

The Quark Model of the Electron and the Vacuum Fabric

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Abstract: We propose a quark model for the electron and the vacuum fabric, where the electron is a non-elementary, non-point like particle comprised of quarks, and the vacuum fabric is comprised of pion tetraquark tetrahedrons. We assume that electrons perform rapid quark flavor exchange reactions with the vacuum pion tetraquark tetrahedrons fabric, which is comprised of the valence quarks and antiquarks, u, d, \bar{u}, \bar{d} . Motion of the electron tetraquark tetrahedron on the vacuum pion tetraquark tetrahedron fabric is performed by a u and d quark flavor exchange reactions by tunneling through a double well potential barrier between the electron tetraquark tetrahedron and the vacuum pion tetraquark tetrahedron sites that transform the electron tetraquark tetrahedrons into a pion tetraquark tetrahedrons and vice versa. We assume that the quark flavor exchanges occur with the extremely high zitterbewegung frequency and hence a single electron cannot be observed since it is part of the electron and pion fabric cloud.

Keywords: QED, QCD, Antimatter, Quantum vacuum.

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1. Problems with the Quantum Electron Theory

C. G. Darwin wrote in 1927 that the width of a gaussian wave-packet representing an electron particle grows linearly with time in free space indicating that the wave-packet spreads rapidly¹. If an electron wave packet is initially localized in a region of an atomic dimension of 10^{-10} meters, the width of the wave-packet doubles in about 10^{-16} seconds, and after 1 milli-second the wave packet width grows to about a kilometer, which is not a reasonable description of the electron².

Dirac proposed a relativistic wave equation and found a new set of negative energy solutions he interpreted as antimatter states³. Dirac thought that since the positive electron solutions would decay to negative energy states there must be an infinite number of invisible electrons that occupy the negative states and prevent the electron decay according to Pauli exclusion principle⁴. Dirac assumed that electrons are not point like particles and proposed a spherical shell electron model with internal oscillations and self-energy⁵. Dirac thought that electrons interact with the vacuum electron-positron virtual pairs and are never bare in contrast to Feynman's QED approach⁶⁻⁷, where in zero-order bare electrons propagate in free space. Dirac thought that better understanding of the vacuum structure is needed for understand QED⁸.

2. The Questions We Address

The questions we address in this paper are:

1. Are electrons non-elementary, non-point like and not a single particle and are they made of quarks and antiquarks?
2. Is the quantum vacuum filled with massive pion tetraquark tetrahedron fabric comprised of 50% quarks and 50% antiquarks?
3. Does electron motion occur via rapid u and d quark flavor exchanges in a first chiral state, and \tilde{u} and \tilde{d} antiquark flavor exchanges in a second chiral state?

3. The Vacuum Pion Tetraquark Tetrahedron Fabric

We assume that the quantum vacuum is filled with pion tetrahedron tetraquark fabric⁹⁻¹³. We note that the vacuum pion tetrahedrons are not ordinary matter particles since they are composed of 50% antiquarks that according to the standard model annihilate the other 50% quarks, however, we assume that the pion tetraquark tetrahedrons condense and remain with a very small mass and internal rotation and vibration energy. We assume that in each site in the vacuum fabric there is a single tetraquark tetrahedron, $u\tilde{d}d\tilde{u}$, composed of two valence quarks, d and u , and their antiquark pairs, \tilde{d} and \tilde{u} . Two pion tetraquark tetrahedron enantiomers may exist obtained by exchanging the positions of two quarks at the tetrahedron vertices as shown below in line with Weyl massless chiral spinors and chiral symmetry of the QCD vacuum ground state¹⁴⁻²⁰.

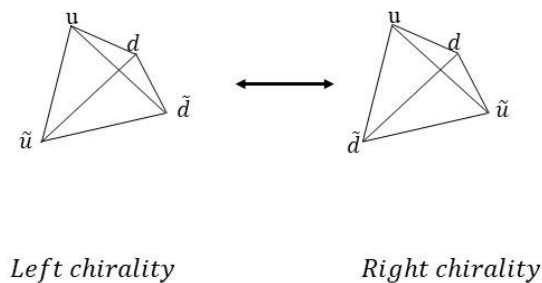


Figure 1 illustrates the two pion tetrahedron enantiomers, where the \tilde{u} and \tilde{d} antiquarks exchanged positions.

