A new equation relating the Hubble parameter to redshift in the linear expansion universe model, R_h = ct

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Abstracts

Hubble's law has been the subject of study in recent years, the problem of the Hubble Tension with respect to the existence of various values of the parameter and the results of dark energy with respect to a change in the trend of the parameter value about 6.5 billion years ago have brought it into play. Through the study of the R_h=ct universe model and by going deeper into it, we have addressed these problems. This model of the linear expansion universe, R_h=ct, studied in recent years by Professor Fulvio Melia, is one of the best to respond to the new challenges posed by experimental research in astrophysics. For this reason, we have opted for this model to apply our hypothesis regarding the use of relativistic Doppler equations in calculating the speed at which galaxies move away from the observation point, the Earth. This has allowed us to develop, within the framework of this model of the universe, a new equation that relates the Hubble parameter with the red shift. The result, in light of their predictions, appears to be consistent with some of the issues raised by the Hubble Tension and Dark Energy.

Keywords = Hubble Parameter, Dark Energy, Hubble Tension, Astrophysics

1.- Development of our equation

The Hubble parameter turns out to be the quotient between the speed at which galaxies are moving away and their distances from the point of observation, i.e. the earth. To calculate the speed at which they are moving away due to the expansion of the universe, we use the data given by the Doppler effect or redshift of the emitted light, due to the movement of the emitting focus. This physical phenomenon can be treated in two ways, from non-relativistic mechanics and from relativistic mechanics. The researcher Samuel Meng [1], has shown that the non-relativistic treatment of this problem was insufficient to solve it correctly and so, in this work we have chosen the relativistic option and have developed, within the linear expansion universe model R_h = ct, an equation that relates the Hubble parameter with the redshift This equation is also valid for large velocities of the emitting focus, i.e. large redshift values.

To obtain the equation, we follow the following steps:

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Speed and non-relativistic Doppler effect:

In the non-relativistic case, the redshift "z" by Doppler effect is determined by the equation:

$$1+z = \frac{c+V}{c}$$

$$V = cz$$

Speed and relativistic Doppler effect:

In the relativistic case, the redshift by the Doppler effect is determined by the equation]:

$$V = \frac{(1+z)^2-1}{(1+z)^2+1} . c$$

According to [2], the linear expansion universe model, Rh = ct, presents the following equations:

$$H(t) = 1/t \qquad (1)$$

$$a(t) = t/t_0 \qquad (2)$$

$$(1+z) = t_0/t_e \qquad (3)$$

H(t) is the Hubble parameter.

a(t)) is the expansion factor of the universe.

t_0 is the time in which the photon was observed

t_e is the time in which the photon was emitted.

Hubble's law relates the speed at which galaxies move away from them with the distance to them through the value of the Hubble parameter, $\mathbf{H_{Hubble}}$.

It is stated as follows:

$$H_{Hubble} = \frac{SPEED}{DISTANCE}$$

We want to calculate the value of the Hubble parameter as a function of the redshift, we proceed as follows:

SPEED, non-relativistic case

$$SPEED = cz$$

SPEED Relativistic case

$$SPEED = \frac{(1+z)^2-1}{(1+z)^2+1} . c$$

DISTANCE traveled by a photon that leaves the galaxy at the instant "t_e" and is observed on Earth at the instant "t_0"-

$$\text{DISTANCE} = c\,(t_0 - t_e)$$

According to equations (1), (2), (3):

$$\text{Distance i} = c(t_o - t) = c(t_0 - t_0/(1+z)) = ct_0\,(z/(1+z)))$$

Hubble's parameter, H(z), if we do not consider relativistic effects for the velocity of galaxies:

$$H_{Hubble} = \frac{SPEED}{\text{DISTANCE}} = \frac{(1+z)}{t_0} = H_0(1+z)$$

Hubble's parameter, H(z), case of considering relativistic effects for the velocity of galaxies:

$$\text{SPEED} = V = c.\,\frac{(1+z)^2-1}{(1+z)^2+1}$$

$$H_{Hubble} = \frac{\text{SPEED}}{DISTANCE} = H_0(1+z).\,\frac{V}{cz}$$

Table 1 shows the calculations made with these formulas for the following values $0 \leq z \leq 11$

