M.E.M.S. which allow the extraction of vacuum energy conform to Emmy Noether's theorem?

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ABSTRACT

This theoretical work corresponds to the hope of extracting, without contradicting EMMY NOETHER's theorem, an energy present throughout the universe: that of the spatial quantum vacuum.
This article shows that it should be theoretically possible to maintain a continuous periodic vibration of a piezoelectric structure, which generates current peaks during a fraction of each vibration period.
These vibrations are obtained by automatically controlling and in predetermined situations, the action of an attractive Casimir force between two electrodes by a repulsive Coulomb force applied to a third return electrode.
The internal electric field of a piezoelectric bridge, deformed by this force of Casimir, attracts from the mass mobile charges of contrary signs. These mobile electric charges of the same sign and blocked on the sources of a MOS transistor as well as on one of the faces of the piezoelectric bridge, suddenly homogenize with a Coulomb electrode located in series .
They generate a temporal peak of current as well as an ephemeral and repulsive Coulomb force.
Electronics without any power supply, then transform these alternating current signals into a usable direct voltage.
To manufacture these different structures, we present an original micro-technology to realize these electronics, and for technologically control and optimize the very weak interfaces between the Casimir electrodes and of the return electrodes.

1: DESCRIPTION OF THE SYSTEM

I1: Introduction

We know that the quantum vacuum, the energy vacuum, the absolutely nothing, does not exist!
This statement has been proven multiple times and noted by:
• Lamb's shift (1947) of atomic emission frequencies. (https://quantummechanics.ucsd.edu/ph130a/130_notes/node476.html)

• By the force of Van der Waals which plays a very important physicochemical role and had an interpretation quantum 1930 [London] when two atoms are coupled to the same fluctuations in vacuum.
https://culturesciences.chimie.ens.fr/thematiques/chimie-du-vivant/les-forces-de-van-der-waals-et-le-gecko

• By Hawking's radiation theory, predicted in 1974 and observed on September 7, 2016. Article Observation of quantum Hawking radiation and its entanglement in an analogue black hole :
https://www.nature.com/articles/nphys3863

• By the experimental verification (1958) of the existence of a force equated by Casimir in 1948. This so-called Casimir force was measured for the first time in 1997 https://arxiv.org/abs/quant-ph/9907076 https://en.wikipedia.org/wiki/Casimir_effect

The term *vacuum energy* is sometimes used by some scientists claiming that it is possible to extract energy from the vacuum - that is, mechanical work, heat….. . But these different hypotheses arouse great scepticism among many scientific researchers because they call into question a principle demonstrated mathematically by the theorem of the mathematician Emmy Noether in 1915.
This important mathematical theorem, which is accepted in physics and has never been faulted until now, stipulate that "Any invariance (of space-time as well as the physical laws used) according to a group of symmetries (continuous and global) is necessarily associated with a physical quantity preserved in all circumstances" . It involves the conservation of energy.
https://fr.wikiversity.org/wiki/Outils_math%C3%A9matiques_pour_la_physique_(PCSI)/Th%C3%A9or%C3%A8me_d%27Emmy_N%C5%93ther#

But the aforementioned effects, the interpretation of which is indisputable, imply a source of energy coming from a sort of "nothing" or more precisely from the quantum vacuum.
Thus, it is certain that this source of energy causing unmistakable physical manifestations exists. We will therefore choose for the remainder of this article a reference frame consisting of the 4-dimensional "Space-time" continuum augmented by those

still unknown in the "quantum vacuum". We will try to show that this physical repository makes it possible to explain, by a constant contribution of vacuum energy, an apparent "perpetual movement" of the MEMS device presented.

In fact, the problem is less to extract energy from the vacuum than to extract it without spending more energy that we cannot hope to recover!

Thus, in a cyclic system on the model of a piston engine move from position n°1 to n°2 then from n ° 2 to n ° 1, the Casimir force is in 1 /zs^4, therefore greater in position n°2 than in n°1, would then imply spending more energy to return to n°1. Which would necessarily require an added energy!! But we know that in the case of a deformation perpendicular to the polarization of a piezoelectric layer, the fixed charges Q_F induced by the deformation of this piezoelectric layer are proportional to F_{CA} and are therefore in $1/z_s^4$.
We have $QF = (d_{31}.F_{CA}.l_P)/a_p$ (Eq.(4)), [5] [6],which not depend on the common width bp = bs = bi of the structures. This point is important and facilitates the technological realization of these structures since it limits the difficulties of a deep and straight engraving of the different structures. The piezoelectric coefficient (CN^{-1}) is d_{31}, l_p, a_p respectively length and thickness (m) of the piezoelectric bridge (figure 5). These fixed electric charges on the two metallized faces of the piezoelectric bridge have opposite signs and attract mobile charges of opposite signs from the mass (figure 8).

Thus, when it is effective, the Coulomb return force F_{CO} is in $1/z_s^{10}$, with z_s = distance, time dependent, between Casimir electrodes and z_0 = initial distance between Casimir electrodes

This is a brief summary of the functioning of a MEMS which presents the hope of extracting a small energy from the quantum vacuum.

It uses the attractive driving force of Casimir between two electrodes that brutally releases a more intense, repulsive, and ephemeral Coulomb force. This Coulomb force is generated and controlled automatically [1,2,3].

The perpetual, isotropic and timeless Casimir F_{CA} force, resulting from quantum vacuum fluctuations, causes the deformation of a microscopic piezoelectric bridge embedded in a silicon wafer. This deformation generates internal ionic electric charges in a piezoelectric bridge, creating an electric field that, on its turn, attracts the moving electrical charges of the mass towards its external metallic surfaces. (Fig 1,2)

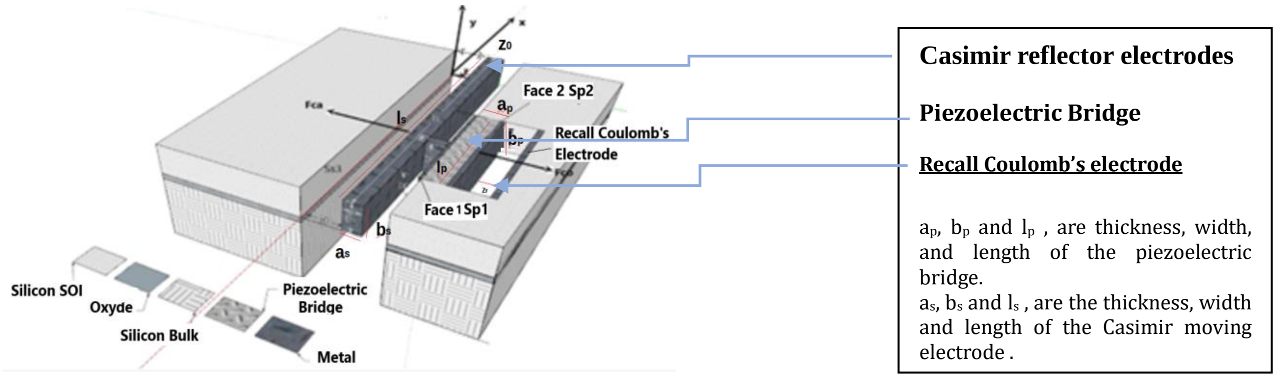
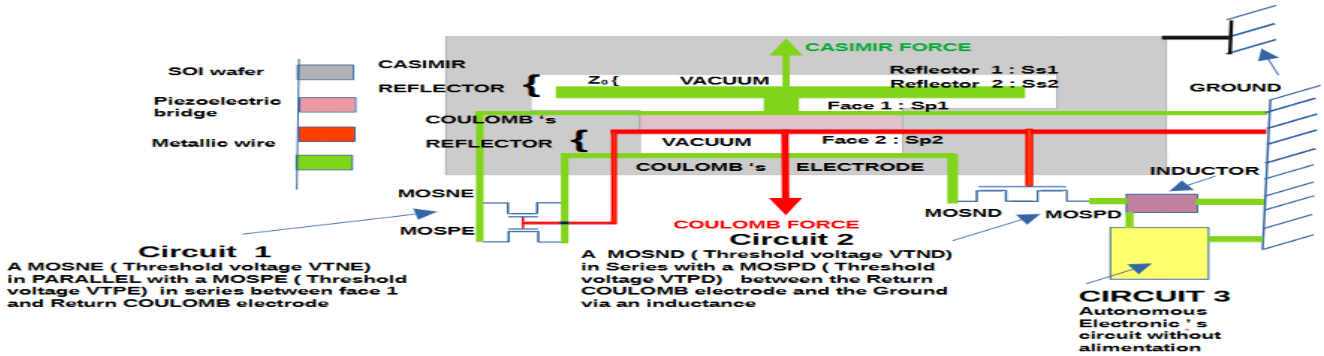


Fig. 1 Vue of the top of device, Axes, Forces , Casimir's Electrodes



Figures 2: general configuration of the device: MOS grid connections (Face 2 of the piezoelectric bridge: red), Source connections (Face 1 of the piezoelectric bridge: green)

These moving electrical charge can move on metal film that connects (see Figures 1, 2):

1/ The sources of MOSNE or MOSPE transistors MOS (Metal Oxide Semiconductor) enriched (N or P) and in parallel in the circuit switch n°1, for moving electrical charges of the metal face n°1 (green line on Fig 2).

We remark on Fig. 2, 4 ,5 that:

The sources of the MOSNE or MOSPE transistors of the circuit n°1 are in series with face n°1 of the piezoelectric bridge (green line). This switch n°1 is itself in series by its drains with an inductor coil and a metal called return Coulomb electrode. The latter is itself in series with a switch consisting of a serial MOSND with MOSPD transistors MOS in depletion.
This switch n°2 defining circuit n°2 is connected to the system ground.

2/ For moving electrical charges of the metal face n°2 (Red line):

2a / The grids of enriched MOSNE or MOSPE transistors MOS parallel to switch circuit n°1.

2 b / The grids of the MOSND or MOSPD transistors in series and in depletion of circuit switch n°2.

We remark:

That when the switch n°1 is OPEN the mobiles charges of face n°1 don't move and keep on this face n°1.

When the switch n°1 is CLOSED and switch n°2 is OPEN, the free moving charges of face n°1 must homogenize between the metallic film of face n°1 and the metallic film of Coulombs electrode (Fig 2).

Then, as the electrical nature of mobiles charges of faces n°1 and n°2 are opposite, a Coulomb's force F_{CO} must appears between the two metallic electrodes. The threshold voltages of the transistors of switch n°1, technologically predetermined, can impose Coulomb's forces much greater than the force of Casimir F_{CA}. The Coulomb force's lifespan is determined by the threshold voltages of the MOSND or MOSPD of switch n°2, which switch with ground.

So, this Coulomb force F_{CO} is ephemeral because it disappears automatically when the switch circuit 2 (Fig 2,3,5,17and 18) is closed or when the deformation bridge becomes null.

If the resulting force $F_{CO} + F_{CA}$, applied on the center of the piezoelectric bridge change of direction or is null, it contributes with the deformation energy memorized in the elastically piezoelectric bridge, to the return toward the initial state of equilibrium of the piezoelectric bridge, therefore without any electrical charges.

This ephemeral Coulomb force suppresses the collapse of the two very close electrodes of the Casimir reflector and reduces, then cancels the deformation of the piezoelectric bridge, and thus its electric charges. The structure returning to its initial state, is again deformed by the timeless and homogeneous Casimir force F_{CA} which always exist and this cycle should reproduce itself.

The system then vibrates, with the vacuum energy transmitted by the F_{CA} force, as a continuous drive source for the deformation of the piezoelectric bridge and with a self-built Coulomb force F_{CO}, superior and opposed to F_{CA} as the counter-reaction force.

At each cycle, the automatic switching of the integrated switches of circuits n°1 and n°2 (Fig. 2,3,4,5 and 17,18) distributes differently the mobile electrical charges located on face n°1 of the bridge.

Notice that initially, the Coulomb's electrode was grounded by the automatically closing of this switch n°2 (Fig 17,18)

The switch 1 return to OFF , so when the switch 2 becomes again ON, then the mobile charges present on the electrode of Coulomb return to the ground by crossing the integrated inductance L, a peak of current ΔI, flowing through an integrated inductance to the Coulomb return electrode. The latter being isolated from the mass by the opening of the switch n°2.

These systematic current peaks ΔI crossing an integrated inductance are transformed into voltage peaks ΔU, without any external input energy (figure 2, 6,7).

The power ΔU. ΔI at the terminals of this inductance, supplies electronics without any external power source, and is automatically transformed into a direct voltage of several volts, usable on a high impedance. See section 2/2/3 and figures 6,7.

An original technological realization is proposed to maximize the chances of manufacturing this MEMS device and in particular regarding a self-controlled realization of the very weak Casimir interface and return electrodes around 200 A° (See chapter 4 page 19).

This presented system, should be able to function as described above, because it includes:

1/ A Casimir reflector (Fig 1 and 2)

2/ A piezoelectric bridge (Fig 1 and 2)

3/ Three integrated electronic circuits with two different electronic switches and a self-contained amplifier transformer (without any power supply) of alternating signals in direct voltage. (Fig 4,5,6)

2/ DESCRIPTION OF SWITCHES N°1 OR N°2 AND AUTONOMOUS ELECTRONIC

2/1: switches and autonomous electronic description

These switches are made with:

a/ Circuit 1 (fig 4): with MOS P and MOS N transistors enriched and in parallel: Threshold voltage V_{TNE} and V_{TPE}

b/ Circuit 2 (fig 5): with MOS P and MOS N transistors in depletion and in series: Threshold voltage V_{TND} and V_{TPD}

An important point is that the threshold voltage values of these transistors are positioned as the Figure 3.

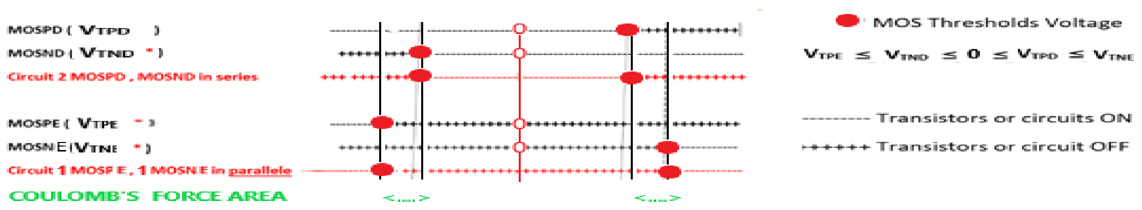


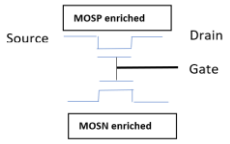
Figure 3: distribution of the threshold voltages of enriched and depleted N and P MOS switches.