We assume that in the vicinity of a massive body, the pion tetraquark tetrahedron fabric may have higher density and a spherical symmetry according to the gravitational or electrical field. The size of the pion tetraquark tetrahedron may be less than a femtometer, while the pion tetrahedron fabric lattice length in outer space far from any massive body may be much larger, for example about Compton lengths, $\sim 10^{-13}$ meter. In extreme gravitational field, in the vicinity of a black hole for example, the pion tetraquark tetrahedron fabric cell size may become extremely small, and far away from any galaxy in the cosmic voids²¹⁻²³, the pion tetraquark tetrahedron vacuum fabric may be extremely diluted with cell size of tens or more Compton lengths.

4. The Electron and Pion Tetraquark Tetrahedron Fabric Double Well Potential Model

We assume that electrons are non-elementary, non-point like particles comprised of tetraquarks having two configurations, a right chiral, $\tilde{u} du\tilde{u}$, and a left chiral, $\tilde{u} dd\tilde{d}$ ⁹⁻¹³. Transforming an electron to a pion tetraquark tetrahedron on the vacuum fabric occurs by quark flavor exchanges between two fabric sites. A pion tetraquark tetrahedron is transformed by the quark flavor exchanges to an electron tetraquark tetrahedron and vice versa. The quark flavor exchanges reaction is symmetric where the reactants and products are identical and hence a double well potential model²⁴ is used below to represent the reaction like in the ammonia molecule inversion²⁵. Motion of the electron tetraquark tetrahedron on the pion tetrahedron fabric occurs via tunneling through the double well potential barrier that represents the potential barrier for exchanging the quark flavors via gluons exchanges between the vacuum fabric sites. The *u* and *d* quark flavors are exchanged as illustrated in figure 2 and equations 1 below for the electron left chiral state.

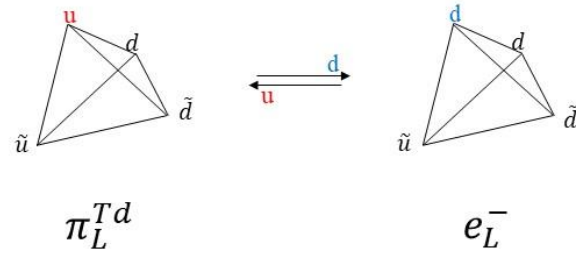


Figure 2 illustrates an electron tetraquark tetrahedron and a pion tetraquark tetrahedron exchanging quark flavors (**u** and **d**).

$$\tilde{u}d\tilde{d}u (\pi^{Td})_i + \tilde{u}d\tilde{d}d (e^L)_j \rightarrow \tilde{u}d\tilde{d}d (e^L)_i + \tilde{u}d\tilde{d}u (\pi^{Td})_j \quad (1)$$

In the case of the electron right chiral state, the \tilde{u} and \tilde{d} antiquarks exchange their flavors and the quark flavor exchange reaction equation is

$$\tilde{u}d\tilde{d}u (\pi^{Td})_i + \tilde{u}d\tilde{u}u (e^R)_j \rightarrow \tilde{u}d\tilde{u}u (e^R)_i + \tilde{u}d\tilde{d}u (\pi^{Td})_j \quad (2)$$

Note that the quark flavor exchange reactions are symmetric, e.g., the reactants on the left-hand-side and the products on the right-hand-side of the equation are identical. The double well potential Hamiltonian model is²⁴

$$\hat{H} = \frac{\hat{p}^2}{2m_e} + m_e \lambda (\hat{x}^2 - a^2)^2 \quad (3)$$

Where m_e is the electron rest mass, $2a$ is the distance between pion tetrahedron sites and the double well potential parameter λ determines the potential barrier height, $V_0 = m_e \lambda a^4$. We assume that the potential barrier height $V_0 = \hbar\omega = 2m_e c^2$, where the frequency $\omega = \frac{2m_e c^2}{\hbar}$ is Dirac's equation free space trembling motion zitterbewegung²⁵⁻²⁶.