Table 1. Values of H(z), z and times obtained

z	H(z), relativistic equation	H(z) = Ho(1+z), non-relativistic equation	chronology
	Km/s per Mpc	Km/s per Mpc	Million years
0	65,0	65	14359
0,05	66,5	68,25	14359
0,1	67,9	71,5	13706
0,15	69,2	74,75	13110
0,2	70,3	78	12564
0,25	71,3	81,25	12062
0,5	75,0	97,5	10051
0,75	77,0	113,75	8615
1	78,0	130	7538
1,25	78,4	146,25	6701
1,5	78,4	162,5	6031
1,75	78,3	178,75	5483
2	78,0	195	5026
2,25	77,6	211,25	4639
2,5	77,3	227,5	4308
2,75	76,9	243,75	4021
3	76,5	260	3769
3,25	76,1	276,25	3548
3,5	75,7	292,5	3350
3,75	75,3	308,75	3174
4	75,0	325	3015
4,25	74,7	341,25	2872
4,5	74,4	357,5	2741
4,75	74,1	373,75	2622

5	73,8	390	2513
5,25	73,5	406,25	2412
5,5	73,3	422,5	2320
5,75	73,0	438,75	2234
6	72,8	455	2154
6,25	72,6	471,25	2080
6,5	72,4	487,5	2010
6,75	72,2	503,75	1945
7	72,0	520	1885
7,25	71,8	536,25	1828
7,5	71,7	552,5	1774
7,75	71,5	568,75	1723
8	71,3	585	1675
8,25	71,2	601,25	1630
8,5	71,1	617,5	1587
8,75	70,9	633,75	1546
9	70,8	650	1508
9,25	70,7	666,25	1471
9,5	70,6	682,5	1436
9,75	70,4	698,75	1403
10	70,3	715	1371
10,25	70,2	731,25	1340
10,5	70,1	747,5	1311
10,75	70,0	763,75	1283
11	69,9	780	1256

2.- The Hubble parameter as a function of redshift

According to the numerical results obtained, the result is:

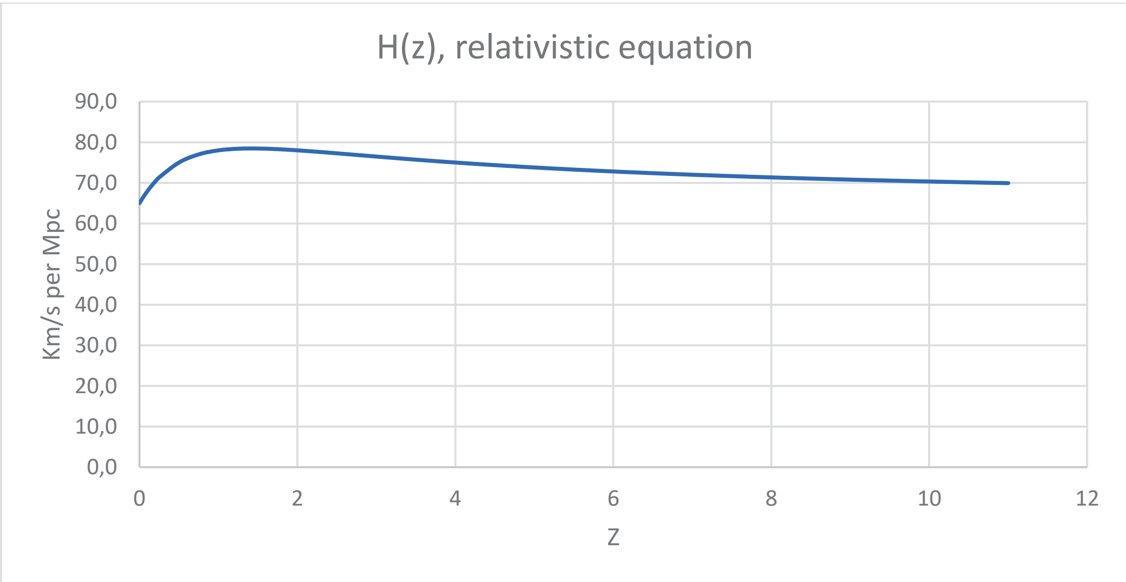

Fig. 1a, Hubble parameter as a function of redshift

$$H(z) = \frac{H_0(1+z).V}{cz}$$

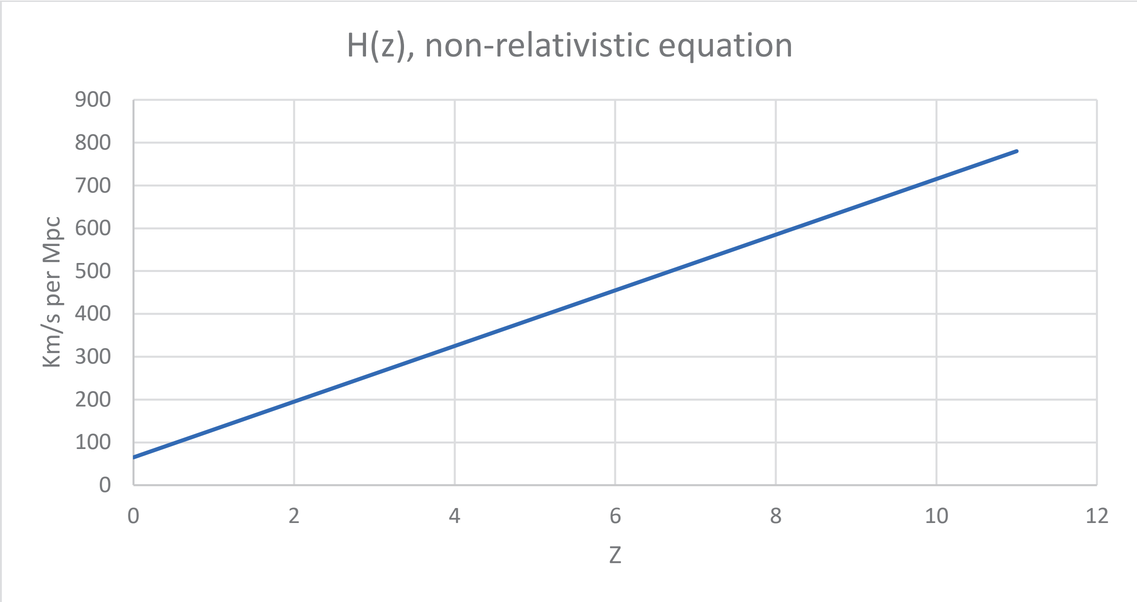

Fig. 1b, Hubble parameter as a function of redshift

H(z) = Ho(1+z)

value of H_0 = 65 km/s per Mpc. according to reference [3]

3.- The Hubble parameter as a function of the age of the universe

The linear expansion model we are studying [2] allows us to determine the times corresponding to the value of the redshift by solving this equation:

$$t = \frac{1}{H_o\,(1+z)}$$

In this way we have drawn the following graphs contained in figures 2a and 2b.