We have, $V_{TPE} < V_{TND} < 0 < V_{TPD} < V_{TNE}$. For the functioning symmetry $|V_{TPE}|$ can equal $|V_{TNE}|$ and $|V_{TPD}|$ can equal $|V_{TND}|$

Consequently, as $|V_{TND}| < |V_{TPE}|$ and $|V_{TPD}| < |V_{TNE}|$, circuit switch n° 2 is open or closed *just before* circuit switch n° 1 is respectively closed or open (see figure n° 18).

2/1/1: Circuit n°1: Switch n° 1

Switch n°1 consists of an enriched P type MOS in parallel with an enriched N type MOS (see fig 1,2 4 and 17,18) with their threshold voltage as positioned in fig 3



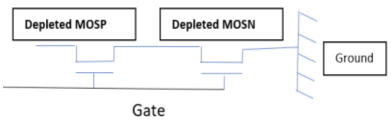
(Fig. 4)

The common gates voltage of these enriched MOS N and P in parallel of this switch n°1, are controlled by the free charges appearing on face n°2 of the piezoelectric bridge (Fig 1,2,4).

The N and P sources of these MOS are connected to face n° 1 of the bridge and the drains to the input of a series inductor. The output of this inductance is connected to the return Coulomb electrode and to the electronic n°3 in parallel, (figure 2). This return Coulomb electrode is itself grounded via switch n°2 (figure 2)

These two types of enriched MOSPE or MOSNE transistors of switch n°1 are in parallel to avoid the exact nature (holes or electrons) of the mobile electric charges appearing on the metal face n°1 of the piezoelectric bridge.
Preferably, their threshold voltages are the same in absolute value $|V_{TNE}| = |V_{TPE}|$

2/1/2: Circuit 2: Switches n°2



(Fig. 5)

Consist of a P type depletion MOSPD in series with an N type depletion MOSND (see figure 1, 2, 3 and 5).

The common gates of these MOS switches are controlled by the free charges appearing on face n°2 of the piezoelectric bridge. (See figure 2)

The input of Switch n°2 is linked to the Coulomb electrode, and its output is ground. Preferably, their threshold voltages are the same in absolute value $|V_{TND}| = |V_{TPD}|$

The values of these threshold voltage of MOS in depletion or enriched are such that $|V_{TND}| = |V_{TPD}| < |V_{TNE}| = |V_{TPE}|$ *(Fig 3)*

2/2/3: Electronics n°3; <u>(See figure 6 and 7)</u>

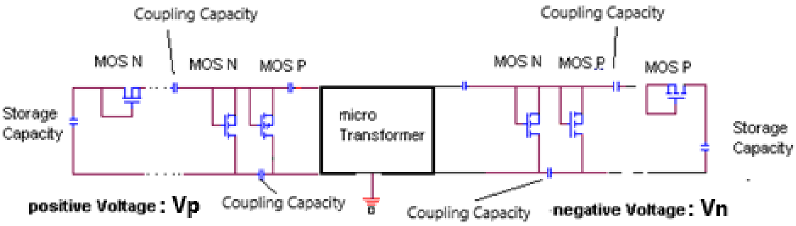


Figure 6: principle of the <u>one stage</u> of the doubler without any power supply electrical diagram

It is an autonomous device operating <u>without any electrical power source</u>. It rectifies and accumulates the repetitive peak power delivered to the terminals of the inductance (replaced, for the SPICE simulation, by the micro transformer in the figure 6) and transforms them into a usable direct voltage source.

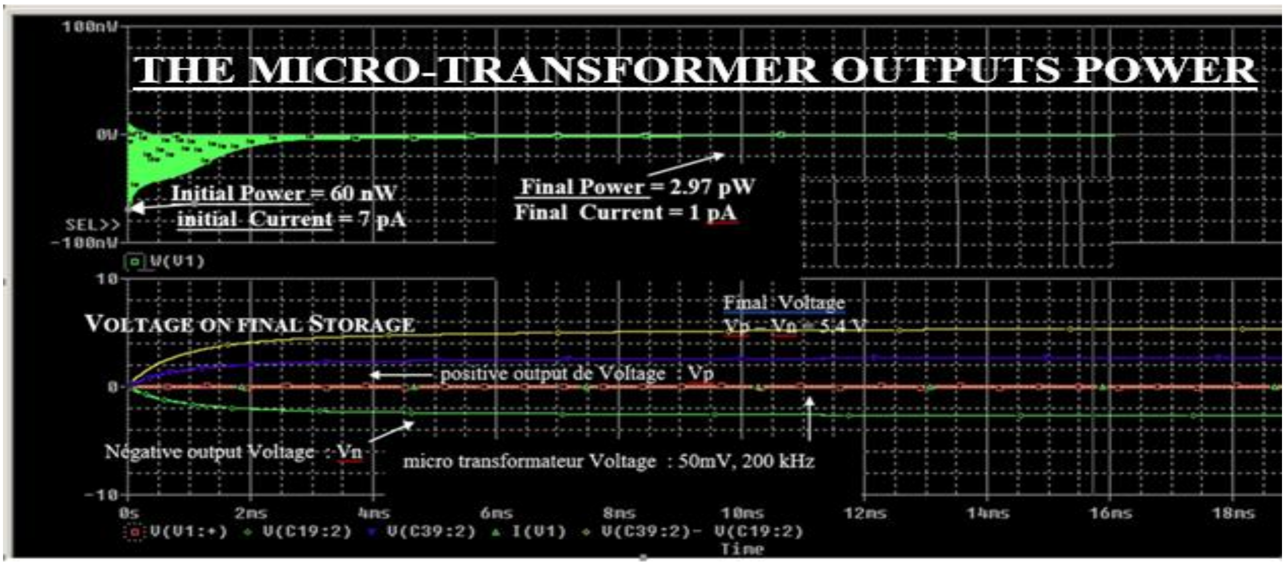


Figure 7: SPICE simulations of the voltages, current, power consumed by the electronics for the transformation into DC voltage (5.4 V) of an AC input signal of 50 mV, frequency= 150 kHz, Number of stages =14, Coupling capacitances = 20 pF, storage capacitance = 10nF

<u>3: DESCRIPTION OF THE DIFFERENT AND CYCLIC SITUATIONS</u>

The following figure 8 present the system and the nomenclature used

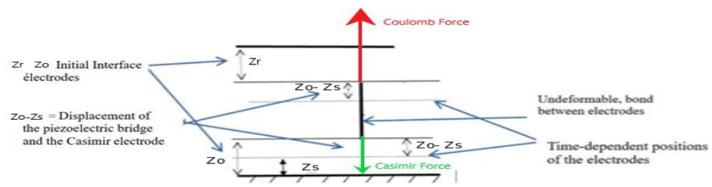


Figure 8: Axes, Forces , Casimir's Electrodes

<u>**3/1: Situation n°1: Initial position:**</u> **$0 < V_{GRIDS} < V_{TPD} < V_{TNE}$ or $V_{TPE} < V_{TND} < V_{GRIDS} < 0$,** Figures: 1,2,3,17,18

<u>The gate voltages appearing on the different MOS of the switches are lower than their threshold voltage:</u>

<u>Switch n°1 of enriched MOSE is OFF (open), switch n°2 of MOSD in depletion is ON (closed).</u>

The metal film covering the 1 or 2 side of the piezoelectric bridge is grounded on one side but electrically isolated by the opening of switch n°1 on the other side for side n°1 or by the isolated gate of MOS for side n°2 (Fig 1,2). At the start of the cycle, the piezoelectric bridge is not deformed, so the barycenter of negative ions is equal to that of positive ions in the piezoelectric bridge. There are no ionic charges so no electric field, and no moving electric charges appear on the film.

Switch n°1 is OFF and therefore isolates the Coulomb's return electrode from the face N°1 of the piezoelectric bridge.

Switch n°2 is ON and therefore connects the Coulomb's return electrode to ground.

The piezoelectric bridge is subject only to the Casimir force which deforms it. This force applies perpetually between the metallic electrodes of the Casimir reflector and is mechanically transmitted on the piezoelectric bridge by the small metallic finger (Fig 1,2). These electrodes of Casimir are separated by a vacuum. (See Fig 2 and 3 ,17 ,18)

We have the Casimir force: $F_{CA} = \frac{d(E_{CA})}{dz} = S\left(\frac{\pi^2 \hbar c}{240\ z_4^4}\right)$ (Eq 1)

With S the area of the Casimir reflector, $\hbar$ the reduce Planck Constant, c the light speed, z the separation of Casimir electrodes and E_{CA} the vacuum quantum energy.

This F_{CA} force transmitted by the connecting finger on the piezoelectric bridge deforms it (Fig 1,2, 17, 18). Thus, the barycenter of positive and negative ionic charges is not the same and an electric field appears inside the bridge.

This electric field inside the bridge attracts, from the ground, mobiles electrical charges (holes or electrons) on its two deformed faces. Both sides of the piezoelectric bridge accumulate opposite mobile electrical charges.

As a result, the gate voltages of all MOS transistors, connected at the face 2 of the piezoelectric bridge, increase but is still less than the threshold's enriched MOS voltages.

In this situation:

Switch 1 is OFF. So, it isolates face 1 of the bridge from the return electrode. (Fig 1,2,3, *17 ,18*)

Switch n°2 is ON, so the returned metallic Coulomb's electrode is grounded and as the electrics charges on it are zero, there is any electrical Coulomb's force on the return Coulomb's electrodes. (Fig 2,3, *17*, *18*)

Obviously, there is no Coulomb's force (Fig 3,17,18)

3/2: Situation n°2: $0 < V_{TPD} < V_{GRIDS} < V_{TNE}$ or **$V_{TPE} < V_{GRIDS} < V_{TPND} < 0$,** Figures: 1,2,3,17,18

The gate voltages appearing on the MOSE of the switch n°1 are lower than their threshold voltage but upper than the threshold voltage of the MOSD of switch n° 2: (figure 1,2,3, *17*, *18*).

As the deformation of the piezoelectric bridge grows up, the voltage on the gate of MOSD switch n° 2 become higher than their threshold voltage, so the switch n°2 toggle to OFF (open), isolating the Coulomb's electrode from the ground.
But as this electrode was previously connected to the ground its electrical charges are still null.

However, this voltage appearing on the grids of the transistors MOS is still lower than the threshold voltage of the MOSE, so the switch n°1 remains OFF (open).

In resume:

Switch n°1 is OFF (open) and still isolates the Coulomb's return electrode from the face n°1 of the piezoelectric bridge.

Switch n°2 is OFF (open)and therefore isolate the Coulomb's return electrode of ground.

There is no Coulomb's force and the return Coulomb's electrode is completely isolated. It implies that it is electrically isolated from ground and from the face n°1 of the piezoelectric bridge.

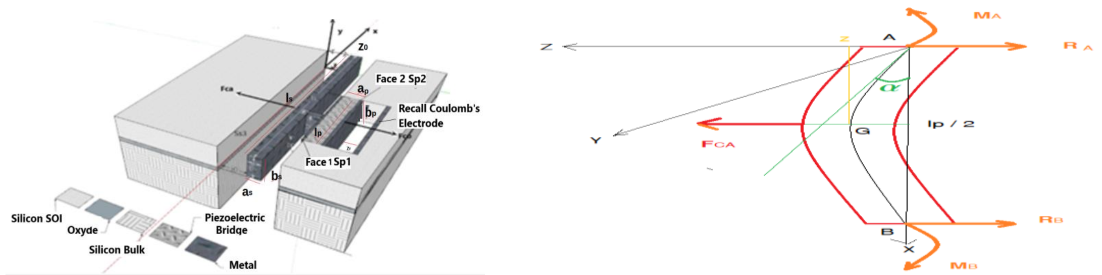


Figure 9: Piezoelectric bridge Cutting Reactions and Bending Moment, Deflection

The differential equation (following the conservation of kinetics' moment) which governs the movement of deformation of this

bridge is: $\overrightarrow{\sigma^S_{Ax,y,z}\ (\ Structure)} = \overline{\mathrm{I}^S_{\mathrm{A}\,x,y,z}}\ \overrightarrow{\theta^S_{A\,x,y,z}}$ With σ^S_{Axyz} the angular momentum vector of the structure, I^S_{Axyz} the inertia matrix of the structure with respect to the reference (A, x,y,z) and θ^S_{Axyz} the rotation vector of the piezoelectric bridge with respect to the axis Ay with α the low angle of rotation along the y axis of the piezoelectric bridge.
All calculations made; we get:

$$\frac{d^2z}{dt^2} = \frac{l_P^2}{8\ I_Y^S} S_S \frac{\pi^2\hbar c}{240} \frac{1}{z^4} = \frac{B}{z^4} \quad with\ B = \frac{l_P^2}{8\ I_Y^S} S_S \frac{\pi^2\hbar c}{240} \qquad \text{Eq 2}$$

With I^S_Y the inertia of the structure relatively to the axe Ay (Fig 9) (Casimir reflector + connecting finger + piezoelectric bridge) and z the Casimir reflector interval and relative. Considering Huygens' theorem. the expression of I^S_Y is:

$$I^S_Y = \rho_P a_P b_P l_P \left(\frac{\left(l_P^2 + a_P^2\right)}{12} + \frac{\left(l_P^2 + a_P^2\right)}{4} \right) + \rho_i a_i b_i l_i \left(\frac{\left(l_i^2 + a_i^2\right)}{12} + \frac{\left(l_P + l_i\right)^2 + \left(a_P + a_i\right)^2}{4} \right) + \rho_s a_s b_s l_s \left(\frac{\left(l_s^2 + a_s^2\right)}{12} + \frac{\left(l_P + l_i + l_s\right)^2 + \left(a_P + a_i + a_S\right)^2}{4} \right) \qquad \text{Eq 3}$$

With ρ_P, ρ_i, ρ_S, respectively the densities of the piezoelectric bridge, the intermediate finger and the mobile electrode of the Casimir reflector, a_p, a_i, a_s ; b_p, b_i, b_s ; l_p, l_i, l_s respectively the thickness, width and length of piezoelectric, intermediate finger, and sole electrode Casimir (Fig 1, 2). We obtain the solution of this differential equation n° 2 with the resolution of this differential equation (MATLAB) presented on figure 10 and 11 (falling part).