Figure 3 below illustrates the double well potential model for the electron tetraquark tetrahedron and the pion tetraquark tetrahedron quark flavor exchange reaction in adjacent lattice sites i and j in the ground state. We assume that the electron motion on the vacuum fabric is via the quark flavor exchange reactions by quantum tunneling of gluons through the potential barrier V_0 where the double well potential exists between all adjacent lattice sites in the vacuum fabric. $V_0 = 2m_e c^2$ is twice the electron rest mass energy and is the threshold for electron-positron pair production. Note that the electron tetraquarks on both sides of the double well is identical and hence the electron chiral configuration (a $\tilde{u} d d \tilde{d}$ or a $\tilde{u} d u \tilde{u}$) is conserved, the two electron tetraquark tetrahedron chiral states are not mixed by the rapid quark flavor exchanges.

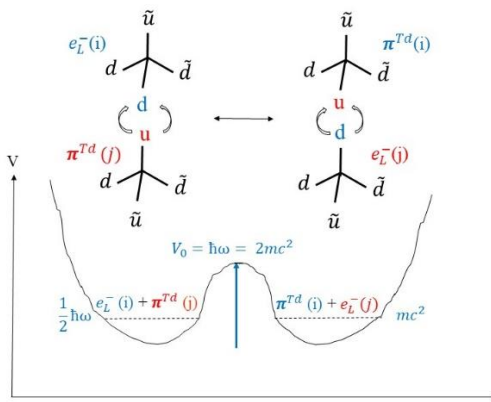


Figure 3 illustrates the double well potential model for the electron tetraquark tetrahedron and pion tetraquark tetrahedron quark flavor exchange reactions via gluons in adjacent lattice sites i and j .

Figure 4 below illustrates the double well potential model with the potential well parameters given above, $V_0 = \hbar\omega = 2m_e c^2$ and $a = \frac{\hbar}{m_e c}$.

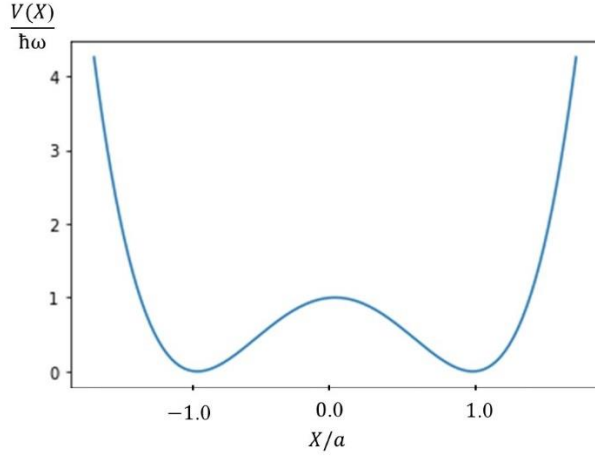


Figure 4 illustrates the double well potential model for the electron tetraquark tetrahedron and pion tetraquark tetrahedron quark flavor exchange reactions via gluons in adjacent lattice sites i and j .

The symmetric ground state and the antisymmetric first excited state energies and wavefunctions are calculated numerically by diagonalizing the Hamiltonian (Eq. 3) using a Fourier basis set. The tunneling time, $T_{tunneling}$, from the left to the right potential well is an inverse function of the energy split between the first anti-symmetric state E_a and the symmetric ground state E_s . With the parameters above, $E_a = 1.0463 \hbar\omega$ is just above the potential well barrier and $E_s = 0.7004 \hbar\omega$ is in the well. The tunneling time is extremely fast, $5.849 * 10^{-21}$ seconds.

$$T_{tunneling} = \frac{\pi\hbar}{E_a - E_s} = 5.849 * 10^{-21} \text{ seconds} \quad (4)$$

In order to describe an electron located in the left well initially, a superposition of the symmetric and antisymmetric eigenstates is taken as the initial state $\psi_{t=0}$.

$$\psi_{t=0} = \frac{1}{\sqrt{2}} (\psi_s + j \psi_a) \quad (5)$$

and after propagating the wavepacket a half period time using the analytical solution for the time-dependent Schrödinger equation

$$\psi_t = \frac{1}{\sqrt{2}} (\psi_s e^{-iE_s t} + j \psi_a e^{-iE_a t}) \quad (6)$$

the electron wavepacket tunneled to the right well. The electron wavepacket will continue oscillating between the two wells with a period of $T_p = \frac{2\pi\hbar}{E_a - E_s}$.

