,33864E+00	5,20567E+00

Table 3: Comparison of the Results by application of different exponents of the related masses

Remarkable relations with the x-values (exponents) of Table 3:

$x_1 = 0.46589815$; $x_2 = 0.6565025$; $x_3 = 0.6611222$

$$x_1^{-1} * x_3^{-1} / \sqrt{\pi * \Phi} = 1.439987 \quad [\approx 1.44]$$
$$x_1 * x_2^{-1} * \sqrt{\pi * \Phi} = 1.60001 \quad [\approx 1.6]$$
$$x_2 * \sqrt{\pi * \Phi} / 0.666 = 2.2224444$$
$$x_1 * \pi * \sqrt{\pi * \Phi} = 3.299967 \quad [\approx 3.3]$$
$$x_2^2 * (1.2 * \Phi^2)^{1.5} = 2.39998 \quad [\approx 2.4; \text{ close to the term } 2 * \pi / \Phi^2 (= 2.39996)]$$

$0.666^{-1} - x_2 = 0.8449990$ [≈ 0.845; 845 = 5 * 13²]

$x_3^2 * \sqrt{\pi} / e = 0.2849994$ [≈ 0.285; 285 = 3 * 5 * 19]

$0.666^{-1} - x_2 - x_3^2 * \sqrt{\pi} / e = 0.5599996$ [≈ 0.56; 56 = 7 * 8]

$5.6^3 - 3 * 12.86 = 137.036$ [≈ α^{-1}; figure 3 used two times, as exponent and as multiplicator]

$137.035999087 = 137.036 * (1 - 6.66 * 10^{-3*3})$ [value is far within the tolerance for α^{-1}]

$6.66 + 5.6 + 2*0.3 = 12.86$; $(e / \Phi) / 3 = 1.679991 / 3 = 0.5599969$ [≈ 0.56]

$1.286^{1.44/0.99} * 0.999^{(1286-56)/1000} = 1.4399967$ [≈ 1.44]; $1.4399967 - 0.5599969 = 0.880000$

Use of figures 0.88 and 14.146 (=11*1.286) for the Avogadro-Constant[3.4] N_A (given at last page):

$N_{A\#} = 14.146^{(1286-666)/30} * [1 - 1.4146/(144*666)]^{2*0.88} = 6.0221\,4076\,8 * 10^{23}$

Another example with the figures 7 and 8: very exact Approximation (MR2) for the mass ratio Neutron/Electron by use of the Fine Tuning Term $(1 - 1/144/666)^{8/7}$ is (see literature [8], page 8):

$$MR_{Pi} = (2*\pi)^4 + (2*\pi)^3 + (2*\pi)^2 - (2*\pi)^1 - (2*\pi)^0 - (2*\pi)^{-1} + (2*\pi)^{-2} + (2*\pi)^{-3} + (2*\pi)^{-4} = 1838.66175070 \qquad (MR1)$$

$$MR_{n/e} = MR_{Pi} \,/\, [1 - 1/(144*666)]^{8/7} = 1838.68366169 \qquad (MR2)$$

Please look at the first three terms of Equation (MR1), which possess only positive exponents and only plus-operators, and here at the figure 2 and its exponents. Connection of the figures:

$2 * (2^4 + 2^3 + 2^2) = 7 * 8 = 56$;   use of term 0.5*(7+8) is given in literature [8], page 8

The mass ratio Neutron/Electron is given by the value 1838.68366173[7] with its tolerance $\pm 89*10^{-8}$.
The absolute ratio "tolerance to set value" is: $|\pm 89*10^{-8} / 1838.68366173| = 4.84*10^{-10}$.
By this tiny ratio the circumstance is emphasized, how exact the ratio m_n/m_e is given and how difficult it might be performing an exact approximation for the ratio m_n/m_e (and also for other ratios and for other values of Physical Constants each with small related tolerance).
Furthermore the result of Equation (MR2) only uses about 5% of the tolerance $|89*10^{-8}|$.
With an exponent "$8/7 * (1 - 4*10^{-5})$" the result of Equation (MR2) lies outside the tolerance range. By that the exponent value "8/7" is clearly visible the appropriate one for Equation (MR2). In [8] it is often shown, that the Fine Tuning Term (1 ± 1/144/666) with appropriate, comprehensible exponents possesses the attributes creating exact approximations for Physical Constants.

Figure 1.5 (= 3/2):

$(m_{e*} * m_{p*} * m_{n*})^{1/3} = 150.0164$ [≈ 100 * 1.5]

$x_3 * (m_{e*}^{x1} + m_{p6*}^{x1} + m_{n6*}^{x1})^{1/x1} = 150.0082$ [≈ 100 * 1.5]; $m_{p6*} = \sqrt{(m_p/m_e)}$; $m_{n6*} = \sqrt{(m_n/m_e)}$

4) Conclusion

By comparison of the two Triples applied at modified Koide-Formulas unexpected informations are won, which lead to the Figure 0.444 or to the Figure 0.666 by the ratio 3/2 (value is close to the Inverse of the Koide Formula result), respectively.
Although the ratios of the figures, which are performed for the single Triples, are quite different, the results of the modified Koide Formulas are pretty close for both Triples over a wide exponent range.