We remark that during the deformation of the piezoelectric bridge between 0 to z_2 (the switching position), <u>a potential electrostatic energy is accumulated in the bridge.</u> But this potential energy suddenly <u>will trigger for a short time,</u> and only when the <u>gate voltage of the MOS transistors of circuit n°1 exceeds their threshold voltage</u>.

<u>As long as the voltage on the gates of the enriched MOS of circuit 1 does not reach their threshold voltages, the state of circuit 1 and 2 do not change and the deformation of the bridge follows the curve of the first part of fig 10,11</u>

<u>3/3: Situation n°3:</u> $0 < V_{TPD} < V_{TNE} <= V_{GRIDS}$ or $V_{GRIDS} <= V_{TPE} < V_{TPND} < 0$, Figures: 1,2,3,17,18

<u>The gate voltages appearing on one of the MOS of the switch n° 1 are higher than its threshold voltage.</u>

<u>Switch n°1 is ON (closed), switch n°2 is still OFF (open)</u>

The face n°2 of the piezoelectric bridge is connected at the grid of the MOS transistors of switch n°1 or n° 2. When the gate voltages, due to the accumulation of moving electrical charges on the face n°2, exceed their threshold voltages, then:

<u>Switch n°1 toggle ON</u> and <u>connects face n° 1 of the bridge with the return electrode</u>, via the inductor L_{IN}.
<u>Switch n°2 remains OFF, thus isolated the Coulomb return electrode from ground.</u> (Figures 2, *<u>17, 18</u>*)

The face n°1 of the bridge is now connected to <u>the return electrode which is isolated from the ground.</u>

3\3\1 / calculation of the peak of current and voltage in the serial inductor.

Now, it is known that the free charges appearing at the terminals of the piezoelectric bridge are [5]:
$Q_F = (d_{31}.F_{CA}.l_P)/a_p$. (Eq 4). With d_{31} one of the values of the piezoelectric tensors of the bridge, F_{CA} the Casimir force, l_p and a_p the length and thickness of the bridge. (fig 1,2)

Knowing that the electric field inside a conductor is zero, the electric mobile charges, accumulated on the faces n°1 of the bridge, <u>must be distributed uniformly between this face n°1 and the return electrode</u> (remember that this return electrode was free of electrical charges because grounded a little time previously).

During this homogenization there is a displacement of mobiles charges in a metallic electrode during a small time, so a current peak appears (Figure 12).

The variation in time of the mobile charges on the initially grounded but now isolated return Coulomb electrode follows, as a first approximation, a law of distribution of the charges on a short-circuited capacitor. This temporal variation of the charges is given by the well-known exponential form of discharge of a capacitor according with the formula:

$$Q_m = Q_F \left[1 - Exp\left(- \frac{t}{R_m C_S} \right) \right] \quad \text{(Eq. (5))}$$

This variation in mobile charges stops when the electrical charges Q_m are uniformly distributed over the two electrodes S_{p1} and Sr and are equal to Q_F / 2 on the two electrodes, therefore, at time $t_e = R_m C_s \ln(2)$ (Eq. (5)), t_e being the time to reach equilibrium, with R_m the ohmic resistance of the metal track, L_{in} of the inductance, C_s the capacitance formed by the electrodes.

- $R_m \approx \rho_m \cdot l_m / S_m$, with ρ_m the resistivity of the self between electrode, l_m its total length, S_m its section.

- $Cs = \varepsilon_0 . \varepsilon_{0m} l_p . b_p / z_r$, the inter-electrode return capacitance, with ε_0 the permittivity of vacuum, ε_{0m} the relative permittivity of the metal oxide, l_p and b_p the geometries of the return electrode.

A calculation of the duration of the homogenization of the electric charges and therefore of the duration of the current peak, based on an estimate to propagate in a L_{IN} inductor of about 10^{-5} Henri, gives $t_e \approx 10^{-9}$ s.

This current peak, passing through a L_{IN} solenoid develops a voltage peak which will be exploited by integrated electronics without any power supply described in chapter IV.

We therefore obtain a current peak during this homogenization with a duration t_e of the order of a nanosecond. This current peak I_{IN} circulating for the duration of time t_e is:

$I_{IN} = d(Q_m)/dt \Rightarrow$

$$I_{IN} = -\frac{Q_F}{R_m C_S} Exp\left(-\frac{t}{R_R C_S}\right) \text{(Eq. (6))}$$

The time t =0 is counted from the closing of one of the transistors of circuit n° 1. This quick current peak ΔI_{IN} is present even if the switch transistors may close after because the mobile charges have already propagated.
This current peak ΔI_{IN}, passing during the time t_e through an integrated inductor L_{IN}, which is in series between switch n° 1 and the return Coulomb's electrode, induce a voltage peak ΔU_{IN} on the terminals of this inductor. (Fig 13)

$$U_{IN} = L_{IN}\frac{dI_{IN}}{dt} = -L_{IN}\frac{Q_F}{\left(R_m C_S\right)^2} Exp\left(-\frac{t}{R_R C_S}\right) = -L_{IN}\frac{Q_F \, Ln(2)}{\left(R_m C \, t_e\right)} Exp\left(-\frac{t \, Ln(2)}{t_e}\right) \quad \text{Eq. (7) , (Fig 13)}$$

The power peak $\Delta U . \Delta I$, electrically feeds the input of an autonomous electronics, which is without any other power supply. This self-contained electronics turns the periodic alternative peak power into a continuous voltage. (Fig 6, 7)

On the other hand, now we have electrics mobiles charges of opposite signs, between the metallic electrode of face 2 of the piezoelectric bridge and the metallic return Coulomb's electrode (electrically isolated from ground because the circuit 2 are OFF).
So, a force of Coulomb opposed to the force of Casimir appears between the face n° 2 of the bridge and the return electrode.

Indeed, this Coulomb force is proportional to the products of the charges accumulated at the terminals of these 2 electrodes and inversely proportional to the square of the interface between the face 2 of the bridge and the return electrode. (Fig 2 ,8)

We get, all calculations made, a Coulomb force whose expression is:

$$F_{CO} = \frac{Q_F Q_F}{2} \frac{1}{4\pi\varepsilon_0 \varepsilon r} \left(\frac{1}{z_r + z_0 - z_s}\right)^2 = \left[\left(S_S \frac{\pi^2 \hbar c}{240}\right)\left(\frac{1}{z_S^4} - \frac{1}{z_0^4}\right)\frac{d_{31} l_P}{a_P}\right]^2 \frac{1}{8\pi\varepsilon_0 \varepsilon r}\left(\frac{1}{z_r + z_0 - z_s}\right)^2 \text{(Eq 8)}$$

With Q_F (Eq 4) the electrical charges on the piezoelectric faces, z_r = initial interface between the face n°2 and the return Coulomb's electrode, z_o = initial Casimir's reflector interval and z_s current Casimir's reflector interval, ε_0, ε_r the vacuum and relative permittivity of the interface.
We note that $F_{CO} = 0$ when the bridge has no deflection ($z_s = z_0$). We note also that this Coulomb Force in $1/z_s^{10}$, is in the opposite direction and can be adjusted by the values of threshold voltage of the MOS enriched transistors to become much greater than that of Casimir which is in $1/zs^4$. (We can have $F_{CO}/F_{CA} > 100000$.)

Subject to the intensity of the resultant force and opposite direction FCO + FCA, the deformation of the piezoelectric bridge, aided by the stored deformation energy of situation #2 decreases very quickly and is cancelled when it returns to situation #1 without any deformation. (Fig 10,11,17,18).

The return from situation n°3 to situation n°1 takes place much faster than from the symmetric situation n°1 to n°3 and the differential equation that governs the movement of the bridge back to the initial position is:

$$\frac{d^2z}{dt^2} = \frac{l_P{}^2}{8\,I_S{}^Y}\left(F_{CA} - F_{CO}\right) = \frac{l_P{}^2}{8\,I_S{}^Y}\left\{\left(l_S\, b_S\, \frac{\pi^2 \hbar\, c}{240\, z_S{}^4}\right) - \frac{1}{2}\left[l_S\, b_S\, \frac{\pi^2 \hbar\, c}{240}\left(\frac{d_{31}\, l_P}{a_P}\right)\left(\frac{1}{z_s{}^4} - \frac{1}{z_0{}^4}\right)\right]^2\left(\frac{1}{4\,\pi\,\varepsilon_0\,\varepsilon_r}\right)\left(\frac{1}{z_r + z_0 - z_S}\right)^2\right\} \quad \text{(Eq 9)}$$

The above equations (Eq 2 and Eq 9) have no analytical solutions, we programmed them on MATLAB and the curves resulting from these numerical solutions for example on fig 10, 11,12

The z_2 position of appearance of the Coulomb force F_{CO} is such that $F_{CO} = p\ F_{CA}$, with p a proportionality coefficient that is technologically chosen and that can be very large and greater than 10^5: This factor depends on the values of the threshold voltages of the MOS transistors and is defined during the technological implementation of the system.
To obtain this value of z_2, we must solve the equation (Eq 9) in z_s:

The position z_2 of the appearance of this force F_{CO} is when:

$$F_{CO} = \left(\left(S_S\, \frac{\pi^2 \hbar\, c}{240}\right)\frac{d_{31}\, l_p}{a_p}\right)^2\left(\frac{1}{8\,\pi\,\varepsilon_0\,\varepsilon_r}\right)\left(\frac{1}{z_s{}^4} - \frac{1}{z_0{}^4}\right)^2\left(\frac{1}{z_r + z_0 - z_s}\right)^2 = p\, F_{CA} = p\left(S_S\, \frac{\pi^2\, \hbar\, c}{240\, z_S{}^4}\right) \quad \text{(Eq 8)}$$

This situation above appears only during a short time because, due to the return of piezoelectric bridge toward its initial position. When the mobile structure reaches the position z_1, the gate voltage of the depleted transistors of switch n°2 falls below their threshold voltage (Fig 3). Switch n° 2 therefore switches from OFF to ON and reconnects the Coulomb electrode to ground again. As a result, at the position z_1 the electric charges disappear on the return Coulomb's electrode. Therefore, Coulomb's force F_{CO} also disappears and only the Casimir force F_{CA} is retained.

So, this force F_{CO} (Eq 8) is present only during the short time when switch n°2 is still OFF after the switching of switch n°1 (Fig 10 ,11, 17 ,18)

In the z_1 position, the mobile system has acquired kinetic energy, which is spent in bringing the mobile system - braked by the Casimir force- at least to its starting position. The mobile bridge can very slightly exceed it because of its kinetic inertia (fig 10 ,11).

The system then returns to the situation n°1 and is only subjected, another time, to the omnipresent force of Casimir.

The values of the threshold voltages of the switches are set during manufacture of the system. (Fig *17* ,*18*)
These situations being repetitive, the system should therefore enter vibrations.

It is important to note that this is not an illusory perpetual movement because it is the perpetual driving force of Casimir which is the source of energy and of these vibrations.

Casimir's FCA force deformation automatically creates the ephemeral Coulomb FCO force, which is in the opposite direction of the FCA force and can be of equal or significantly higher intensity. Thus, for situation n°3, the resulting $F_{CO} + F_{CA}$ force is in the opposite direction or at least zero. Aided by this resultant force, the system uses the potential energy previously stored in the bridge to regain a position without deformation.

The frequency of these vibrations depends on the physico-geometrical characteristics of the piezoelectric bridge and the values of the threshold voltages of the switches.

This position and evolution z_2 in function of the coefficient of proportionality p is numerically calculable. (Fig 16)
The creation of a Coulomb force superior and opposite to that of Casimir, is a consequence of the creation of a potential and memorized energy W_{BRIDGE} by the energy of Casimir applied on an elastic piezoelectric bridge.

This Coulomb's force avoids the collapse of the Casimir electrodes and cancels the deformation of the bridge.
The electrics charges disappears because the switches of circuits 1 returning to the OFF position, isolate the return electrode from the bridge electrodes and circuits 2 in the ON position ground the return electrodes.

The system regains its initial characteristics. The only force that applies is again the force of Casimir F_{CA} and everything starts again. The bridge then should begin to vibrate. (Fig *17* ,*18*)

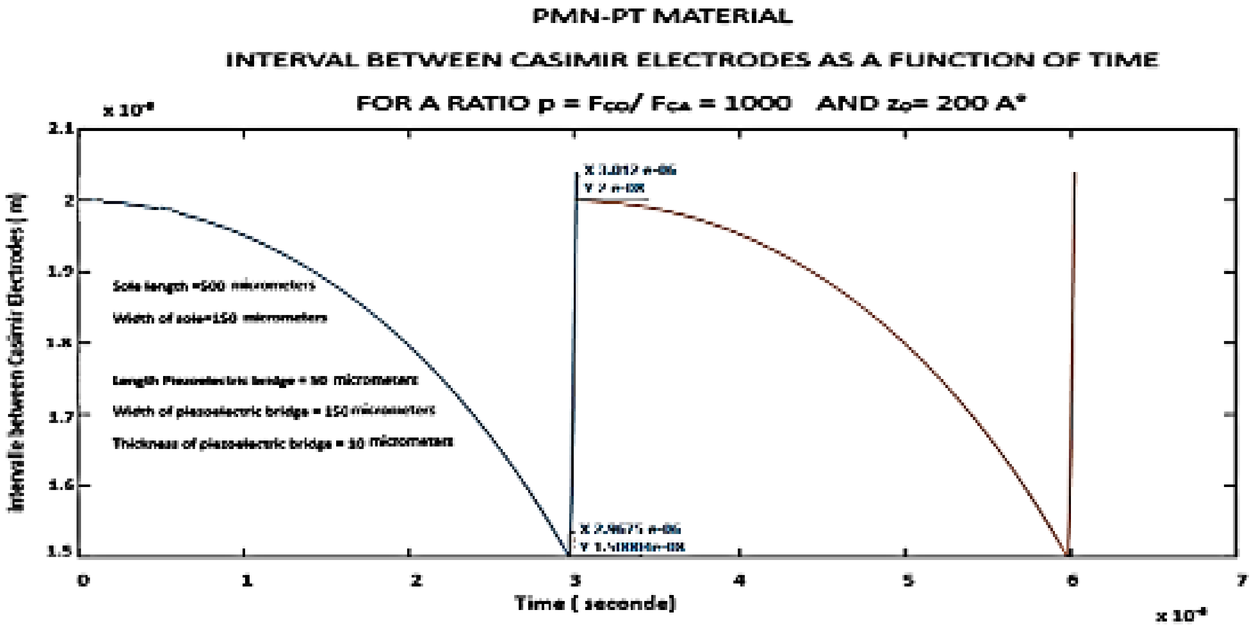


Figure 10: Plot of the evolution of the Casimir inter-electrode interval as a function of time over two periods and an Fco / Fca ratio = 1000: Casimir inter-electrode interface = 200 A°. Note the little overlap of the elastic bridge

