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# New approach to asserting the Riemann hypothesis

**Abstract**: we will establish a relationship between the classic Riemann Zeta function and Gauss's estimate for the prime numbers for the sequence of $x_n = e^{n^s}$ where s is a real s> 1. Then we will use the equivalence of the Riemann hypothesis. At the end some increase of the prime number theorem that we will render as density to affirm the Riemann hypothesis.

## I) New relationship between Riemann's Zeta function and Gauss's estimate for prime numbers P(x) $=\frac{x}{Lnx}$

For the following $x_n = e^{n^s}$ where n is a non-zero natural number and s > 1

For Gauss estimation of prime numbers P(x) $=\frac{x}{Lnx}$

$P( e^{n^s}) = \frac{e^{n^s}}{\log(e^{n^s})} = \frac{e^{n^s}}{n^s}$.

$\frac{P(e^{n^s})}{e^{n^s}} = \frac{1}{n^s}$ which gives $\sum_1^{+\infty} \frac{P(e^{n^s})}{n^s} = \sum_1^{+\infty} \frac{1}{n^s} = \xi(s)$ for s > 1.

**Conclusion: for s > 1 $\xi(s) = \sum_1^{+\infty} \frac{P(e^{n^s})}{e^{n^s}}$ where P is the Gaussian prime number count function and ξ is the classical Riemann function.**

**II) Best estimate of density sum error**

**of the prime number theorem for a sequence of $x_n$=$e^{n^s}$**

$\pi(x)$: the function of counting prime numbers

The Dusarat 1999 inequality gives $\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+ \frac{1.2762}{Lnx})$, the reduction is true for x ≥ 599and the increase for x > 1. We ax = 599≃ $e^{6.39}$.In the following we will always take x ≥ $e^7$and P(x) =$\frac{x}{Lnx}$

$\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+ \frac{1.2762}{Lnx})$ which gives

$\frac{x}{Lnx\,Lnx} \leq \pi(x) - P(x) \leq \frac{1.2762\,x}{Lnx\,Lnx}$divide by the real x with x ≥$e^7$

$\frac{1}{Lnx\,Lnx} \leq \frac{\pi(x)}{x} - \frac{P(x)}{x} \leq \frac{1.2762}{Lnx\,Lnx}$replace x by $x_n$= $e^{n^s}$.with n ≥ 7and s > 1.

$\frac{1}{n^{2s}} \leq \frac{\pi(x)}{x} - \frac{P(x)}{x} \leq \frac{1.2762}{n^{2s}}$ With n ≥ 7and s > 1 let's go to the sum between 7 and +∞

$\sum_7^{+\infty} \frac{1}{n^{2s}} \leq \sum_7^{+\infty} (\frac{\pi(x)}{x} - \frac{P(x)}{x}) \leq \sum_7^{+\infty} \frac{1.2762}{n^{2s}}$

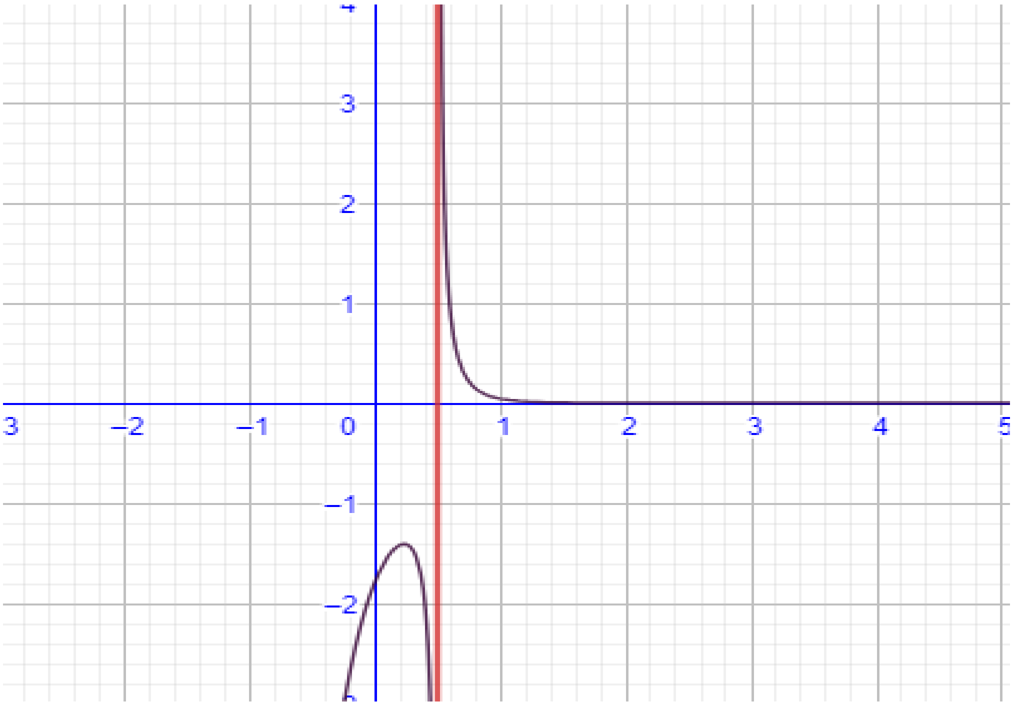
$\xi(2s)$- $(1+ \frac{1}{2^{2s}}+ \frac{1}{3^{2s}}+ \frac{1}{4^{2s}}+ \frac{1}{5^{2s}}+ \frac{1}{6^{2s}}) \leq \sum_7^{+\infty} (\frac{\pi(x)}{x} - \frac{P(x)}{x}) \leq 1.2762(\xi(2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))$