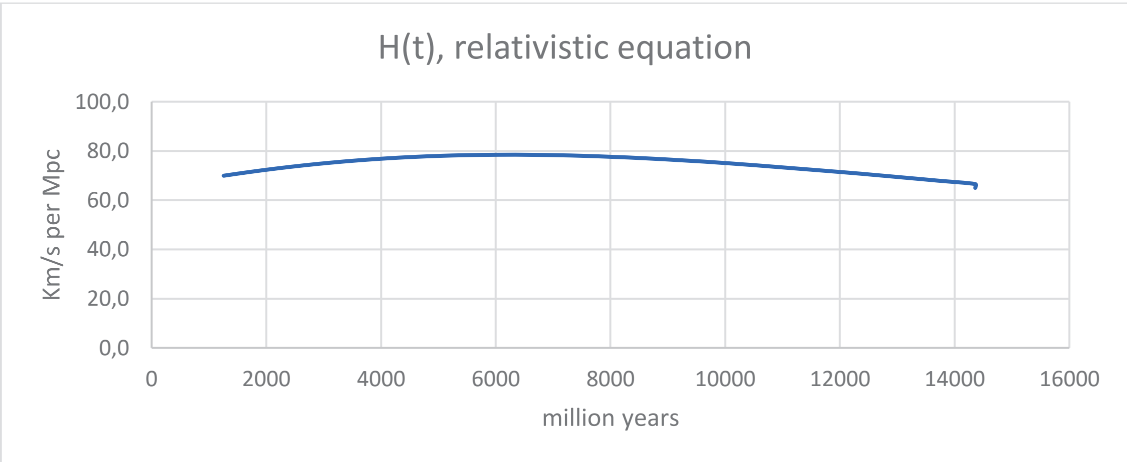

Fig. 2a. Hubble parameter as a function of the age of the universe

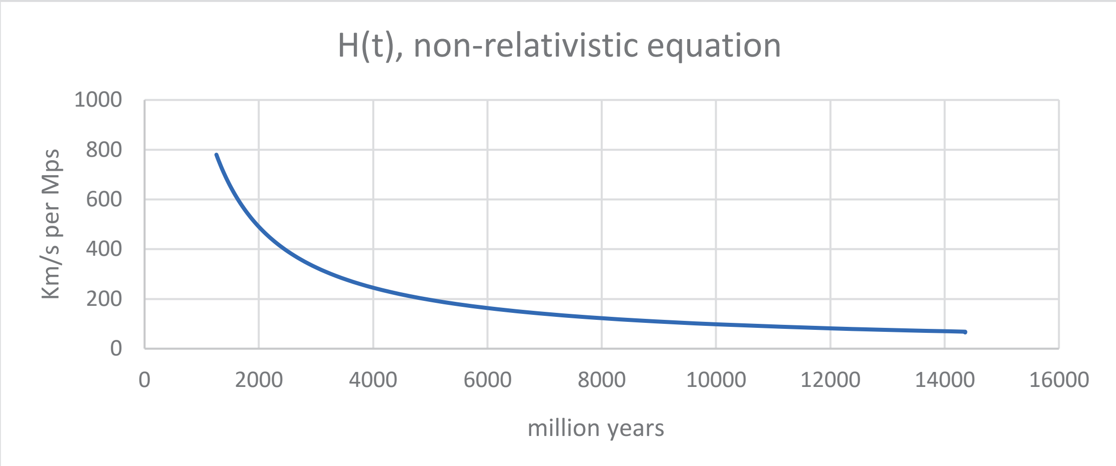

Fig. 2b. Hubble parameter as a function of the age of the universe

4.- Discussion

The first thing we notice from Figures 1a, 1b, 2a, 2b is the remarkable difference in the results of the Hubble parameter when applying the relativistic correction of the Doppler effect on velocity. The values of the Hubble parameter of the new equation are very different for z>1 from those obtained with the old equation, that is, the equation that does not apply the relativistic correction of the Doppler effect for velocity. This indicates the need to apply the correction in the calculations.

The parameter $\frac{V}{cz}$ is the multiplicative factor that differentiates the equations of the relativistic and non-relativistic Hubble parameter. Physically, this parameter is the quotient between the relativistic and non-relativistic velocities of the Galaxy we are observing.

We have studied a redshift range $0 \leq z \leq 11$, in which the values of the relativistic corrected Hubble parameter H(z) Fig 1a have remained within a 10% difference between them, a very narrow range compared to the values obtained by the non-relativistic equation, Fig 1b. This 10% range of values predicted by this equation is consistent with the most recent experimental data of the Hubble parameter measurement [3],[4],[5], and could be an explanation provided by this R_h = ct universe model for the Hubble Tension problem.

This model of the universe R_h=ct allows us to relate redshift to time through the equation:

$$t = \frac{1}{H_o(1+z)}$$

thus, we have established a chronology that has allowed us to relate the Hubble parameter to the different ages of the universe, Fig 2a and Fig 2b. Again, the results are very different in the relativistic and non-relativistic cases. It is worth noting the change in trend observed in the Hubble parameter in the relativistic case approximately 6.5 billion years ago, which could be related to the problem of dark energy. This model of the universe therefore reflects this change in trend of the Hubble parameter 6.5 billion years ago.

Thus, although the expected results are markedly different between the two approximations referring to the values of the Hubble parameter, the relativistic equation seems to respond better to large values of "z" according to the challenges of the Hubble Tension and Dark Energy that physics has posed today.

5.- Conclusion

We have carefully studied the Hubble parameter as a function of redshift. To do so, based on the linear expansion universe model, $R_h = ct$, and through a detailed study of the velocity of the galaxies we are observing, taking into account that their observation velocity may be high enough to require relativistic corrections, we have established an equation, taking into account these corrections, that relates the Hubble parameter H(z), first with the redshift and second with the age of the universe. We believe that the equations obtained improve those already existing in the $R_h = ct$ universe model, when we determine the value of the parameter for $z \geq 1$, the results being equivalent for small z.

Finally, we have used the results obtained to analyze the problem of the Hubble Tension and the Dark Energy, concluding that our results are consistent with the range of values predicted by our equation and those obtained in the latest experimental data [3], [4], [5]. As for the Dark Energy problem, we see that our equation predicts a change in the value of the Hubble parameter about 6.5 billion years ago, as has been observed experimentally.

For all these reasons, we believe that our equation does not invalidate the existing equations for this model if it is extended to higher values of "z".

6.- References

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