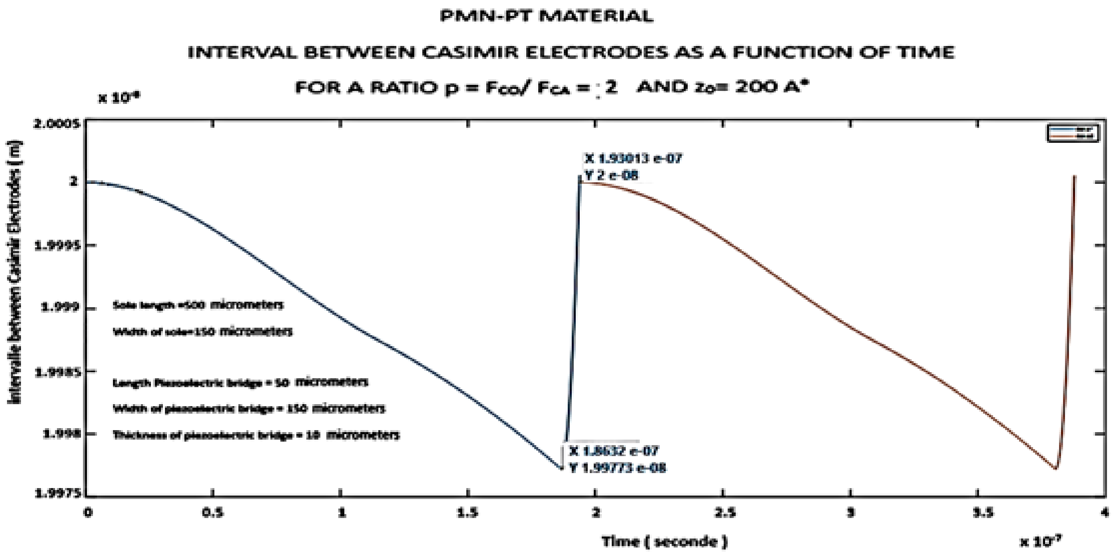


Figure 11: plot of the evolution of the Casimir inter-electrode interval as a function of time over two periods and a Ratio Fco / Fca = 2. Casimir inter-electrode interface = 200 A Note the little overlap of the elastic bridge

For a parameter F_{CO} / F_{CA}=2 (Fig 11), we notice that the time for the piezoelectric bridge to reach its limit position z_2 (1.8 10^{-7} s) is lower than for F_{CO} / F_{CA}=1000 (time 2.9$10^{-6}$ s) (Fig 10)
But that on the other hand, for F_{CO} / F_{CA}=1000 the time the mobile system to find its starting position is :

a/ a little longer (4.5 10^{-8} s Fig 10), than for F_{CO} / F_{CA}=2 (0.66 10^{-8} s Fig 11),

b/ exceeds the starting position z_0 a little more. This is consistent with the fact that the power of the F_{CO} reaction force is 500 times lower.

We observed in Figure 13 the current peak due to the homogenization of the mobile charges between face 1 and the Coulomb electrode.

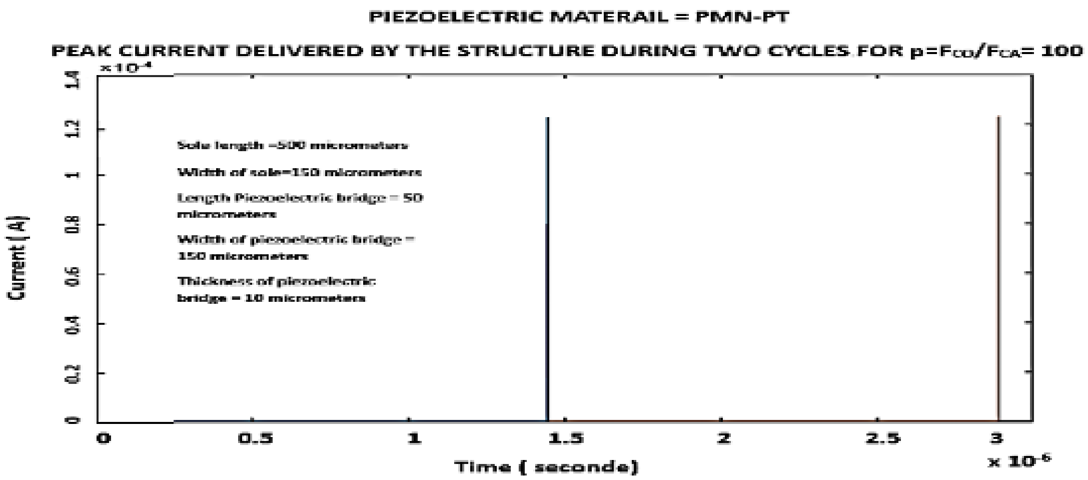


Figure 12: Materials = PMN-PT: Current peak (1.2 10^{-4} A) and Voltage peak (4 V) across the 4 * 10^{-5} H solenoid as a function of time obtained over 2 cycles. Starting interface = 200A Ratio p=Fco /Fca=1000

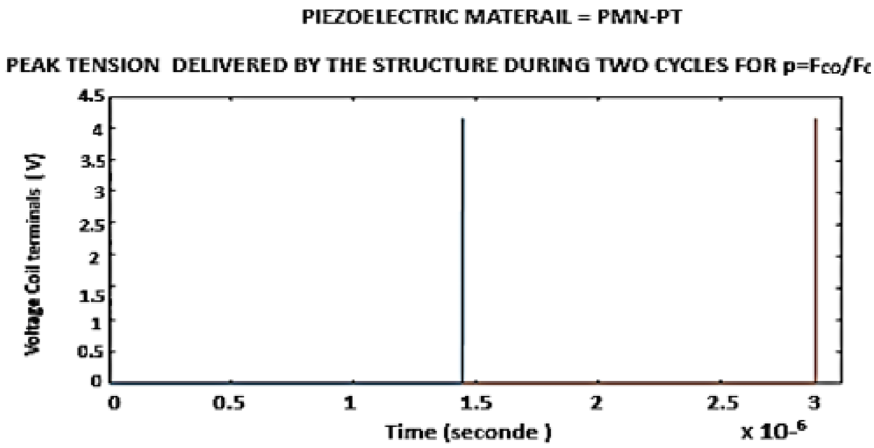
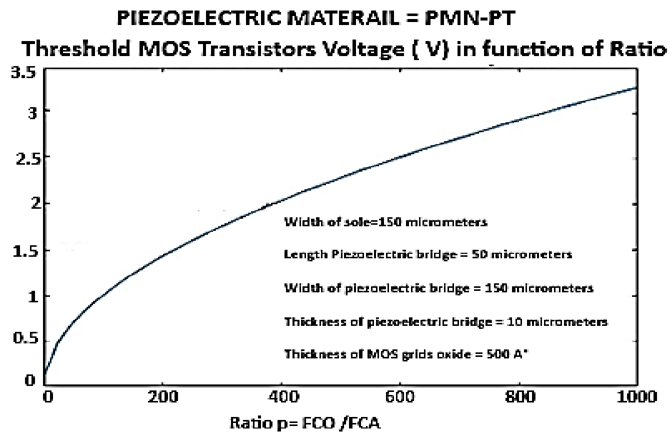


Figure 13: Materials = PMN-PT: Voltage peak across the 4 * 10^{-5} H solenoid as a function of time obtained over 2 cycles. Starting interface = 200A Ratio p=Fco /Fca=1000

We can observe in Figure 14 the evolution of the threshold voltage of enriched MOS transistors necessary to ensure a ratio p = F_{CO}/ F_{CA}, and Figure 15, the rapid fall of peak current with the increase of the interval in function of the interval of Casimir's Electrodes.



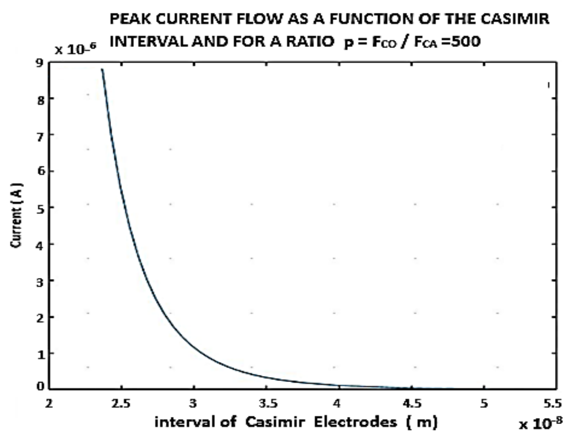


Figure 14: Materials = PMN-PT: Threshold voltage of the Enriched or Depleted MOTS according to the F_{CO} / F_{CA} Ratio. Start interface = 200A °

Figure 15: Materials = PMN-PT: Current peak across the 2. 10^{-4} H inductance as a function of the starting interval between Casimir electrodes. Start interface = 200A °

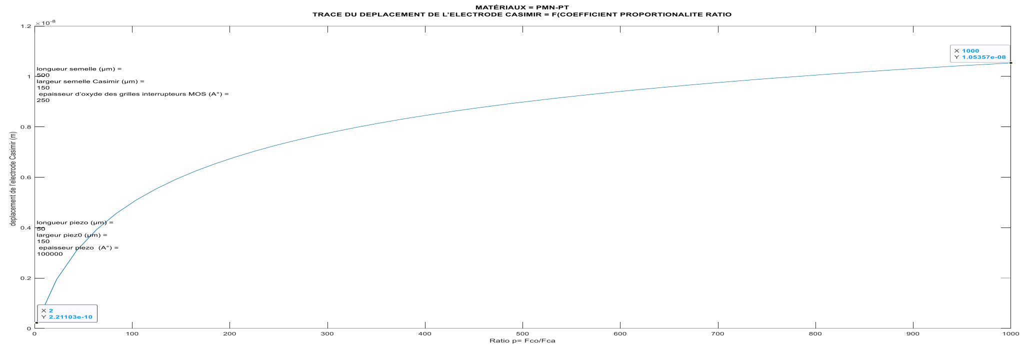


Fig 16: Displacement of the mobile Casimir electrode during the appearance of the Coulomb force for $z_r = z_0 = 200$A° ls = 500 ❍m, bs = 150❍m, lp = 50❍m, bp = 150❍m, ap = 10❍m

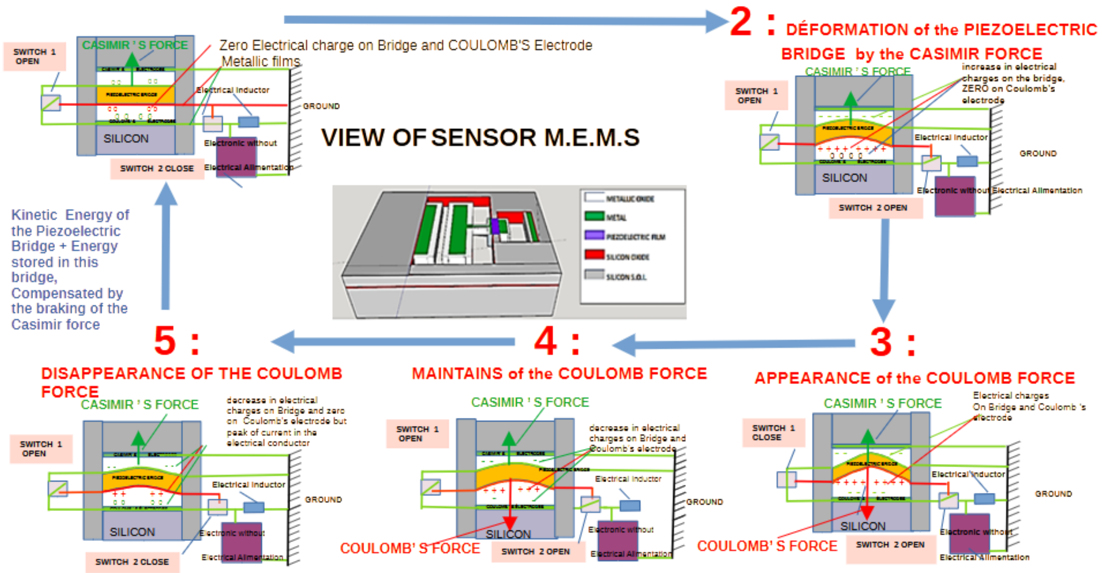


Figure 17 : Different dispositions of the piezoelectric bridge

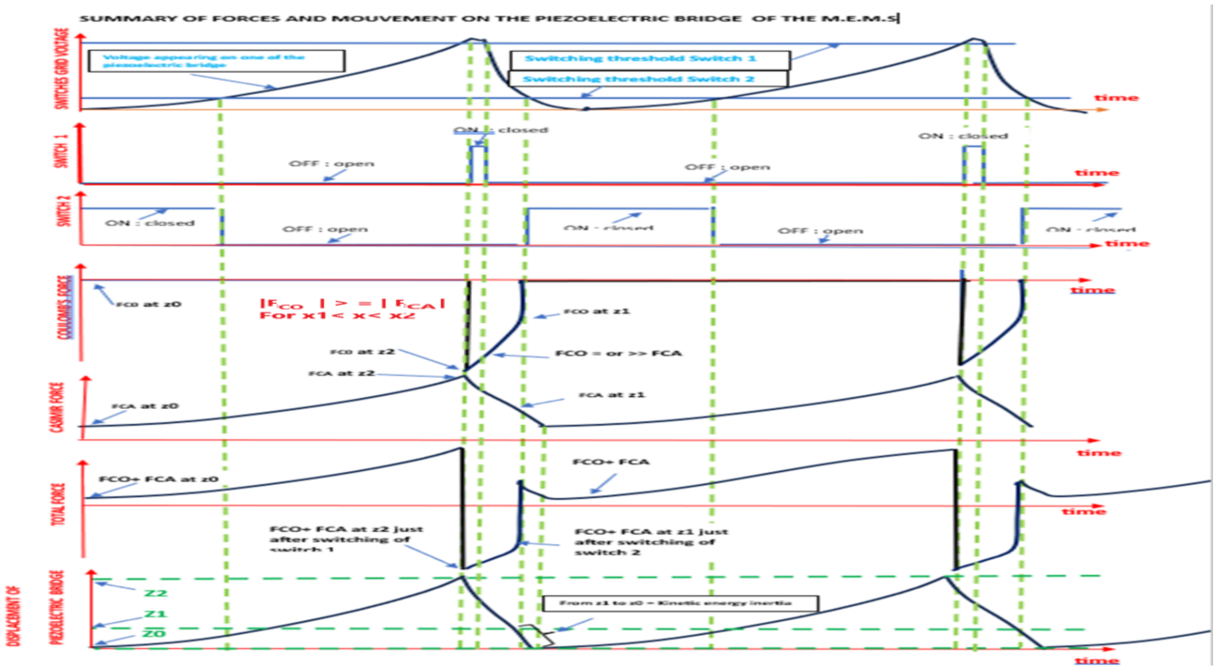


Figure 18 summary of force and movement of the piezoelectric bridge

4: ENERGY BALANCE

In this chapter we assess the energies that pass through this system. In particular we show that the CASIMIR force is blocked in its evolution towards a collapse of its two electrodes, by a COULOMB force. This Coulomb's force can be much more intense than the Casimir's one but lasts an extremely short time because it is cancelled very quickly.