It can be a task for expert mathematicians to examine the results and informations, which are won by the comparison of the Triples "Φ, e, π" and "π, 4, 6" set as Input parameters in the modified Koide Formulas presented in this report.

Question: why leads the x-value, which fulfills the equalization of the two Formulas (FK1) and (FK2), to a result close to the figure 0.444? Is it random or another piece of an unknown system?
Figure 666 - or Figure 444 divided by the Koide Ratio 2/3 - in combination with 10-powers is an often used input value for very exact approximations of many Physical Constants, which are presented in the author's report [8].

Literature and wikipedia.de- or other Internet-Entries:

The data of the physical Constants and the data of the celestial bodies of our sun system are taken in the mayority from the entries of Wikipedia Germany. The physical constants given in the corresponding entries refer mostly to CODATA2018.

[1] Mathematical Formulas, which follow to the Koide-Formula, using the Input Values “Data of the Celestial Bodies of our Sun System and Physical Constants“, and the Connection of the Results; Andreas Ball
Published at viXra: viXra submission 2405.0080v1; 2024-05-15

[2] Wikipedia.de-Entry “Yoshio Koide“; Status March 2024
1. Piotr Żenczykowski: Elementary Particles and Emergent Phase Space. WORLD SCIENTIFIC, 2013, ISBN 978-981-4525-68-8, S. 66-68, doi:10.1142/8918 (https://doi.org/10.1142/8918).

[3] Wikipedia.de-Entry “Physikalische Konstante“; Status May 2024

[3.1] Electron Mass m_e:
41. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?me) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Elektronenmasse in Kilogramm).

[3.2] Fine Structure Constant α:
25. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?alph) NIST, abgerufen am 20. April 2020 (englisch, Wert für die Feinstrukturkonstante)

[3.3] Inverse of Fine Structure Constant α^{-1}:
26. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?alphinv) NIST, abgerufen am 20. April 2020 (englisch, Kehrwert der Feinstrukturkonstante)

[3.4] Avogadro-Constant N_A:
61. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?na) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Avogadro-Konstante).

[4] Wikipedia.de-Entry “Myon“; Status March 2024
1. Die Angaben über die Teilcheneigenschaften der Infobox sind, wenn nicht anders angegeben, entnommen aus der Veröffentlichung der CODATA Task Group on Fundamental Constants: CODATA Recommended Values. (http://physics.nist.gov/cgi-bin/cuu/Results?search_for=muon) National Institute of Standards and Technology, abgerufen am 4. Juli 2019 (englisch).

[5] Wikipedia.de-Entry “Tauon“; Status March 2024
1. Die Angaben über die Teilcheneigenschaften (Infobox) sind, wenn nicht anders angegeben, entnommen aus der Veröffentlichung der CODATA Task Group on Fundamental Constants: CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Results?search_for=tau) National Institute of Standards and Technology, abgerufen am 4. Juli 2019 (englisch).

[6] Wikipedia.de-Entry “Proton“; Status March 2024
1. Die Angaben über die Teilcheneigenschaften (Infobox) sind, wenn nicht anders angegeben, entnommen aus: CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Category?view=html&Atomic+and+nuclear.x=114&Atomic+and+nuclear.y=16) National Institute of Standards and Technology, abgerufen am 21. Juli 2019.

[7] Wikipedia.de-Entry “Neutron“; Status March 2024
1. Die Angaben über die Teilcheneigenschaften (Infobox) sind, wenn nicht anders angegeben, entnommen aus der Veröffentlichung der CODATA Task Group on Fundamental Constants: CODATA Recommended Values. (http://physics.nist.gov/cgi-bin/cuu/Results?search_for=neutron) National Institute of Standards and Technology, abgerufen am 7. Juli 2019 (englisch).

[8] Exact Approximations of Physical Constants using the Figures Φ, π, 144 and 666; Andreas Ball
Published at viXra: viXra submission 2406.0018v1; 2024-06-13

Used Data of Physical Constants:

Avogadro-Constant N_A[3.4]	$6.0221\,4076 * 10^{23}$ mol^{-1}
Fine Structure Constant α[3.2]:	$7.297\,352\,5693(11) * 10^{-3}$
Inverse of Fine Structure Constant $1/\alpha$[3.3]:	137.035 999 084(21)
Mass of Electron m_e[3.1]:	$9.109\,383\,7015(28) * 10^{-31}$ kg
Mass of Neutron m_n[7]:	$1.674\,927\,498\,04(95) * 10^{-27}$ kg
Mass Ratio Neutron/Electron $MR_{n/e}$[7]:	1838.683 661 73(89)
Mass Ratio Neutron/Proton $MR_{n/p}$:	1.001 378 41931
Mass of Protons m_p[6]:	$1.672\,621\,923\,69(51) * 10^{-27}$ kg
Mass Ratio Proton/Electron $MR_{p/e}$[6]:	1836.152 673 43(11)
Mass of Myon m_μ[4]:	$1.883\,531\,627(42) * 10^{-28}$ kg
Mass Ratio Myon/Electron $MR_{\mu/e}$[4]:	206.768 2830(46)
Mass of Tauon m_τ[5]:	$3.167\,54(21) * 10^{-27}$ kg
Mass Ratio of Tauon/Electron $MR_{\tau/e}$[5]:	3477.23(23)

The figures in the brackets behind the data descibe the uncertainty referring the last places of the given value.[3]

Selected Modifications[1] of the Koide Formula[2] and their Connections:

$(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 0.66666056$ [≈ 2/3] (Koide Formula)

$Exp = (3/4)^2 = (0.75)^2 = 0.5625$
$(m_e + m_\mu + m_\tau) / (m_e^{Exp} + m_\mu^{Exp} + m_\tau^{Exp})^{1/Exp} = 0.7500633$ [$\approx 0.75 = (2/3)^{-1} / 2 = \sqrt{Exp}$]

$Exp_\Phi = (3/4)^{(1.2*\Phi*\Phi)} = 0.40503017$
$(m_e + m_\mu + m_\tau) / (m_e^{Exp_\Phi} + m_\mu^{Exp_\Phi} + m_\tau^{Exp_\Phi})^{1/Exp_\Phi} = 0.50001$
$[(m_e + m_\mu + m_\tau) / m_e] / [(m_e^{Exp_\Phi} + m_\mu^{Exp_\Phi} + m_\tau^{Exp_\Phi}) / m_e^{Exp_\Phi}] = 99.99994$ [≈ 100]

$(m_e + m_p + m_n) / \sqrt{(m_e^2 + m_p^2 + m_n^2)} = 1.414598$ [$\approx 0.1 * 14.146$; $14.146 = 11 * 1.286$]

$1 * \pi^4 + 4 * \pi^2 + 1 * \pi^{-2} + 5 * \pi^{-4} - 4 * \pi^{-6} = 137.035999087$ (result far within the tolerance of α^{-1})
$1 * 10^1 + 4 * 10^0 + 1 * 10^{-1} + 5 * 10^{-2} - 4 * 10^{-3} = 14.146$ (equal multiplicators at both equations)

$\Phi^{2/3} + e^{2/3} + \pi^{2/3} + 1.44^{2/3} + 6.66^{2/3} = 1.286 + 9.000028 = 1.286028 + 9$ [1.286 = 14.146/11]
$\Phi + \pi + 1.44 + 6.66 = 12.8596$ [$\approx 10 * 1.286$; $0.666^{1./1.286} = 0.7290087 \approx 0.729 = 0.9*0.9*0.9$]

Used figures π, 4 and 6 are quantities for determination of the circle surface and sphere volume.
$Exp_a = 0.72559092$ [$\approx \Phi^{-2/3} = 0.72556263$]; Exponent Exp_a is derived by a set Result Value 2/3.
$(\pi + 4 + 6) / (\pi^{Exp_a} + 4^{Exp_a} + 6^{Exp_a})^{1/Exp_a} = 0.66666666$ [Set Result Value 2/3]

$Exp_b = 1 / Exp_a = 1 / 0.72559092 = 1.37818704$ [$\approx \Phi^{2/3} = 1.3782408$; with Exponent 2/3]
$(\pi + 4 + 6) / (\pi^{Exp_b} + 4^{Exp_b} + 6^{Exp_b})^{1/Exp_b} = 1.333358$ [≈ 4/3]

Remarkable: with the exponent Exp_b, which is the inverse of exponent Exp_a, one gets nearly the double value compared to the result value won by the exponent Exp_a.

Remarkable relations with exponent x_w presented in this report:

$F_{K1}(x) = (\Phi + e + \pi) / (\Phi^x + e^x + \pi^x)^{1/x}$

$F_{K2}(x) = (\pi + 4 + 6) / (\pi^x + 4^x + 6^x)^{1/x}$

$F_{K1}(x_w) = F_{K2}(x_w) = 0.444025902$ with $x_w = 0.570365150$

$Rel_1 = 0.444 * \Phi / (x_w * \Phi - 0.444) = 1.500213$ [≈ 3 / 2]

$Rel_2 = (0.444 * e - x_w) * \pi = 1.999787$

$Rel_1 + Rel_2 = 3.4999996$ [$\approx (m_\mu / m_e)^{2/3} / 9.99 = 3.50081$]

$Rel_3 = [\pi + 1 / (0.444 * x_w)] / \sqrt{\pi} = 4.000320$