The electron velocity is calculated by dividing the distance between the wells, $2a$, by the tunneling time.

$$v_e = \frac{2a}{T_{tunneling}} = \frac{2a (E_a - E_s)}{\pi\hbar} = 0.44 c \left[\frac{m}{sec} \right] \quad (7)$$

Note that the electron velocity is about half of the speed of light, on the time scale of the free space trembling motion, the zitterbewegung, and electron semi-classical models²⁶⁻²⁷. The extremely fast electron wavepacket dynamics in the vacuum fabric may be observed in the future with attosecond electron microscopy²⁸.

We propose that the electron cloud is an extremely dense pion tetraquark tetrahedron fabric sphere, where in the center of the sphere the double well potential model length a may be as extremely small below the Compton length. Away from the cloud center, the distance between pion tetraquark tetrahedron increases and the pion tetraquark tetrahedron density is reduced. After about few Compton lengths, the distance between pion tetraquark tetrahedron is such that the quark exchange reactions stop, the electron tunneling is exponentially decreased and the electron is trapped in the cloud by the lack of quark exchange reactions. The electron is confined by the pion tetraquark tetrahedron fabric.

The following table summarizes results with increasing distance between the two potential wells, $2a$, $4a$ and $6a$ keeping the potential barrier at the same value, $V_0 = 2m_e c^2$, by changing the value of λ , $\lambda = \frac{2c^2}{a^4}$, $\lambda = \frac{2c^2}{16a^4}$ and $\lambda = \frac{2c^2}{81a^4}$.

Distance between the two wells	$E_a/\hbar\omega$	$E_s/\hbar\omega$	$T_{tunneling}(\text{sec})$
$2a$	1.0463	0.7004	5.849510^{-21}
$4a$	0.4741	0.4502	8.457710^{-20}
$6a$	0.318497	0.31698	1.34040^{-18}

The electron tunneling time between the two wells is reduced significantly with the growth of the distance between the wells. With the $6a$ distance the tunneling is about 229 times slower than with the smaller $2a$ distance (a is defined as the electron Compton length $\frac{\hbar}{m_e c}$).

Next, to simulate the electron and pion tetraquark tetrahedrons fabric cloud in one dimension, we duplicated the double well potential barrier and created a periodic potential with 10 wells and added a longer-range harmonic potential term, $\frac{\hbar\omega x^2}{L^2}$, that represents the longer length scale of the electron and pion tetraquarks cloud, L , as shown below with $L = 60a$.

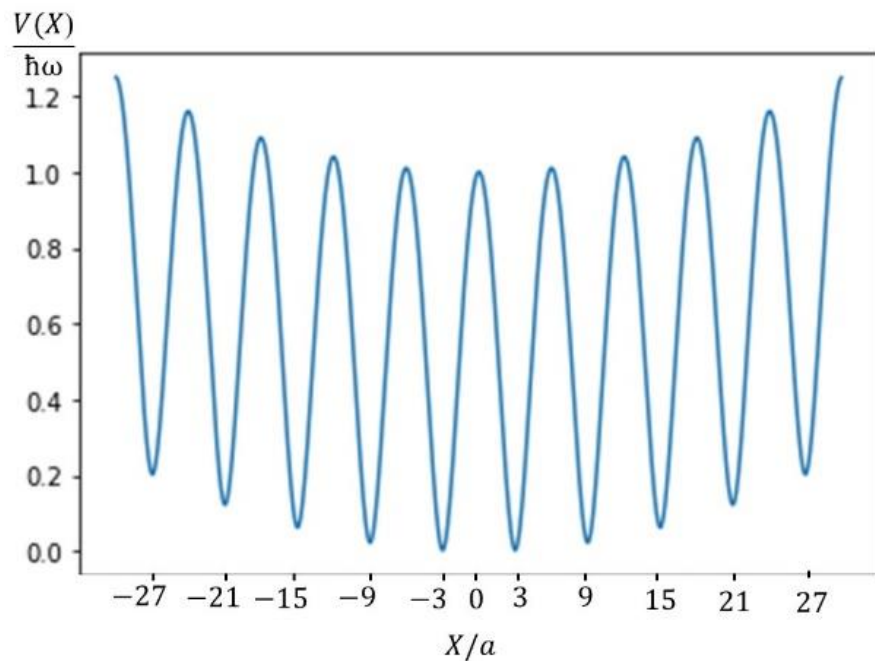


Figure 5 illustrates the electron and pion tetraquark tetrahedron quark cloud potential.