Let us recall that:

a/ the voltage thresholds of the MOS are by technological design as: **$0 < VT_{PD} < VT_{NE}$, and $VT_{PE} < VT_{ND} < 0$.**

b/ The piezoelectric bridge is elastic which implies that, as with any elastic structure, the energy expended by a mechanical deformation of the positions from 1 to 2 is integrally restored when returning from the positions from 2 to 1, without any deformation. Thus, the structure being assumed to be perfectly elastic and the amplitudes of the vibrations being very low, we neglect the mechanical energy losses in this device.

1/ Note also that the mobile parallelepipedal metal electrode remains parallel to the fixed metal electrode when it moves. It simply transmits its movement to a piezoelectric bridge which deforms by bending.

2/ The expulsion of entropy ΔS from the vibrating electrode of Casimir is transmitted to the piezoelectric bridge but causes an extremely small increase in its temperature ΔT. This one expels this heat to the outside.
Let's calculate an order of this magnitude ΔT. Let's call ΔQ_{vib} the heat transmitted by the vibrations of the piezoelectric bridge. In first approximation, we can use the well-known formula $\Delta Q_{vib} = \Delta S.\ \Delta T$, with ΔS entropy variation (J °K^{-1}) and Δ T = temperature variation (° K)

However, we also know that [10]: $\Delta\ Q_{vib} = \dfrac{M_{Structure}\left(2\pi f_{vib}\right)^2 z_e^{\,2}}{2}$ Eq. (10)

With: f_{vib}= Vibration frequencies of the piezoelectric bridge, m = mass of this bridge, z_e = maximum deflection of the bridge.

This heat, expended at the level of the piezoelectric bridge, causes its temperature increase.

As a first approximation we can say: $\Delta Q_{vib} = m_{STRUCTURE}.\ C_{piezo}\ \Delta T$, with:

C_{piezo} = Specific heat capacity of the piezoelectric bridge (J Kg^{-1} °K^{-1}), ΔT = Temperature variation (°K).

Consequently $\Delta\ T = \dfrac{2\left(\pi\ f_{vib}\right)^2\ z_e^{\,2}}{C_{piezo}}$ = Temperature variation of the bridge. Eq. (11)

For example, for a PMN-PT piezoelectric film: $C_{piezo} = C_{PMN\text{-}PT} = 310$ (J Kg^{-1} °K^{-1}), $f_{vib} \approx 10^6$ Hz, $z_e = x_{max} \approx 100*10^{-10}$ m, we then obtain: $\Delta\ T \approx 10^{-3}$ °K which is negligible!

The expulsion of entropy from the vibrating Casimir Electrode is negligible!

So, the energy associated with the F_{CA} force, is only used to deform the piezoelectric elastic bridge with a W_{DEFCA} energy and also create Q_F fixed charges in this structure. This energy is stored in the deformed bridge as a potential energy!!

The fixed charges Q_F attract moving charges from the mass. When circuit 1 closed, the circuit 2 of switches MOSD is already opened (see fig 18).
The free charges Q_{mn}, stored on the metal electrodes of face 1, passing through one of the MOSE transistors, are uniformly distributed on the Coulomb metal electrode of the same surface S_{P1}.This metallic Coulomb's electrode therefore has approximately a mobile charge Q_{mn} / 2.

The free charges Q_{mp}, which are stored on the other grids electrode (face 2) don't move. So, grids electrode and return electrode have opposite free charge. (Fig 2).

A Coulomb's force then appears between these two electrodes during a very short time during which switch n°2 is still open isolating the Coulomb electrode from the ground. This time is of the order of a few tens of nanoseconds (fig 5, 12,17,18).

The position z_2 of appearance of this force is such that $F_{CO} = p\ F_{CA}$ and is numerically calculated by MATLAB (fig. 16). So, this position z_2 depends on the values of the interface's z_o of Casimir's electrodes and of Coulomb's electrodes z_r.
We describe below the energy the cycle of the moving positions of the piezoelectric bridge.

4/1: From z_0 to z_2 (going phase): $0 < V_{GRIDS} < VT_{MOSD} < VT_{MOSE}$ or $VT_{MOSE} < VT_{MOSD} < V_{GRIDS} < 0$ and $F_{CO} / F_{CA} < p$

No moving electric charge appears on the return side of the Coulomb electrode, which is connected to ground by switch 2, which is ON, and isolated from the piezoelectric bridge by switch 1, which is OFF.
Therefore, there is any electricals charges on this return electrode and the Coulomb force does not exist!

In a cycle from z_0 to z_2, thus, just during the displacement " going " the quantum vacuum energy $W_{CASIMIR}$ is used for four different energies:

1/ The mechanical energy for the deformation of the elastic bridge: W_{DEFCA}
2/ The electrical energy accumulation because the bridge is piezoelectric: W_{BRIDGE}
3/ The displacement of the point of application of the Casimir force in the middle of the bridge : $E_{CASIMIR}$
4/ The expulsion of entropy ΔS energy, expended in heat due to the friction of the atoms in the half of the vibration of the bridge Heat : $\Delta Q_{vib}/2$

$W_{CASIMIR} = W_{DEFCA} + W_{BRIDGE} + E_{CASIMIR} + \Delta Q_{vib}/2$. This quantum vacuum energy $W_{CASIMIR}$ is bigger than $E_{CASIMIR}$

1/ The **deformation** energy of the piezoelectric bridge is W_{DEFCA}.

We know that the deformation energy of an elastic system is the energy that accumulates in the solid body during its elastic deformation. Yet, the deformation energy of the piezoelectric bridge is more important than the simple displacement energy $E_{CASIMIR}$ of the F_{CA} force. $E_{CASIMIR} = \int_{z_0}^{z_e} F_{CA}\, dz = E_{CA} = S\left(\dfrac{\pi^2 \hbar c}{740\, z_s^3}\right)$.

If we neglect the quantities of energy lost by the losses of entropy during the deformation ΔQ_{vib}, then a part of this work $W_{CASIMIR}$ is transformed into the elastic deformation energy of the body W_{DEFCA}

Let us carry out a simplified calculation to obtain an order of magnitude of this deformation energy which is spent by the Casimir force in its path from z_o to z_2.
Consider an intermediate state during a deformation and consider that the stress σ_z ($N.m^{-2}$) along the z-axis is constant during an elementary deformation dz. The total elongation in the z-direction is: ε_z dz, with ε_z the strain along the z axis.

If dV = dx. dy. dz is the elementary volume element then the elementary work dW_Z performed is:

$dW_Z = \sigma_z\, d\varepsilon_z$ (dx. dy. dz). Knowing that the material obeys Hooke's law with $\sigma_z = E_p.\ \varepsilon_z$, the total work W_{DEFCA} of the external forces becomes, with ε_{zm} the maximum deformation at the centre of the bridge, E_p Young Modulus of bridge, and σ_z the stress in the bridge.

$$W_{DEFCA} = \int_0^{\varepsilon_{zm}} \left(\sigma_x d\varepsilon_x + \sigma_y d\varepsilon_y + \sigma_z d\varepsilon_z\right) dV \approx V\int_0^{\varepsilon_{zm}} \left(\sigma_z d\varepsilon_z\right) dV \approx \int_0^{\varepsilon_{zm}} E_p \varepsilon_z\, d\varepsilon_z \approx V\frac{E_p\, \varepsilon_z^{\ 2}}{2} \quad with\ V = l_p b_p a_p$$

This is the approximate expression of the strain energy caused by the external force acting on this element of volume V. We have neglected the deformations and stresses along the x, and y axes.

We know (APPENDIX X) that the maximum deflection of the piezoelectric bridge is Eq. (22).

$z_e = \dfrac{F_{CA}\, l_P^{\,3}}{192\, E_P\, I_P}$ in x= $l_P/2$. With I_p = bending moment of inertia along the z-axis of the section of this parallelepiped bridge which is: $I_P = \dfrac{b_P\, a_P^{\,3}}{12} = Ctc$

Considering this arrow of the bridge subjected to a force F_{CA}, the deformation ε_z can be written in first approximation by :

$$\varepsilon_x = \frac{\Delta z}{a_P} = \frac{z_e}{a_P}\ so\ in\ x = \frac{l_P}{2} \Rightarrow \quad \varepsilon_x = \frac{F_{CA}\ \ l_p^{\,3}}{192\ E_p\ I_P\ \ a_P}$$

So, the deformation energy can be written as:

$$W_{DEFCA} \approx l_P\, b_P\, a_P\, \frac{E_P}{2}\left[\frac{F_{CA}\; l_P^{\,3}}{192\; E_P\; I_P\; a_P}\right]^2 = \frac{F_{CA}^{\;2}\, b_P\, l_P^{\,7}}{73728\; a_P\; E_P\, I_P^{\,2}} =$$

$$= \frac{l_s\, b_S\, b_p\, l_P^{\,7}}{73728\; a_P\; E_P\; I_P^{\,2}}\left(\frac{\pi^2 \hbar\, c}{240\left(z_0 - z_e\right)^4}\right)^2 \qquad \text{Eq. (12)}$$

2/ During the displacement " going " the energy $W_{CASIMIR}$ is also used also to generates a <u>potential energy W_{BRIDGE} accumulated in the capacity of this piezoelectric bridge</u> which follows the equation:

$$d\left(W_{BRIDGE}\right) = Q_F\, d\left(V_{PIEZO}\right) \Rightarrow W_{BRIDGE} = \int_0^{Q_e} \frac{Q_F}{C_{PIEZO}}\, \mathrm{d}Q_F = \left[\frac{Q_F^{\;2}}{2\, C_{PIEZO}}\right]_0^{Q_e} =$$

$$= \frac{a_P}{2\, l_P\, b_p\, \varepsilon_0\, \varepsilon_{PIEZO}}\left(\frac{d_{31} l_P}{2\, a_P}\right)^2 F_{CA}^{\;2} = \frac{a_P}{2\, l_P\, b_p\, \varepsilon_0\, \varepsilon_{PIEZO}}\left(\frac{d_{31}\; l_p\; l_s\, b_S\; \pi^2\, \hbar\, c}{480\, a_P}\right)^2\left(\frac{1}{z_e}\right)^2 \qquad \text{Eq. (13)}$$

In this equation, we have $d(Q_F)= C_{pi}\, d\,(V_{pi})$, where C_{pi} = electrical capacity, $C_{PIEZO} = \varepsilon_0\, \varepsilon_{PIEZO}\, \frac{l_P\, b_P}{a_P}$ and V_{pi} the voltage across the piezoelectric bridge , z_e the position of the very brief appearance of the Coulomb force, ε_{pi} the piezoelectric relative permittivity, l_p ,b_p, a_p the length, width, and thickness of the bridge. With Q_F the naturally creating fixed charges on this piezoelectric structure. $Q_F = \frac{d_{31}\; F_{CA}\; l_P}{a_P}$ Eq. (3), and $Q_e = - Q_F$ = the accumulated mobile charges, coming from the mass, on the surface of the "return" electrode when coulomb's force is triggered.

So, during the phase "going" from z_0 to z_e the total energy $G_{oing} = W_{CASIMIR}$ is used to deform the piezoelectric bridge and to produce the electrical charges as potential energy use during the return phase + a part of the heat **$\Delta Q_{vib}/2$**

This part W_{BRIDGE} of $W_{CASIMIR}$ is stored in the piezoelectric bridge and is the usable energy $W_{ELECTRIC}$ appearing during a cycle. $W_{ELECTRIC}$ is not due to any electrical energy applied but produced by the potential energy W_{BRIDGE} accumulated in the piezoelectric bridge.
We have: $E_{GOING} = W_{CASIMIR} = W_{DEFCA} + E_{CASIMIR} + \Delta Q_{vib}/2 + W_{BRIDGE}$

Since the elasticity conditions of the deformed piezoelectric bridge apply, and that we are not entering the plasticity domain, W_{DEFCA} and W_{BRIDGE} are potential energies that will be used when the bridge returns to its equilibrium position, i.e. without deformation.

