“Infinitely often”=“Infinity” — a statistics approach to small gaps between primes

Yung Zhao
Tianjin University
E-mail: yzhao@tju.edu.cn *

Abstract

We construct a sequence of consecutive primes. From the perspective of statistics, we analyze and handle them by the combination of the fundamental property of primes with James Maynard's result. It reveals that there are infinitely many pairs of primes which differ by 2.

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MSC 2010: 11A41, 62P05, 11N05

1. INTRODUCTION

One of the most famous problems in mathematics is Twin Prime Conjecture. Up to now, Y. Zhang [1], James Maynard, Terence Tao and dozens of mathematicians [2] have succeeded in making dramatic new progress. However, there is a key limitation inherent in standard sieve method. The conjecture remains unsolved and new ideas are needed for the final proof.

2. PROOF

Fundamental property of primes *Every prime greater than 3 must be of either the form “6K−1” or the form “6K+1” (K, integer ≥ 1)*

We construct and consider the following number sequence:

$$\{p_n,\ p_{n+1}\}$$

where p_n and p_{n+1} are consecutive primes, p_n is the n-th prime.

By **fundamental property of primes** , every p_n must be of either the form “$6K_1 - 1$” or the form “$6K_1 + 1$”, every p_{n+1} must be of either the form “$6K_2 - 1$” or the form “$6K_2 + 1$”.

The number sequence $\{p_n,\ p_{n+1}\}$ must be only four cases with different K_1 *and* K_2,:

- ① $\{p_n = 6K_1 - 1,\ p_{n+1} = 6K_2 - 1 \geq 6K_1 + 5\}$

$$(K_2 \neq K_1,\quad K_2 \geq K_1 + 1)$$

- ② $\{p_n = 6K_1 - 1,\ p_{n+1} = 6K_2 + 1 = 6K_1 + 1\}$

$$(K_2 = K_1)$$

- ③ $\{p_n = 6K_1 + 1,\ p_{n+1} = 6K_2 - 1 \geq 6K_1 + 5\}$

$$(K_2 \neq K_1, \quad K_2 \geq K_1 + 1)$$

- ④ $\{p_n = 6K_1 + 1,\ p_{n+1} = 6K_2 + 1 \geq 6K_1 + 7\}$

$$(K_2 \neq K_1, \quad K_2 \geq K_1 + 1)$$

In comparison, there is no difference between ③ and ④ . For every n and every p_n, $\{p_n,\ p_{n+1}\}$ must be one of the three cases: ① , ② and ③ . By the Statistical theory: as $n \to \infty$, each case $\to$ infinitely often, which means infinity. James Maynard has proved that there are infinitely many consecutive primes with a distance of 246 at most.

Hence,

$$(\ \textit{case}\ ①\ \textit{and}\ ③\ )\quad 2 < \liminf_{n\to\infty}(\ p_{n+1} - p_n\ ) \leq 246$$

$$(\ \textit{case}\ ②\ )\quad \liminf_{n\to\infty}(\ p_{n+1} - p_n\ ) = 2$$

Therefore, Twin Prime Conjecture is true.

References

[1] ZHANG Y., *Bounded gaps between primes.* Ann. of Math. 179 (2014), 1121–1174

[2] D. H. J. POLYMATH. *The "bounded gaps between primes" polymath projects—a retrospective.* arXiv:1409.8361, 2014.