The two lowest symmetric, ψ_1 , and antisymmetric, ψ_2 , eigenfunctions are shown below.

The eigenfunctions peaks are localized in the fabric wells.

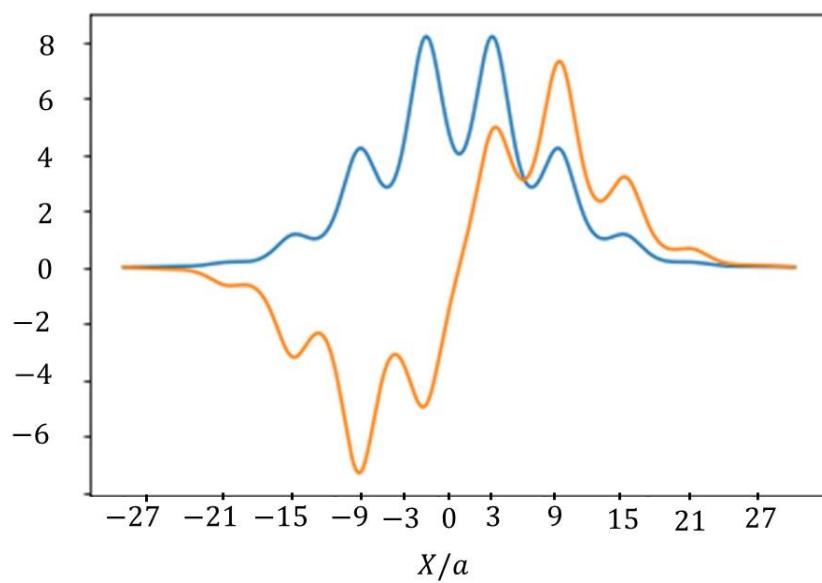


Figure 6 illustrates the first and second symmetric ψ_1 and antisymmetric ψ_2 eigenfunctions.

We form the initial wavepacket $\psi_{t=0} = \frac{1}{\sqrt{2}}(\psi_1 + j\psi_2)$ in orange below. The electron has high probability to be found in the first and second wells on the left initially. After half a period, the wavepacket tunnels to the right-hand side well (in blue) and the electron has high probability to be found in the first and second wells on the right-hand side.

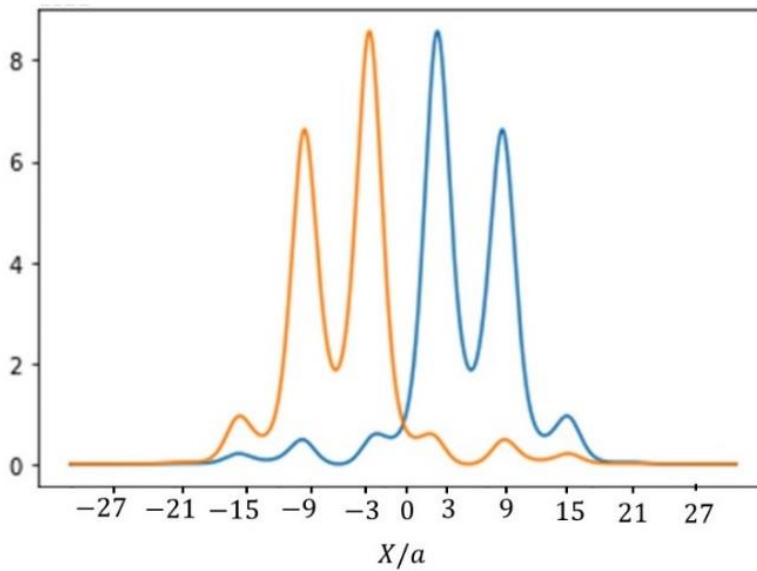


Figure 7 illustrates the electron wavepacket at $t=0$ (in orange) and after half time period (in blue).

