An Induction Proof
For Goldbach's Conjecture

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August 8, 2024

Abstract

We use a series of Tables with an induction argument to show Goldbach's conjecture is true.

Introduction

We will argue that the $2k$ case showing Goldbach's conjecture is correct implies the $2(k+1)$ case. It isn't necessary to use an abstract k value; we can use $k = 8$ implies $k = 9$, that 16 implies 18. The structure will be the same. We use tables.

Goldbach tables

16	15	14	13	12	11	10	9	8
0	1	2	3	4	5	6	7	8

Table 1: Goldbach table for $k = 8$.

Make a table that counts from 0 to 8 on the bottom row and, going right to left from 8 to 16 on the top row. This is done in Table 1; a Goldbach table for $k = 8$. Next use a sieve to determine the composite and prime numbers in this table. This is done in Table 2. All primes between 2 and 16 have been determined; they are underlined. Slash marks indicate composites; those numbers divisible by one or more primes residing on the first row. In Table 3 we indicate details governing these composites.

Table 3 gives the reasons for a number being crossed out from Table 2: row 2 gives primes from the table and below these primes are composites that are divisible by the corresponding row 2 prime. Thus below 2 are even numbers between 2 and 16; below 3 are multiples of 3; below 5 are multiples of 5; below 7 is just 14; there are no multiples of 11 or 13 below these numbers as twice these exceeds 16, the table's maximum value.

Row 1 from Table 3, currently blank, is to be filled in with corresponding row numbers from Table 1. These numbers are greater than $k = 8$ and can be composites in which case they are located between row 3 and row 9 in Table 3 or they can be primes. The latter case validates Goldbach's conjecture for this $k = 8$ case. The second row has such prime pairs: $(13, 3)$ and $(11, 5)$. The top primes come from the far right of row 2.

~~16~~	~~15~~	~~14~~	13	~~12~~	11	~~10~~	~~9~~	~~8~~
~~0~~	~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~

Table 2: Sieve applied for Goldbach table for $k = 8$.

	A	B	C	D	E	F
1						
2	2	3	5	7	11	13
3	4	6	10	14		
4	6	9	15			
5	8	12				
6	10	15				
7	12					
8	14					
9	16					

Table 3: Prime and composite distribution table for $k = 8$ case.

Induction

18	17	16	15	14	13	12	11	10	9
0	1	2	3	4	5	6	7	8	9

Table 4: Goldbach table for $k = 9$ case.

Per an induction argument, we get to assume we know all the primes and composites discerned from the early $k = 8$ tables just presented. For $k = 9$ the primes will remain the same as will the prime distributions.

To see this consider Table 4, the Goldbach table for this new case and Table 5, its prime and composite distribution table: the latter is identical to Table 3, the one for the previous case. The general case will have that prime pairs exist; going from k to $k + 1$, provided $k + 1$ is not a prime, introduces one new odd and one new even: 17 and 18 – both of which we can ignore. They are the $(18, 0)$ and $(17, 1)$ pairs which are not of interest. If $k+1$ should be a prime, we immediately have (k, k) as a prime pair.

	A	B	C	D	E	F
1						
2	2	3	5	7	11	13
3	4	6	10	14		
4	6	9	15			
5	8	12				
6	10	15				
7	12					
8	14					
9	16					

Table 5: Prime and composite distribution table for $k = 9$ case.

Proving the $k + 1$ case

Row 2 of Table 5 has been left blank. Can we fill it in with numbers that add to 18. Are we forced to use prime numbers? If so then the induction proceeds;

we have an induction proof. First all 2 through 16 numbers are present. So all pairs in Table 4 are possible. If bottom row numbers can be added to unique top row numbers to yield 16 then they can be added to other top row numbers to get 18. So the top row is $18 - 2 = 16$, $18 - 3 = 15$,$18 - 5 = 13$, and $18 - 7 = 11$, $18 - 11 = 7$. We are forced to use two primes 11 and 13, completing the induction.

References

[1] Apostol, T. M. (1976). *Introduction to Analytic Number Theory*. New York: Springer.

[2] Hardy, G. H., Wright, E. M., Heath-Brown, R. , Silverman, J. , Wiles, A. (2008). *An Introduction to the Theory of Numbers*, 6th ed. London: Oxford Univ. Press.