<u>4/2: From z_2 to z_0 (returning phase)</u>: There are <u>two phases: from z_2 to z_1 and 2/ from z_1 to z_0</u>

We use MATLAB to find the position z_2 of commutation of circuit 1 to create the Coulomb's Force F_{CO}, see (Eq 6) and figure 16

$$F_{CO} = \left(\left(S_S\, \frac{\pi^2 \hbar\, c}{240}\right)\frac{d_{31}\, l_p}{a_p}\right)^2\left(\frac{1}{8\,\pi\, \varepsilon_0\, \varepsilon_r}\right)\left(\frac{1}{z_s^{\,4}} - \frac{1}{z_0^{\,4}}\right)^2\left(\frac{1}{z_r + z_0 - z_s}\right)^2 = p\, F_{CA} = p\left(S_S\, \frac{\pi^2\, \hbar\, c}{240\, z_S^{\,4}}\right)$$

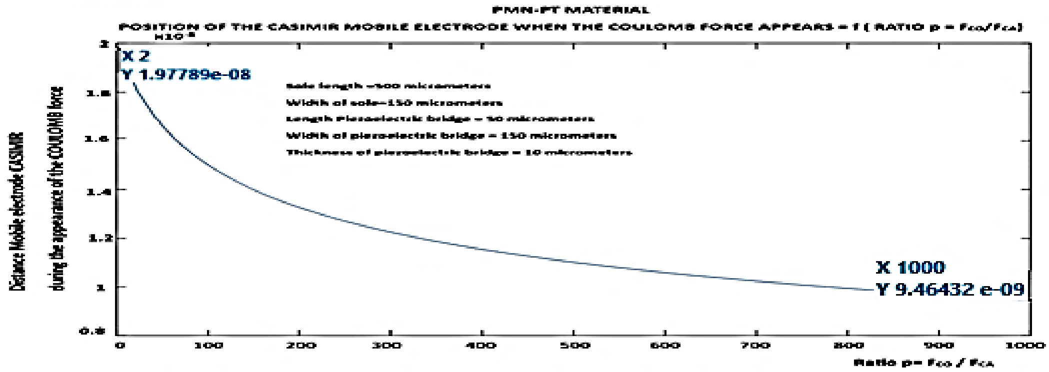


Figure 19: Position of the mobile Casimir electrode when the Coulomb force occurs for $z_r = z_0$ =200A°
ls = 500 µm; bs = 150 µm; lp = 50µm; bp = 150µm; ap = 10µm

4/2/1: First return phase of return movement: From z_2 to z_1: $0 < VT_{PD} < V_{GRIDS} < VT_{NE}$ or $VT_{PE} < V_{GRIDS} < VT_{ND} < 0$

The position z_1 is characterized by the fact that $0 < VT_{PD} < V_{GRIDS} \leq VT_{NE}$ or $VT_{PE} \leq V_{GRIDS} < VT_{ND} < 0$
The sudden and ephemeral appearance of this Coulomb force is because circuit 1, returning very quickly to the OFF state, disconnects the Coulomb electrode from the piezoelectric bridge. But as circuit 2 is still OFF, so the Coulomb's electrode is electrically isolated.
Shortly afterwards, at position z_1, circuit 2 connect back to the ON state, bringing the Coulomb electrode to ground.
This cancels the F_{CO} force. (See Fig 16)

The have already predetermined the position z_2 of appearance of this force such that $F_{CO} = p\, F_{CA}$. Note that it's extremely small value, since it goes from 1 A ° for an F_{CO} / F_{CA} ratio = 2 to 10 A ° for a ratio of 1000. (Eq 6, Fig 19)

When the equilibrium position z_2 is reached, just after the closure of circuit no. 1 and the circulation of the current peak ΔI, circuit no. 2 still being open, the electrical charges of the mobile are:

1/ At the position z_2 , on the isolated Coulomb's electrode in series with the electrode's piezoelectric bridge .

$$Q_{mn} = \frac{1}{2}\left(\frac{S_S\, \pi^2\, \hbar\, c\;\; d_{31}\, l_P}{240\, a_P}\right)\left(\frac{1}{z_2{}^4} - \frac{1}{z_0{}^4}\right)$$ Eq. (14) because the area of Coulomb's electrode $\approx$ area of bridge

2/ At the position z_2 , on the moving piezoelectric electrode connected to the transistor gates, the charge Q_{mp} is time-dependent, as it depends also on the bridge position over time.

$$Q_{mp} = \left(\frac{S_s\, \pi^2\, \hbar c\, d_{31} l_p}{240\, a_p}\right)\left(\frac{1}{z_2{}^4} - \frac{1}{z_0{}^4}\right)$$ Eq. (15):

Now the Coulomb force becomes $$F_{CO} = Q_{mp}\, Q_{mn}\left(\frac{1}{4\pi\, \varepsilon_0\;\; \varepsilon_r}\right)\left(\frac{1}{z_r + z_0 - z_s}\right)^2$$
We observe that Q_{mp}, Q_{mn} therefore F_{CO} disappear when $z_s = z_0$. We have noticed that this force only exists and follows this law for the short time during which switch 2 is not closed to ground.
After this time, i.e. when $zs = z_1$, the charge on the Coulomb electrode is zero, so the Coulomb force disappears.

The energy associated with the displacements of these forces therefore considers that relating to the displacement of the Coulomb force F_{CO} from z_2 to z_1 (where it cancels out) and that of the displacement of the Casimir force F_{CA} from z_e to z_0.

During the return phase, therefore, we have the energy of the displacement induced by F_{CA} $E_{CASIMIR} = \int_{z_2}^{z_0} F_{CA}\, dz_s = S_S\left(\frac{\pi^2\, \hbar\, c}{740\, z_s{}^3}\right)$

and the Coulomb's energy $W_{Coulomb} = \int_{z_2}^{z_1} F_{CO}\, dz_s$ (Eq 16)

As we have seen above, the expression of the current peak ΔI related to the homogenization of the mobile charges, inducing voltage peak ΔV. Eq 6 and Eq 7. So, during one cycle, the only energy which is effectively used outside is associated with these power $U_{IN}\, I_{IN}$ peaks and becomes:

$$W_{ELECTRIC} = ABS\left(\int_0^{t_e} I_{IN}\, U_{IN}\; dt\right) = \frac{L_{IN}}{2}\left(\frac{d_{31}\, F_{CA}\;\; l_P\;\; Ln(2)}{a_P\;\; t_e}\right)^2 \{1 - exp\,(\,-2\,Ln(2))\}$$ Eq. (17)

4/2/2 : Second phase of return movement From z_1 to z_0 $0 < V_{GRIDS} < VT_{PD} < VT_{NE}$ or $VT_{PE} < VT_{ND} < V_{GRIDS} < 0$

When the piezoelectric bridge reaches the z_1 position, circuit 2 switches ON, which connects the Coulomb electrodes to ground, so $F_{CO} = 0$. Only the Casimir force F_{CA} remains.

But during the previous phase from z_2 to z_1, the whole mass structure $M_{structure}$, has acquired a speed $V_{structure}$

This speed gives to it a kinetic energy $W_{CIN} = ½\, M_{structure}\, V^2_{structure}$.

So, this inertia must be spent until the stop of the structure only submit to the force F_{CA}. This kinetic inertia induces, for the bridge, a position which can slightly exceed the starting position z_0 because of the inertia. (Fig 10 ,11)

We have:

$W_{RETURNING} = W_{COULOMB} + W_{CIN} + E_{CASIMIR} + W_{ELECTRIC} + \Delta Q_{vib}/2 = W_{DEFCA} + E_{CASIMIR} + W_{ELECTRIC} + \Delta Q_{vib}/2$

Recall that: $W_{BRIDGE} = W_{ELECTRIC}$ and $E_{GOING} = W_{CASIMIR} = W_{DEFCA} + E_{CASIMIR} + W_{BRIDGE} + \Delta Q_{vib}/2$

Thus, we see that in the balance $E_{GOING} - W_{RETURNING}$, $= W_{CASIMIR} - W_{CASIMIR} = 0$

The energy expended by the Coulomb force F_{CO} remains weaker than the Casimir energy. This Casimir energy consumed by the deformation of the piezoelectric bridge and the creation of the electrics charges is partly restored in the form of usable electrical energy.

So, the energy $W_{CASIMIR}$ is conservated over a complete cycle. ***There is not a created new energy.***

Casimir energy is used differently in the "GOING" and "RETURN" part of the vibration.

In the "GOING" part, the omnipresent force of Casimir not only moves in a translation but also deforms a piezoelectric bridge, which implies a mechanical deformation energy but also an electrical energy.

Mechanical deformation energy is higher than displacement energy. This deformation energy is stored in the elastic bridge and will be used in the "RETURNING" part of the vibration.

Casimir's force naturally induces a large but ephemeral opposing Coulomb force that remains low energy but high power. This Coulomb force blocks the action of the Casimir force, imposes the "RETURN" part of the vibration, communicates kinetic energy to the moving structure but quickly disappears.

The deformation energy stored in the "GOING" part is used during the "RETURN" part of the ELASTIC structure.
The electrical energy part of the "GOING" phase is used to operate autonomous electronics.

The energy balance of a cycle therefore seems to satisfy Emmy NOTHER's theorem, but it seems also that we can recuperate and use a small electrical energy $W_{ELECTRIC}$.

4/3: SOME IMPORTANT REMARKS

In fact, energy expended by the Coulomb force is lower than that calculated by the previous expression (Eq 16).

Indeed, this $W_{COULOMB}$ energy is maximized for at least three reasons because:

1/ The previous formula Eq16 presupposes that all the points along the length of the piezoelectric bridge move on a distance $z_2 - z_1$ and in parallel (Fig 8) . This consideration is wrong because the piezoelectric bridge is recessed at both ends.

The ends of this bridge do not move at all, only the points in $x = l_p/2$ move on a distance z_0-z_2 and those between the ends of bridge and the middle move on a distance shorter than z_0-z_2.
In fact (Appendix and Fig 33), for a bridge recessed in its two extremities and subjected to a force F_{CA} in its middle, we know that the form z(x) of this bridge follows the law for 0 < x < lp/2 (Appendix X Eq 21 and Fig 33)

$$z = \frac{F_{CA}\, x^2 \left(l_P - \frac{4x}{3} \right)}{16\, E_P\, I_P} \qquad \text{(Eq 21)}$$

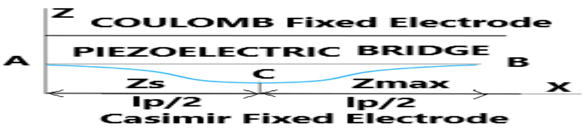


fig 20

The maximum $z_{max} = z_e$ is in x= $l_P/2$ and is $z_{max} = \frac{F_{CA}\, l_P^3}{192\, E_P I_P}$ (Eq 22)

So, the calculation in equation 23 assuming that the piezoelectric bridge is completely free to move parallel to the Coulomb electrode is wrong and maximize the energy $E_{COULOMB}$.

2/ Piezoelectric grid electrode and Coulomb electrode are not parallel. So, the Coulomb forces which appears at each point depend on the longitudinal position considered along the bridge and is of the form $F_{CO}(x,z)$ and $F_{CA}(x,z)$!

3/ We did not consider these first two reasons because the fact that the Coulomb force was of greater intensity than the Casimir force but especially of a very short duration, was preponderant.

This duration depends on the stiffness and speed of the switching of the MOS transistors in depletion and enriched.

An estimate of this duration is not simple because it depends on the technology used to produce these electronic components, but this time should be on the order of some nanoseconds.

As the Coulombs force F_{CO} only exists for a very short time, its power can be significant, but its energy remains low and does not exceed that of Casimir (following figure 21)!

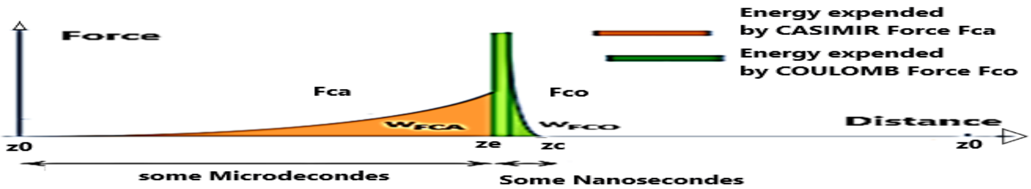


Fig 21 Comparison between Coulomb's and Casimir's energy

For these 3 reasons, the energy expended by the Coulomb force is lower than the simple equation 16 suggests and lower than the Casimir energy which induced it. There is of course conservation of energy.

Note that the $W_{COULOMB}$ approximate expression of equation 16 depends on the term z_r and decreases according to an increase in z_r with a power of z_r^{-1} . We can therefore adapt the interval z_r to minimize the energy $W_{COULOMB}$.

For example, WE OBTAIN AT EACH VIBRATION of the mobile structure,

For an interface z_0 = 200 A, $z_r = z_o$ =200 A°, dimensions of the Casimir electrodes (length = 500 µm, width = 15 µm, thickness = 10 µm), dimensions of the piezoelectric bridge in PMN -PT (length = 50µm, width = 15µm, thickness = 10 µm), a proportionality factor $p = F_{CO} / F_{CA} = 1000$, an inductance $L_{IN} = 1.10^{-6}$ H:

• $z_2 = 9.46\ 10^{-09}$ (m) i.e., a displacement of the mobile Casimir electrode of about 105A° (fig 16)

• $E_{CASIMIR} = \int_{z_0}^{z_e} F_{CA}\,dz$ = $3.4\ 10^{-11}$ (Joule) = Energy of vacuum = Energy dispensed by the force of Casimir just *to just to move* from z_0 to z_2

• $W_{DEFCA} = 5.25\ 10^{-11}$(Joule) = deformation energy of the piezoelectric bridge, embedded at both ends, to obtain a deflection of z2 at its center = energy produced by the Casimir force to deform the bridge and to give it this deflection z_2. This energy is greater than $E_{CASIMIR}$: $W_{DEFCA} > E_{CASIMIR}$

• Peak current = $1.20\ 10^{-4}$ A (figure 12)

• Voltage peak across the inductance = 0.1V (Fig 13).

• Structure vibration frequency = 750 kHz

• Threshold Voltage of enriched MOSE = 3.25 V (Fig 14)

• $W_{BRIDGE} = 2.7\ 10^{-11}$ the potential energy accumulated in the piezoelectric bridge

• $W_{ELECTRIC} = 2.7\ 10^{-11}$ (Joule) = Usable energy associated with current and voltage peaks.

• Maximum power during one cycle of about 1 nano second during a repetitive signal of about 1µ second = P_{max}

$P_{max} = 2.7\ 10^{-11}\ J/10^{-9}\ s = 27$ mW

•Average power during the repetitive signal of about 1µ second of this signal of about 1 nano second is then Pmed with
P_{med} =27 mW/1000 = 27 µW

We see also in figure 7 p 5 that the maximum power required on the micro transformer is very few ie :

* 60 nW , at the beginning of the transformation of this periodic signal in a continuous signal of several volts

* 3 picoW, at the end of it, i.e. after about 5 milliseconds later

So, the power of the presented device seems sufficient to aliment this electronic, without any external electrical alimentation, of transformation of small repetitive signals into a continuous voltage of several volts

• $W_{CASIMIR} = W_{DEFCA} + E_{CASIMIR} + W_{BRIDGE} + \Delta\ Q_{vib}/2 = 5.25\ 10^{-11} + 3.4\ 10^{-11} + 2.7\ 10^{-11} + 7.8\ 10^{-14} = 11.421\ 10^{-11}$ (Joule).

