**Modeling Dark Matter Through the Effects of Relativistic Mass**

**John Hogarth**

**September 2024**

**Abstract**

Dark matter remains a conundrum within the field of cosmology. While the behavior is well-understood, the underlying cause remains mysterious. Many models have been proposed to explain the phenomenon, whether it be new particles or modifications to the law of gravity.

This document attempts to explain dark matter through the effects of relativistic mass and the effects of a relativistic gravitational field. The goal is that this model might be able to fit with observations of dark matter as well as Modified Newtonian Gravity (MOND) within galactic orbits.

**Model for Dark Matter from Relativistic Mass**

Due to the expansion of the universe, a particle moving in a circular orbit will see the length of its path of motion dilate in the following way over the course of an orbital period:

$$\frac{dx}{dt} = Hd \quad [\boldsymbol{Eqn.1}]$$

“H” is the known value of the Hubble’s constant, and “d” is the distance/length of the particle’s circular path (Pössel, 2017). Thus, for the radial component of the particle’s circular path, dilation is as follows:

$$\frac{1}{2\pi}\frac{dx}{dt} = \frac{Hd}{2\pi} = \frac{Hct}{2\pi} \quad [\boldsymbol{Eqn.2}]$$

The relativistic acceleration that will result along the radial component is therefore:

$$a_{expansion} = \frac{1}{2\pi}\frac{dx}{dt^2} = \frac{Hc}{2\pi} \quad [\boldsymbol{Eqn.3}]$$

This relativistic acceleration of spacetime can be added to the standard centripetal acceleration seen on a particle due to gravitational force.

$$a_c = \frac{\Delta x_0}{\Delta t^2} = \frac{v^2}{r} \quad [\boldsymbol{Eqn.4}]$$

$$a_c + a_{expansion} = \frac{\Delta x_{relativistic}}{\Delta t^2} \quad [\boldsymbol{Eqn.5}]$$

The change in acceleration due to relativistic effects can be used to find a relativistic dilation factor during orbit:

$$\gamma = \frac{\Delta x_{relativistic}}{\Delta x_0} = \frac{a_{expansion}}{a_c} + 1 \quad [\boldsymbol{Eqn.6}]$$

If one subscribes to the concept of relativistic mass, mass should therefore also be expected to dilate by this factor:

$$\gamma m_0 = m_{relativistic} = m_0(\frac{a_{expansion}}{a_c} + 1) \quad [\boldsymbol{Eqn. 7}]$$

While relativistic mass is a controversial topic in special relativity, there are researchers who defend it as holding validity (Sandin, 1991). With the goal of providing a possible explanation for dark matter, this document will assume relativistic increases in mass to be valid.

Moving on from equation 7, the expansion of the universe is an external influence from outside the closed system of the galaxy. Thus, the change in acceleration must be taken to be a change in the force seen by the object. The following force relation can be made:

$$F = m_0\left(a_c + a_{expansion}\right) = m_{relativistic}a_c \quad [\boldsymbol{Eqn. 8}]$$

This force relation can be reinforced using special relativity. In special relativity, relativistic centrifugal force seen by an object is known to take the following form (Das, 2021):

$$F = \gamma m_0 a_c = m_{relativistic}a_c \quad [\boldsymbol{Eqn. 9}]$$

Building from here, it can also be conjectured that the gravitational potential energy changes by the factor $\gamma$. This is due to its direct dependence on the potential energy within mass:

$$\frac{GMm}{r} \to \frac{GM\gamma m}{r} \quad [\boldsymbol{Eqn. 10}]$$

A commonplace energy relation to find circular orbital velocity is as follows:

$$E = \frac{1}{2}mv^2 = \frac{GMm}{2r} \quad [\boldsymbol{Eqn. 11}]$$

Relativistic expressions for kinetic energy (LibreTexts, 2021) and potential energy (equation 10) should yield the following modified form for orbital velocity:

$$E = (\gamma m - m)c^2 \approx \frac{GM\gamma m}{2r} \quad [\boldsymbol{Eqn. 12}]$$

However, observed orbital velocities in a galaxy are ~1% the speed of light (), which is far below relativistic speeds. The relativistic correction to kinetic energy will be approximately equal to the Newtonian form of kinetic energy:

$$KE = (\gamma m - m)c^2 \approx \frac{1}{2}mv^2 \quad [\boldsymbol{Eqn. 13}]$$

Plugging the Newtonian form of kinetic energy into equation 12 yields the following:

$$\frac{1}{2}mv^2 \approx \frac{GM\gamma m}{2r} \quad [\boldsymbol{Eqn. 14}]$$

Thus, the orbital velocity can be said to change as follows:

$$v = \sqrt{\frac{GM}{r}} \to v = \sqrt{\frac{GM\gamma}{r}} \quad [\boldsymbol{Eqn. 15}]$$

Having taken orbital mechanics into account in equation 15, the force of gravity seen on the particle mass by an outside observer (distinct from the force given by equation 9) should conform to the following expressions:

$$F = \frac{m_{relativistic} v^2}{r} = m_{relativistic} a_c \ \ [\boldsymbol{Eqn.16}]$$

$$F = \frac{GM\gamma m_{relativistic}}{r^2} = m_{relativistic} a_c \ \ [\boldsymbol{Eqn.17}]$$

$$\frac{GM_0\left(1 + \frac{a_{expansion}}{a_c}\right)}{r^2} = a_c \ \ [\boldsymbol{Eqn.18}]$$

$$\frac{GM_0}{r^2} = \frac{a_c}{1 + \frac{a_{expansion}}{a_c}} \ \ [\boldsymbol{Eqn.19}]$$

For $a_{expansion} \gg \ a_c$:

$$\frac{GM_0 a_{expansion}}{r^2} \approx a_c^2 \approx \frac{v^4}{r^2} \ \ [\boldsymbol{Eqn.20}]$$

$$GM_0 a_{expansion} \approx v^4 \ \ [\boldsymbol{Eqn.21}]$$

Equations 19-21 align with astronomical observations and equations outlined by the models of MOND (Scarpa, 2006). Since the effects of relativistic mass imply an increase in the observed mass of matter, this model also holds some agreement with the astronomical observations and equations outlined by models of dark matter (Garrett, Duda, 2011).

**References**


1. Pössel, M. (2017, December 29). The Expanding Universe: An Introduction. *arXiv.org*. https://arxiv.org/abs/1712.10315
2. Sandin, T. R. (1991). In defense of relativistic mass. *American Journal of Physics*, 59(11), 1032–1036. https://doi.org/10.1119/1.16642
3. 5. 10: Relativistic energy. (2021, August 11). *Physics LibreTexts*. Retrieved September 13, 2024, from https://phys.libretexts.org/Courses/Muhlenberg_College/MC%3A_Physics_121_-_General_Physics_I/05%3A__Relativity/5.10%3A_Relativistic_Energy
4. Das, D. For the Love of Physics. (2021, July 4). *Relativistic force* [Video]. YouTube. https://www.youtube.com/watch?v=fYfdIs7GMrI
5. Scarpa, R. (2006). Modified Newtonian Dynamics, an Introductory review. *AIP Conference Proceedings*. https://doi.org/10.1063/1.2189141
6. Garrett, K., & Duda, G. (2011). Dark Matter: A primer. *Advances in Astronomy*, 2011, 1–22. https://doi.org/10.1155/2011/968283