On the Bifurcation Structure of Particle Physics

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Abstract

It is nearly universally accepted that the Standard Model (SM) of particle physics, despite its remarkable predictive power, remains an *incomplete framework*. Among the many long-standing puzzles confronting SM, its flavor composition, the origin of three generations, the spectrum of particle masses and charges, and the chirality of electroweak interactions continue to resist explanation. In line with our previous investigations, the goal of this exploratory work is to further bridge the gap between the *universal behavior of nonlinear dynamics*, on the one hand, and the flavor composition and SM chirality, on the other.

Key words: Physics Beyond the Standard Model, nonlinear dynamics, bifurcations, far from equilibrium critical phenomena, three fermion generations, chiral symmetry breaking.

Cautionary remarks

We caution from the outset that the sole intent of this provisional report is to lay the groundwork for further exploration of the topic. Independent work is needed to develop, validate, or reject the hypotheses presented here. Readers unfamiliar with these topics are encouraged to carefully review the enclosed references prior to drawing premature conclusions.

1. The dynamic content of SM: a brief account

SM is a non-Abelian gauge field theory built from the symmetry group $SU(3)_c \times SU(2)_L \times U(1)$. This group has $8+3+1=12$ generators with a non-trivial commutator algebra and describes the electroweak coupling of leptons and quarks, as well as the strong interaction of quarks and gluons. SM contains an octet of gluons associated with the $SU(3)$ color generators, and a quartet of gauge bosons W^+, W^-, Z^0, and γ. Gluons and photons γ are massless because the symmetry induced by the other three generators is spontaneously broken through the *Higgs mechanism*. SM fermions are grouped into three replicated generations. The first generation contains left-handed $SU(2)$ doublets of leptons and quarks,

$$L=\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L;\quad Q_{L\alpha}=\begin{pmatrix}u_\alpha\\ d_\alpha\end{pmatrix}_L \tag{1}$$

and corresponding right-handed $SU(2)$ singlets

$$e_R^-;\ d_{R\alpha},\ u_{R\alpha} \tag{2}$$

where $\alpha=(r,g,b)$ labels the color index, namely *red*, *green* and *blue*. Here, the barred quantities are anti-color states so that,

$$\alpha=(r,g,b) \tag{3}$$

$$\overline{\alpha}=(\overline{r},\overline{g},\overline{b}) \tag{4}$$

Gluon and quark colors are $SU(3)$ charges complying with the overall *neutrality* (or colorless) condition written symbolically as

$$(\alpha,\overline{\alpha})\Rightarrow r\overline{r}+g\overline{g}+b\overline{b}=1 \tag{5}$$

Condition (5) reflects invariance under rotations in color space. Since leptons do not participate in strong interactions, they do not carry color charges. By the same token, since strong interactions are insensitive to chirality, gluons are neither left nor right-handed.

Neutrinos come in three flavors, they are either Dirac or Majorana fermions, undergo oscillations on account of their nonvanishing masses and violate the CP (charge-parity) symmetry. All experimentally observed neutrinos (ν) are known to be left-handed and antineutrinos $(\bar{\nu})$ right-handed.

2. Mean-field theory of dimensional fluctuations

By analogy with (1) – (2) and with reference to [1], we now introduce the notion of *dimensional condensate,* a pair of states defined on a spacetime whose continuous dimensions flow with the observation scale. The model considered in [1] acts on a two-dimensional lattice $(d=2)$, whose local variables are clusters of time-varying *dimensional fluctuations* $[\delta\varepsilon(t)=\delta(2-d(t))] << 1$ generated in the evolution of the primordial Universe.

A key observation here is that dimensional condensates are of *topological nature* and distinct from the SM doublets or singlets. In its most basic form, it can be shown that the mean-field theory of evolving dimensional fluctuations reduces to the logistic map [1],

$$x_{n+1} = r\,x_n(1 - x_n) \tag{6}$$

3. Universal bifurcations of low-dimensional maps

A key concept in the theory of nonlinear dynamical systems is the *universality* of transition to deterministic chaos. Universality implies, for example, that quadratic and cubic maps follow an identical bifurcation route to chaos described by the *period-doubling scenario*. For pedagogical reasons, we begin this section with a short review of the universality concept.