• Approximative duration of one peak of current, $t_{peak} \approx 10^{-9}$ s

• Maximum power during one cycle of about 1 nano second during a repetitive signal of about 1 microsecond = $P_{max} = W_{ELECTRIC} / t_{peak}$

$P_{max} = 2.7\ 10^{-11}\ J/10^{-9}\ s = 27$ mW . We notice that this $W_{ELECTRIC}$ usable energy and the W_{BRIDGE} energy are lower than the Casimir energy.

This usable energy is not brought by an external source but is caused by the deformation of the piezoelectric bridge caused by the omnipresent and perpetual Casimir force, itself controlled by a Coulomb force of opposite direction.

•Average power during the repetitive signal of about 1µ second of this signal of about 1 nano second is then Pmed with $P_{med} = 27\ mW/1000 = 27\ \mu W$

• $\Delta\ Q_{vib}$ = heat transmitted by the vibrations of the piezoelectric bridge $=7.8\ 10^{-14}$ J is very small and negligible!

• We notice that $\Delta\ Q_{vib} + W_{ELECTRIC} < W_{CASIMIR}$

• $W_{COULOMB} + W_{CIN} = W_{DEFCA} - (\Delta\ Q_{vib} + W_{ELECTRIC}) = 5.22\ 10^{-11} - (7.8\ 10^{-14} + 2.7\ 10^{-11}) = 2.51\ 10^{-11}$

We observed that in the referential of our 4 dimensions Space-Time plus the Quantic Vacuum the energy is conserved which is consistent with Noether's theorem and a perpetual small vibration seems exist!

4/4: SIMPLE REMARK and RESUME.

Remember that energy is defined as the "physical quantity that is conserved during any transformation of an isolated system"!

However, the system constituted by simply the MEMS device in space is not an isolated system while the system constituted by the MEMS device plus the space plus the energy vacuum seems an isolated system.

The part of the MEMS energy sensor vibrates at frequencies depending on the size of the structure and operating conditions, but with an amplitude of just a few tens of Angstroms.

These vibrations should not be confused with an impossible perpetual motion, as the system can be continuously powered by the vacuum energy responsible for the Casimir force.

5 / TECHNOLOGY OF REALIZATION OF THE CURRENT EXTRACTOR DEVICE USING THE FORCES OF CASIMIR IN A VACUUM

5/1: INTRODUCTION

For the structures presented above, the space between the two surfaces of the reflectors must be of the order of 200 A °, …. which is not technologically feasible by engraving!
Yet ***it seems possible,*** **to** *be able to obtain this parallel space of the order of 200 A ° between Casimir reflectors, not by etching layers but by making them thermally grow!*

Indeed, the S_{S3} and S_{S2} surfaces of the Casimir reflector must.

- be metallic to conduct the mobile charges.
- insulating as stipulated by the expression of Casimir's law who established for surfaces without charges.
- This should be possible if we grow an insulator on the z direction of the structure, for example Al_2O_3 or TiO_2 or other oxide metal which is previously deposited and in considering the differences in molar mass between the oxides and the original materials.

For example, silicon has a molar mass of 28 g/mol and silicon dioxide SiO2 of 60 g/mol. It is well known that when a silicon dioxide SiO2 grows by one unit, a silicon depth of about 28/60 = 46.6% (figure 22)

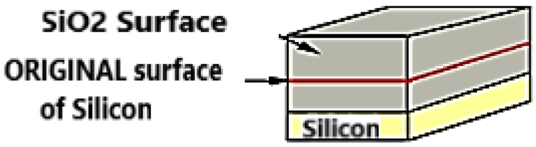


Figure 22: Growth of SiO2 oxide on silicon

This means that the fraction of oxide thickness "below" the initial surface is 46% of the total oxide thickness according to S.M. Sze. [9]

The same must happen, for example for thermal growth of alumina. The molecular masses of Alumina and aluminium are M_{Al2O3} = 102 g/mole and M_{Al} = 27 g/mole. We obtain an aluminium attack ratio of 27/102 = 26%, which implies that the original surface of this metal has shifted by 26% so that 74% of the alumina has grown out of the initial surface of the aluminium….

As regards the technological manufacture of electronics and structure, it therefore seems preferable:

1 / For electronics to choose Titanium Oxide because of its high relative permittivity ε_r =114 allowing to minimize the geometries required for the different capacities

2 / For the Casimir structure, the choice of aluminium, because its low density increases the resonant frequency of the structure and that 74% of the Alumina Al2O3 is outside the metal, allowing to reduce the interface between Casimir electrodes. A simple calculation shows for example that for aluminium gives:

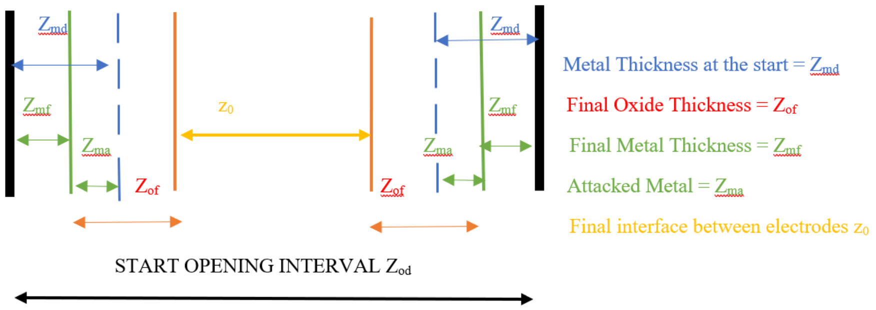


Figure 23 DISTRIBUTION OF THICKNESSES

$$z_{od} = 2\left(z_{md} + z_{of} - z_{ma}\right) + z_0 = 2\left(z_{md} + z_{of} \cdot (1 - 0.26)\right) + z_0$$
$$\Rightarrow z = z_{od} - 2\left(z_{md} + 0.74\, z_{of}\right)$$

For example, we start from an opening z_{od} = 3 µm. We deposit a metal layer of aluminium that is etched leaving a width z_{md} = 1 µm on each side of the reflector. Then an Alumina Al_2O_3 can grow, the thickness of which is precisely adjusted, simply by considerations of time, temperature, and pressure to increase a necessary thickness to have the desired interface z_0.!

For example, if z_0 = 200A °, z_{od} = 3 µm, z_{md} = 1 µm, then z_{of} = 0.662 µm. So, we obtain a Casimir interface of 200 A°. The final remaining metal thickness will be z_{mf} = 0.338 µm and will act as a conductor under the aluminium oxide.

Obviously, the growth of this metal oxide between the electrodes of the Casimir reflector modifies the composition of the dielectric present between these electrodes, therefore of the mean relative permittivity of the dielectric.

Let: ε_0 be the permittivity of vacuum and $\varepsilon_0 . \varepsilon_r$ the metal oxides one (ε_r = relative permittivity ≈ 8 in the case of Al_2O_3), z_{of} the final oxide thickness on one of the electrodes and z the thickness of the vacuum present between electrode, (initially we want $z = z_0$).

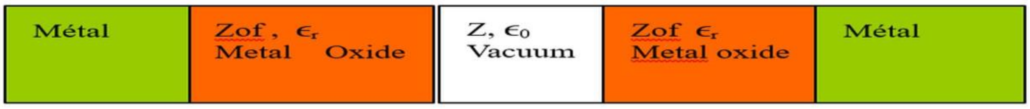


Fig 24: structure of Casimir's reflector or Coulomb's electrodes

Then the <u>average permittivity</u> e_{0m} of the dielectric is:

$$\varepsilon_{Om} = \frac{z_{qf}\,\varepsilon_0\,\varepsilon_r + z_0\,\varepsilon_0 + z_{qf}\,\varepsilon_0\,\varepsilon_r}{\left(2\,z_{qf} + z_0\right)} = \varepsilon_0\,\frac{\left(2\,z_{qf}\,\varepsilon_r + z_0\right)}{\left(2\,z_{qf} + z_0\right)} \simeq \varepsilon_0\,\varepsilon_r$$

, because z_0 is << $z_{of.}$

For example, z_{of} = 6620 A° is large compared to z <= 200A ° therefore $\varepsilon_{0m} \approx 8 * \varepsilon_0$ in the case of Al_2O_3.

<u>We have taken into account this change in permittivity in the preceding simulations.</u>

<u>5/2: STEPS FOR THE REALIZATION OF THE STRUCTURE AND ITS ELECTRONICS</u>

We use an SOI wafer with an intrinsic silicon layer: The realisation start with voltage "doubler" is obtained by using CMOS technology with 8 ion implantations on an SOI wafer to make :

1 / The sources, drains of the MOSNE, MOSND of the "doubler" and of the Coulomb force trigger circuits and of the grounding

2 / The source, drains of the MOSPE, MOSPD of the "doubler" and of the Coulomb force trigger circuits

3 / The best adjust the zero-threshold voltage of the MOSNE of the "doubler" circuit

4 / The best adjust the zero-threshold voltage of the MOSPE of the "doubler" circuit

5 / To define the threshold voltage of the MOSNE of the circuit n°1

6 / To define the threshold voltage of the MOSPE of the circuit n°1

7 / To define the threshold voltage of the MOSND of the circuit n°2

8 / to define the MOSPD threshold voltage of the circuit n°2

<u>This electronic done, we take care of the vibrating structure of CASIMIR.</u>

9 / engrave the S.O.I. silicon to the oxide to define the location of the Casimir structures (figure 25)

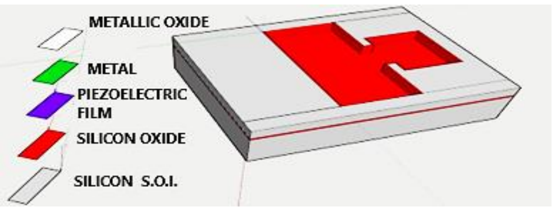


Figure 25:
9/ etching of S.O.I silicon

10/ Place and engrave a protective metal film on the rear faces of the S.O.I wafer (figure 26)

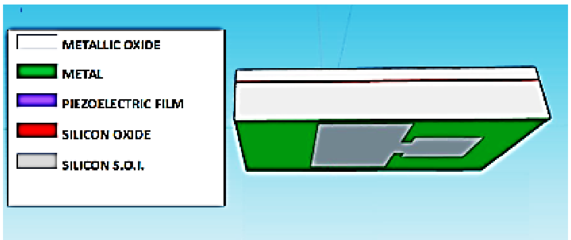


Figure 26:
10/ Engraving of the protective metal rear face of the S.O.I. silicon

11 / Deposit and engrave the piezoelectric layer (figure 27)

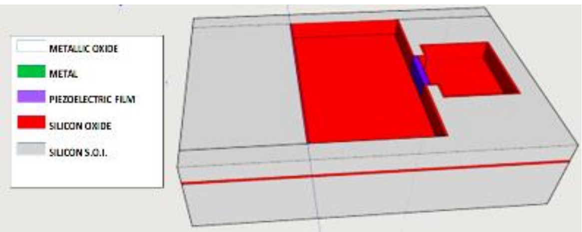


Figure 27:
11/deposition and etching of the piezoelectric layer e 61 deposition and etching of the piezoelectric layer

12/ Depose and etch the metal layer of aluminium (figure 28).

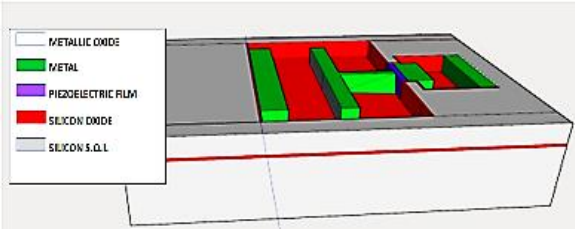


Figure 28:
12/ Metal deposit, Metal engraving etching of the piezoelectric layer

13 / Plasma etching on the rear side the silicon of the Bulk and the oxide of the S.O.I wafer protected by the metal film to free the Casimir structure then very finely clean both sides (figure 29)

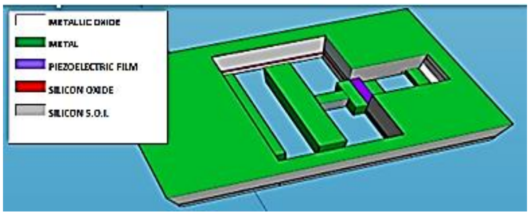


Figure 29:
13/ view of the Casimir device on the rear face, engraving on the rear face of the structures.

14 / <u>Place the structure in a hermetic integrated circuit support box and carry out all the bonding necessary for the structure to function.</u>

15 / Carry out the thermal growth of aluminium oxide Al_2O_3 with <u>*a measurement and control of the circuit under a box. The electronic circuit should generate a signal when the interface between the Casimir electrodes becomes weak enough for the device to vibrate*</u> ... and then stop the oxidation. (Figure 30)

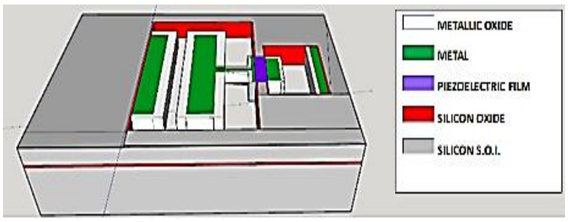


Figure 30:
14 /Adjusted growth of metal oxide under the electronic control, front view of the Casimir device

16 / Create a vacuum in the hermetic box
In the case where the 2 metal electrodes of Casimir, adhere to one another, they can be separated by the application of an electrical voltage on the Coulomb's electrodes!