The position expectation value of the electron wavepacket for 5 time periods is shown below.

$$X(t) = \langle \psi_t | \hat{X} | \psi_t \rangle \quad (8)$$

The electron oscillates between the first two left-hand side wells to the first two right-hand side wells.

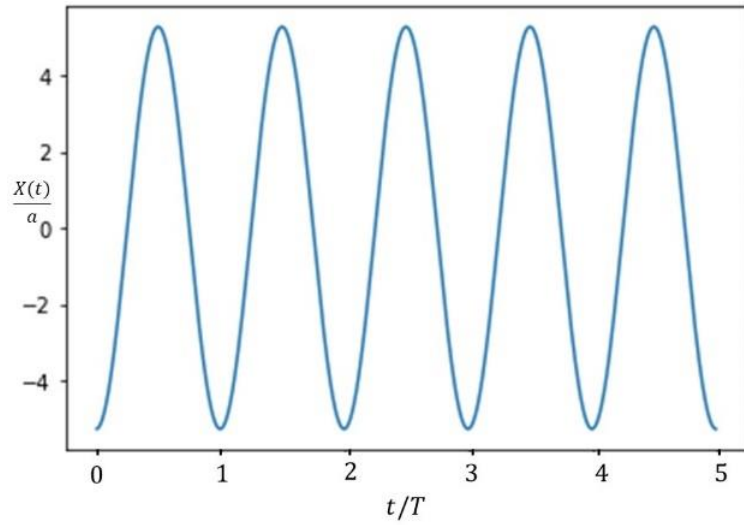


Figure 8 illustrates the position expectation value of the electron wavepacket for 5 time periods.

The two highest symmetric ψ_9 and antisymmetric ψ_{10} eigenstates are localized mainly in the outer wells as shown below.

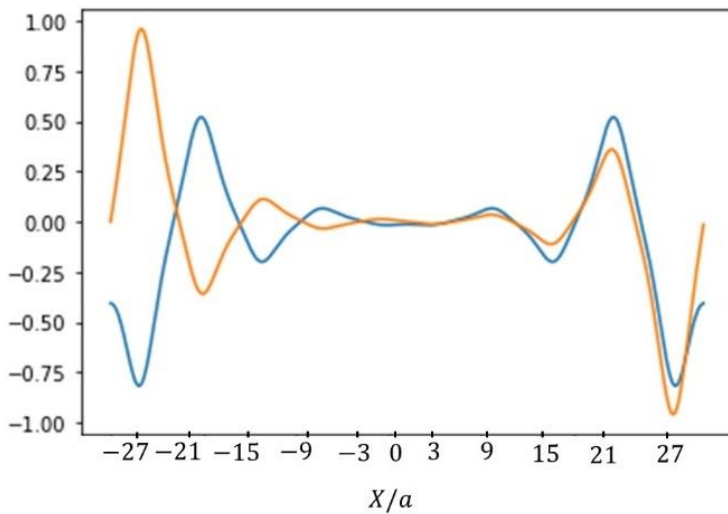


Figure 9 illustrates the 9th and 10th symmetric ψ_9 and antisymmetric ψ_{10} eigenfunctions.

The superposition, $\psi_{t=0} = \frac{1}{\sqrt{2}} (\psi_9 + j\psi_{10})$, is shown below where the tunneling occurs

between the outer wells. The wavepacket velocity is $v_e = \frac{L(E_{10}-E_9)}{\pi\hbar} = 0.211 c \left[\frac{m}{sec} \right]$.

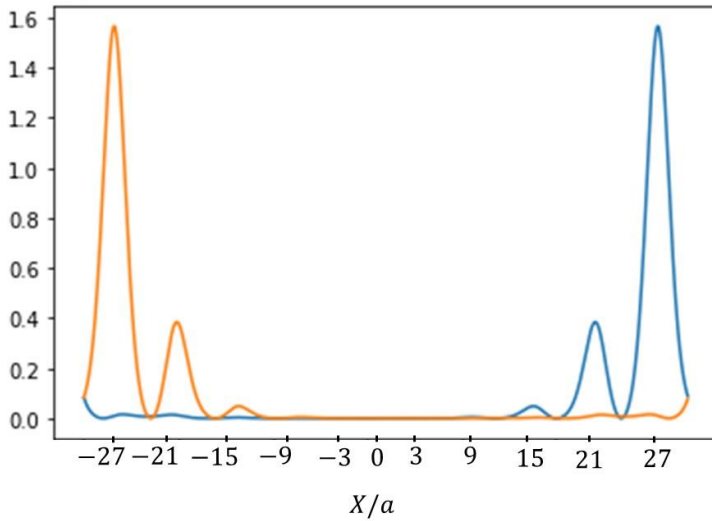


Figure 10 illustrates the electron wavepacket at $t=0$ (in orange) and after half time period (in blue).