The deference between the two framing terms gives the errors

$$\mathbf{R(s)= 0.2762(\xi(2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))}$$

Examples for s=1 a calculation with Géogebra gives R (1) = 0.04

For s large enough R(s) ≃ 0



Graphical representation of R(s) with Géogebra

### III) Calculation equivalent to the Riemann hypothesis

Schönefeld, the Riemann hypothesis is true equivalent for all x ≥ 2657we have

$$|\pi(x) - Li(x)| < \frac{1}{8\pi}\sqrt{x}\, Lnx.$$

**Calculations (using Wolfram Alpha)**

$|\pi(x) - Li(x)| < \frac{1}{8\pi}\sqrt{x}\, Lnx$. Let's divide the inequality by x

$\left|\frac{\pi(x)}{x} - \frac{Li(x)}{x}\right| < \frac{1}{8\pi}\frac{Lnx}{\sqrt{x}}$ let's replace x subsequently $x_n$=$e^{n^s}$

$$\left|\frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}}\right| < \frac{1}{8\pi}\frac{Lne^{n^s}}{\sqrt{e^{n^s}}}$$

$$\left|\frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}}\right| < \frac{1}{8\pi}\frac{n^s}{e^{0.5n^s}}$$

For x ≥ 2657 i.e. x≥ $e^{7.88}$

**Conclusion**

**For s = 1 we have** $\sum_{8}^{+\infty} \frac{1}{8\pi} \frac{n^1}{e^{0.5n^1}} \simeq 0.0176721$

**Which gives** $\sum_{8}^{+\infty} \left| \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}} \right| < 0.0176721$

**For s big enough** $\sum_{8}^{+\infty} \frac{1}{8\pi} \frac{n^s}{e^{0.5n^s}} \simeq 0$ **which gives** $\sum_{8}^{+\infty} \left| \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}} \right| \simeq 0$

**IV) Conclusion**

Π(x) the function that counts prime numbers

Li(x) is the logarithm integral function

P(x) Gaussian estimate which is worth $\frac{x}{Lnx}$

The sequence of $x_n = e^{n^s}$ or n is a natural number and s> 1.

***Under the Riemann hypothesis**

For s =1 gives $\sum_{8}^{+\infty} \left| \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}} \right| < 0.0176721$

For s big, enough $|\pi(x) - Li(x)| < \frac{1}{8\pi}\sqrt{x}\, Lnx$ we have $\sum_{8}^{+\infty} \left| \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{Li(e^{n^s})}{e^{n^s}} \right| \simeq 0.$

***Without Riemann hypothesis**

The framework of Dusart 1999: $\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+ \frac{1.2762}{Lnx})$

The error R(x) = 0.2762( $\xi$ (2s) $-\left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right)$)

For s = 1 $R(x) \simeq 0.04$
For s big enough $R(x) \simeq 0$
**Question: can we say true for the Riemann Hypothesis?**

**References**

-Numbers, TA (2022). *Accreditation to Direct Research* (Doctoral dissertation, University of Limoges).
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