In order to obtain a current peak greater in intensity, the Casimir cells can be positioned in a series and parallel network at the 2 terminals of a single inductance. For example, 20 Casimir cells can be placed in parallel and 10 in series. (Figure 31)

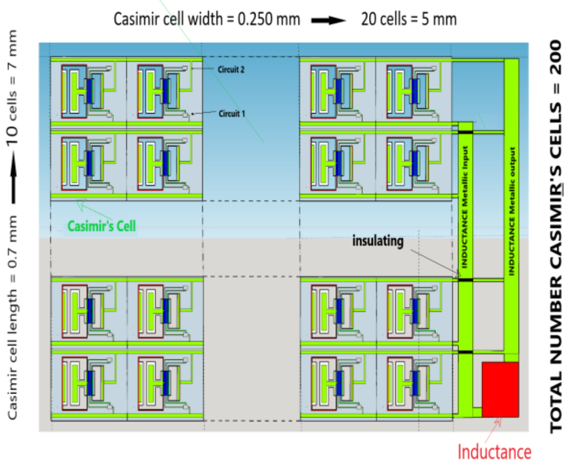
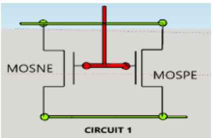
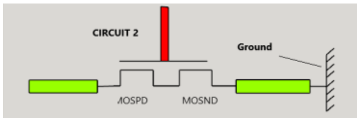


Figure 31: Positioning of 20 Casimir cells in parallel and 10 in series. **Circuit 1, Circuit 2 and Switches of circuit n°1 and n°2**

Total of Casimir cells delivering a periodic current during a small part of the vibration frequency of the devices = 200
Total des cellules = 200,
Width = 5mm, Length = 7 mm

<u>Important increase in current intensity, time duration of the peak, as for the voltage peak</u>





Circuit 1: parallel MOSPE and MOSNE; see Fig 4

Circuit 2: serial MOSPD and MOSND; see Fig 5

A system of 200 of the above structures gives a usable energy by second with a frequency of 750000 peaks. We calculate then that $W_{electric} \approx 60\ 10^{-3}$ (Joule) in one second for a coefficient of proportionality $p = F_{CO}/F_{CA} = 10^6$ and all switch transistors (Width = Length =100 µm, SiO_2 grid thickness = 250 A°), thresholds voltage = 3 V!

I am looking for a microelectronics laboratory with sufficient technological and design resources to confirm or deny this idea extraction of energy from vacuum dreamed by a retired dreamer.

In the universe, everything is energy, everything is vibration, from the infinitely small to the infinitely large" Albert Einstein."

A person who has never made mistakes has never tried to innovate." Albert Einstein

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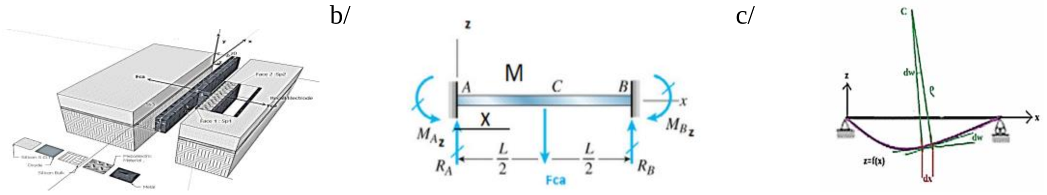
6/ APPENDICES A FEW REMINDERS FROM RDM

5.1 / Calculation of the deflection of a bridge recessed at its 2 ends

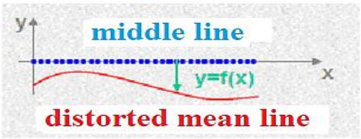
Note: We take the case of pure bending, the shear force T is such that With M the bending moment applied to the piezoelectric bridge. The Casimir force in the z axis is applied in lp / 2 at the centre of the bridge. (Figure 32)

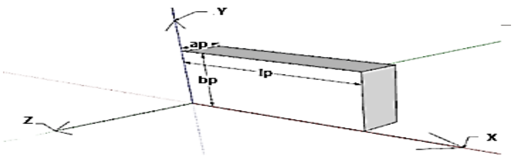
Figure 32: general appearance a/ b/ of the device studied, forces and applied moments, c/ of the deformed bridge

a/ b/ c/



As stated in all RDM books, the equation of the distorted mean line is: $\frac{d^2(f(x))}{dx^2} = -\frac{M_z(x)}{E_P I_P(x)}$ Eq (18)





We know that the radius of curvature ρ is

$$\frac{1}{\rho} = \frac{\frac{d^2 f(x)}{dx^2}}{\left(1 + \frac{df(x)}{dx}\right)^{\frac{3}{2}}} \approx \frac{d^2 f(x)}{dx^2} \approx \frac{d\varpi}{dx} = -\frac{M_z(x)}{E_P I_P(x)}$$

Eq (19)

Since the bridge is parallelepiped in shape, the bending moment of inertia along the z axis of the section of this bridge is: $I_{GSZ} = \frac{b_P \; a_P^3}{12} = Cte$ Eq (20) [10]

So: $$\frac{d^2(f(x))}{d(x^2)} = \frac{d\varpi}{dx} = \frac{M_z(x)}{E_P \; I_P(x)} \Rightarrow f(x) = -\iint \frac{M_z(x)}{E_P \; I_P(x)} d(x) = -12 \iint \frac{M_z(x)}{E_P \; b_P \; a_P^3} d(x)$$

In the case of a <u>beam recessed at both ends</u>, we have a hyperstatic system.

<u>However, we know</u> (See works on Resistance of Materials) that at equilibrium, the sum at all points of the forces and bending moments is zero. Because of the symmetry of the system, we therefore have $R_{AZ} = R_{BZ}$ and $M_{BZ} = M_{Az}$ and the computational reasoning for the deformation equation is identical for $0 \le x \le lp / 2$ or $lp2 \le x \le lp$ so.

For the forces and reactions:_ $\overrightarrow{R_A} + \overrightarrow{R_A} + \overrightarrow{F_{CA}} = \overrightarrow{0} \Rightarrow +F_{CA} = 0 \Rightarrow R_{AZ} = R_{BZ} = \frac{F_{CA}}{2}$

And for bending Moments:

M_x = the bending moment at a point $x < lp / 2$

M_{AZ} = the bending moment in A

F_{CA} = the force of Casimir applied in lp / 2 see (9)

Mz (x) = Bending moment depending on the position x on the bridge

I_P (x) the bending moment of inertia of the bridge section

$$M_z(x) = -E_P I_P \frac{d^2 z}{dx^2} = \frac{F_{CA}}{2} x - M_{AZ} \Rightarrow E_P I_P \; z(x) = -\left(\frac{F_{CA}}{12} x^3 - M_{AZ} \frac{x^2}{2} + C_1 x + C_2\right)$$

$$So\ FOR\ 0 \le x \le \frac{l_P}{2} \Rightarrow z(x) = -\frac{\left(\frac{F_{CA}}{12} x^3 - M_{AZ} \frac{x^2}{2} + C_1 x + C_2\right)}{E_P I_P}$$

However, we have the boundary conditions which impose:

$$In\ x = 0 \Rightarrow z(0)\ and\ \left(dz/dx\right)_{x=0} = 0 \Rightarrow C_{(M)1} = C_2 = 0$$

$$In\ x = \frac{l_P}{2} \Rightarrow \left(\frac{dz}{dx}\right)_{x=\frac{l_P}{2}} = 0 \Rightarrow (M_{AZ})_{x=\frac{l_P}{2}} = -\frac{F_{CA} l_P}{8}$$

$$\Rightarrow (M_{AZ})_{x=0} = -\frac{F_{CA} \; l_P}{8}$$

$$\Rightarrow z(x) = -\frac{\frac{F_{CA} \; l_P}{12} x^3 - \frac{F_{CA} \; l_P}{16} x^2}{E_P I_P} = \frac{F_{CA} \; l_P \; x^2}{16 \, E_P \; I_P} \left(l_P - \frac{4x}{3}\right) for\ 0 \le x \le l_P/2$$ Eq. (21)

The maximum deflection in x = lp / 2 gives an arrow : $z_{max} = \frac{F_{CA} \; L_P^3}{192 \, E_P \, I_P}$ for $x = \frac{L_P}{2}$ Eq(22)

d

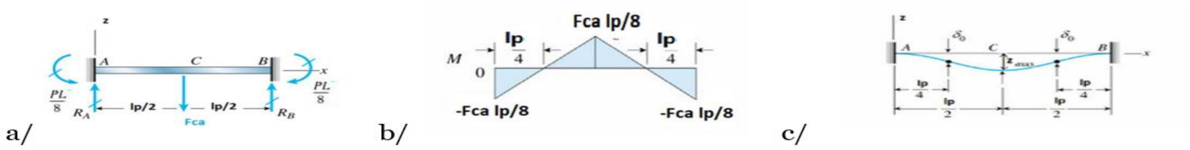


Figure 33: a / Forces, shear forces and Moments applied on the bridge. b / Variation of bending moment. c / Shape and arrow of the bridge recessed at both ends. With: ♎0 = inflection points, z_{max} = arrow of the bridge

6.2 / Calculation of the resonant frequency of the piezoelectric bridge

It is demonstrated (*see for example: Vibrations of continuous media Jean-Louis Guyader (Hermes*)) that the amplitude z (x, t) of the transverse displacement of a cross section of the beam is given by the partial differential equation , if one neglects the internal damping! $\frac{\delta^4 z}{\delta x^4} + \frac{\rho S}{E_P I_P}\frac{\delta^2 z}{\delta t^2} = 0$

With k = (❐S◆^{2}/ E_p I_p)$^{1/4}$,the solution of this differential equation is written in the general form:

Z (x) = A1 exp (k x) + A2 exp (-k x) + A3 exp (i k x) + A4 exp (i k x) and in the more convenient form:

$$z(x) = a\sin(kx) + b\cos(kx) + c\, sh(kx) + d\, ch(kx)$$

Keeping into account the boundaries conditions: $\left[sh(k\, l_P)^2 - \sin(k\, l_P)^2\right] - \left[ch(k\, l_P) - \cos(k\, l_P)\right]^2 \Rightarrow \cos(k\, l_P) = \frac{1}{ch(k\, l_P)}$

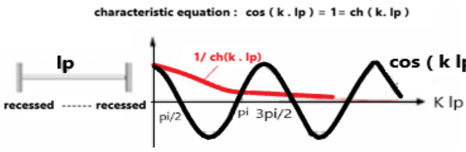


Figure 34: Numerical solution of equation 18

The numerical resolution (for example by the dichotomy method in figure 34 of this equation gives for the first 5 solutions:

a1 = 4.7300; a2 = 7.8532; a3 = 10.9956; a4 = 14.1317; a5 = 17.2787

So, the first resonant frequency of the piezoelectric bridge is:

$$\varpi_{P1} = (4.73)^2\sqrt{\frac{E_P I_P}{M_S\, l_P^{\,3}}} \Rightarrow f_{P1} = \frac{1}{2\pi}(4.73)^2\sqrt{\frac{E_P I_P}{M_S\, l_P^{\,3}}} \quad \text{Eq (23)}$$

$$M_{Stucture} = \rho_P\, a_P S_P + \rho_i\, a_i S_i + \rho_s\, a_s S_s$$

For example, for a bridge recessed at both ends with the following characteristics,

<u>For geometries</u>: lp=50 µm, bp= 150 µm, ap =10 µm; Sp=1,5 10^{-9} m^2, Ip= bp.ap^3/12= 1.25 10^{-20} m^4 ; li = 10 µm ; bi=150 µm ; ai=10 µm ; ls = 1000 µm; bs=150 µm; as=10 µm

<u>For the material:</u> PZT: density r = 7600 (kg/m^3), Young's modulus Ep = 6 * 10^{10} (Pa) (Kg m s^{-2})

<u>For the section inertia</u>: $I_p = I_{GSZ} = \frac{b_P\, a_P^{\,3}}{12} = 1.25\ 10^{-20}\ m^4$

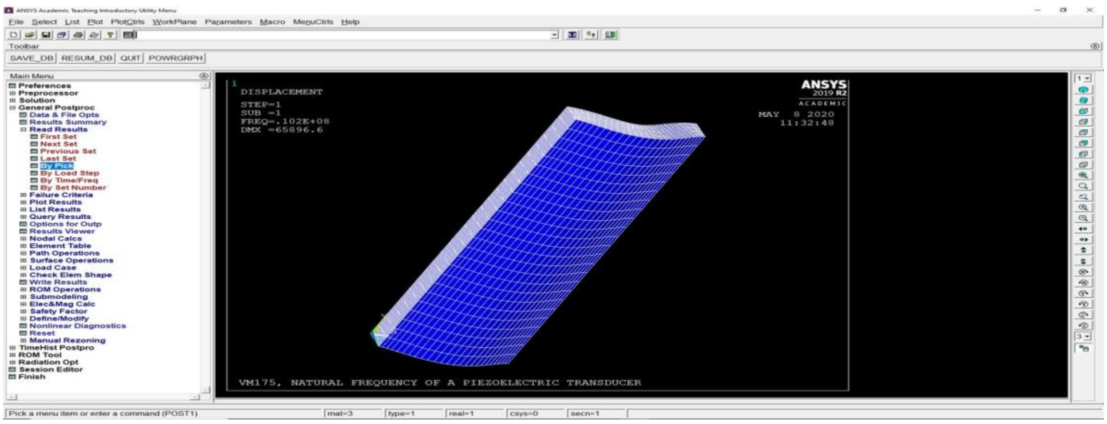


Figure 35: ANSYS simulation of the resonant frequency of the piezoelectric

Then the calculated first resonance frequency is for the PZT material: $f_{p1} = 1.1553 * 10^7$ hertz.

For this recessed bridge, an ANSYS simulation (figure 35) gives a resonant frequency of f1 = $1.02.\ 10^7$ Hz which is close to that calculated in the draft calculation presented in this report and validates the orders of magnitude obtained with the equations for these preliminary calculations.
If one carries out the calculation of the resonant frequency of the structure of figure 1 which comprises a free sole of Casimir S_{S2} parallel to a fixed surface S_{S3} and transmitting by a mechanical link finger the force of Casimir, one finds that the resonant frequency has the same form but with Ms a fixed mass applied in the middle of the bridge.

$M_{structure}$ = the total mass of the structure! $M_{Stucture} = \rho_S\, a_S S_S + \rho_i\, a_i S_i + \rho_P\, a_P S_P$

ρ_P, ρ_i , ρ_S , the medium density of the piezoelectric material, of the connecting finger, of the Casimir electrode sole and S_p, S_i, S_s the longitudinal surfaces of this bridge. Indeed, the presence of the Casimir sole connected by the Casimir force transmission finger in the middle of the piezoelectric bridge, modifies the resonant frequencies of this bridge.

The calculated resonance frequency then becomes for the same geometries and materials. $f_{s1}=2.509\ 10^6$ hertz. With these characteristics, an ANSYS simulation of this structure gives a close resonance frequency: $f_{s1} = 2.62\ 10^6$ Hertz.

This approach greatly simplifies these preliminary calculations because otherwise the curvature of the piezoelectric bridge makes the Casimir force strongly depend on the longitudinal and transverse positions x and z of the facing surfaces!