The electron wavepacket position expectation value oscillates between the outer wells as shown below, from about $-27a$ to $+27a$.

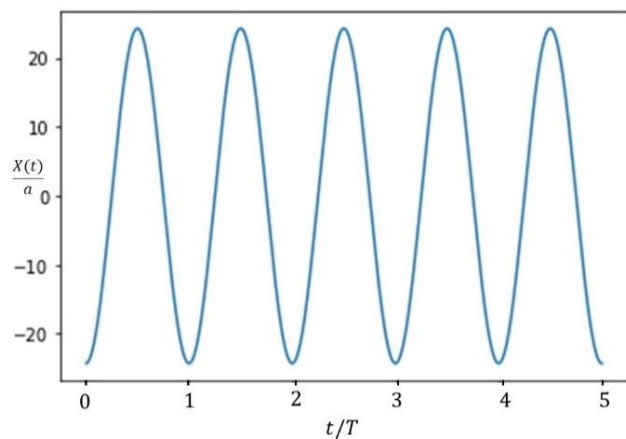


Figure 11 illustrates the X expectation value of the electron wavepacket for 5 time periods.

We assume that after the electron tunneling occurs and the electron tetraquark tetrahedron tunneled to the right-hand side well, the pion tetraquark tetrahedron fabric rearranges with the speed of light, and the potential shown in figure 5 induced by the new electron position is recreated around its new position. For a given configuration of the pion tetrahedron potential, we can adjust numerically the length a ($\sim 0.46 \frac{\hbar}{m_e c}$ for example) such that the calculated electron velocity according to equation 7 will be close to the speed of light. In this limit, the rearrangement of the pion tetrahedrons and the tunneling of the electron wavepacket are maximal and the electron speed will be maximal, but according to this model, it will not exceed c since the pion tetrahedrons fabric cannot rearrange faster than c .

Schrödinger found that a Gaussian wavepacket formed by a linear combination of plane waves gets wider linearly with time in free space and the electron becomes delocalized in contrast to the case of a harmonic oscillator potential, where the wavepacket remains coherent as expected¹. We show above that if the electron wavepacket is formed by a superposition of eigenfunctions formed by the vacuum pion tetraquark tetrahedron fabric, the electron wavepacket remains coherent and localized in the electron and pion fabric cloud, where the electron wavepacket is tunneling from site to site inside the cloud. The electron wavepacket simulations does not prove the underlying quark based electron and pion tetraquark tetrahedron vacuum model, which can the axioms of the model, however, it shows that a coherent electron wavepacket is obtained with duplicated double wells that represent the vacuum fabric cloud model.

We further note that with the proposed quark model of the electron and the vacuum fabric, the electron is never bare. The electron wavepacket superposition represents the underlying rapid quark flavor exchange reaction of the electron with the vacuum pion tetraquark tetrahedrons similar to the motion of the nitrogen atom in the ammonia molecule inversion²⁵. The zero-order

free particle planewave-based propagator may not be the optimal starting point for quantum electron dynamics⁶⁻⁷.

5. The Positron Tetraquark Tetrahedron

The positrons tetraquark tetrahedrons have a positive charge of the u and \bar{d} quarks replacing the negative charge of the \bar{u} and d quarks of the electron tetraquark tetrahedrons as shown below in figures 6 (a-b) for the electrons on the left and for the positrons on the right in figures 6 (c-d). Two positron enantiomers, e_R^+ and e_L^+ , may exist with right and left chirality like the electron tetraquark tetrahedron enantiomers e_R^- and e_L^- . In the four cases, an exchange of two quark flavors, transform the electrons, or the positrons, to a pion tetraquark tetrahedron π^{Td} conserving charge and chiral state.

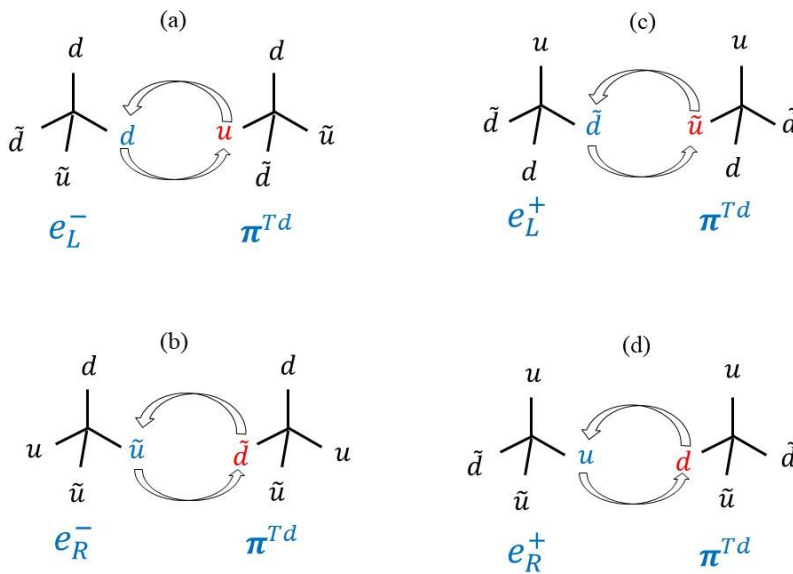


Figure 8 illustrates electron tetraquark tetrahedron enantiomers (a) and (b) and positron tetraquark tetrahedron enantiomers (c) and (d) exchanging quarks with pion tetraquark tetrahedrons with symmetric reactions such that the electrons and positrons transform to pion tetraquark tetrahedrons and vice versa conserving charge and chiral state.

6. Electron-Positron Annihilation on the Vacuum Fabric

Electron-positron tetraquarks tetrahedrons annihilation on the pion tetraquark tetrahedron fabric may occur by a collision of an electron tetraquark tetrahedron and a positron tetraquark tetrahedron on the quantum fabric that form two pion tetraquark tetrahedrons that become part of the quantum vacuum pion tetrahedron fabric as shown in equation 9.

$$\tilde{u}d\tilde{d}d(e_L^-) + u\tilde{d}\tilde{u}u(e_R^+) \rightarrow \tilde{u}du\tilde{d}(\pi^{Td}) + \tilde{d}d\tilde{u}u(\pi^{Td}) \quad (9)$$

Hence if an electron tetrahedron in site i on the fabric collides with a positron on adjacent site j, the outcome is that in both sites i and j after the collision there will be two pion tetraquark tetrahedrons, where the electron and positron charges and spins were annihilated. The extra energy of the electron and positron annihilation may be transferred to the vacuum pion tetraquark tetrahedron fabric as electromagnetic wave excitations. Note that in equation 9 the number and flavor of the quarks are conserved. The quarks are not destroyed or created in the proposed quark exchange reactions⁹⁻¹³.

7. Summary

We assume that the answers to the three questions raised in section 1 are positive and consider them as the axioms of the new quark model of the electron and the vacuum fabric. Accordingly, the electron is not an elementary, point like and not a single particle. The electron tetraquark tetrahedron is comprised of quarks and antiquarks, and it forms with the vacuum pion tetraquark tetrahedrons fabric a cloud. The quantum vacuum has a structure formed by massive pion tetraquark tetrahedrons fabric with varying density. The massive pion tetraquark tetrahedrons are made of 50% matter and 50% antimatter particles and hence the vacuum fabric is not made of regular matter. The electron motion occurs via *u* and *d* quark flavor exchange tunneling between electron tetraquark tetrahedrons and pion tetraquark tetrahedrons through a double well potential in a first electron chiral state, and the exchanges of \tilde{u} and \tilde{d} antiquark

flavors in the second electron chiral state. A conclusion of the proposed quark model is a new quark conservation law where quark number and flavors are conserved in quark exchange reactions like the conservation of atoms and electrons in molecular reactions⁹⁻¹³.

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