According to [2-3], iterated maps of the unit interval are generic models of dynamical systems in discrete time. The standard representation of these models is based on first order difference equations having the generic form

$$x_{n+1} = f(x_n, r) \tag{7}$$

with

$$r = \Delta\lambda = \lambda - \lambda_0 \tag{8}$$

$$r_c = \Delta\lambda_c = \lambda_c - \lambda_0 \tag{9}$$

Here, r measures departure of parameter λ from its critical point λ_c and λ_0 represents an arbitrary reference value.

The dynamics of iterated maps can be either *conservative* or *dissipative*. In the former case, the function (7) is monotonic and describes a one-to-one mapping, whereas in the latter case is non-monotonic and describes a two-to-one mapping. Typical examples of dissipative systems include one-dimensional non-invertible maps such as the logistic map. Feigenbaum discovered in 1978 that the onset of chaos in these maps occurs through *period-doubling bifurcations* driven by changes of the parameter r as in $r_1, r_2, \ldots, r_m$ [4 - 6].

Non-invertible maps exhibit the following behavior: for small values of r (7) has a single stable fixed point and all nearby points converge to it under multiple iterations of (7). Ramping up r to a critical value r_1 renders the fixed point unstable and produces a new stable pair of points of period 2. Further increasing r to another value r_2 bifurcates this cycle into a cycle of period 4. The bifurcation process continues with a new sequence of cycles of period 2^j, $j \geq 3$, eventually leading to a Cantor set structure that attracts almost all the points of the interval $[-1,1]$. On letting r increase beyond an endpoint value r_∞, stable periodic orbits surface again and split up in a similar way. In the new sequence, r scans another series of critical values corresponding to cycles of period $3 \cdot 2^j$, $j = 0,1,2,...$ and so on.

When applied to the flavor composition of SM, period-doubling bifurcations hint to the content of Tab. 1. The first branch of the fermion sector represents the set of ***3 left-handed neutrinos*** emerging at $j = 0$, a finding which naturally accounts for triplication of fermion families.

It is instructive to note that, according to this scenario, the composition of Dark Matter appears to echo the properties of *anyons in 3D space* [9].

4. Working assumptions / conventions

A1) All dimensional condensates generated by bifurcations are invariant under the CPT (charge-parity-time reversal) symmetry of Quantum Field Theory.

A2) The cubic equation is regarded here as minimal extension of quadratic equation supplemented by the cubic term. In this interpretation, the quadratic and cubic equations are *unified* in the general form,

$$\dot{x} = r_1 x - r_2 x^2 - r_3 x^3 \tag{10}$$

where r_1, r_2, r_3 are three independent parameters. The motivation for this ansatz is rooted in the *universal* period-doubling route to chaos of one-dimensional non-invertible maps and nonlinear differential equations.

A3) Unobservable states, such as left-handed antineutrinos or right-handed Z bosons, behave in a similar way with the *unstable fixed points* of (7). These points exist, for example, in *transcritical bifurcations* [7, Appendix].

A4) Left-handed and right-handed states associated with dimensional condensates are labeled as "L" and "R", respectively.

Parameter	**Flavor Content**	**Bifurcation Pattern**	**Spin**
r_1	Higgs scalar	2^j	0
$r_m < r_{DM}$	Gauge Bosons	2^j	1
$r_{DM} < r_\infty$	Dark Matter	2^j	undefined
$r_\infty < r_m$	Fermions	$3 \cdot 2^j$	$1/2$

Tab. 1: The flavor structure of visible and Dark Matter sectors [1]

5. From bifurcations to the hierarchical structure of SM

An intriguing outcome of [8] is that, taking complex-scalar field theory as baseline model, the electroweak sector of SM unfolds sequentially from bifurcations driven by the observation scale. Starting from the conventional form of the Higgs potential, the cubic map defining the evolution of the complex-scalar field may be shown to represent the cubic map,

$$x_{n+1} = f(r, x_n) = r x_n (1 - x_n^2) \tag{11}$$

Considering previous sections in conjunction with [8], we display below a plausible, (*yet not unique!*) configuration of the overall bifurcation diagram reflecting the splitting of SM fields out of the Higgs vacuum.

To better visualize the bifurcation process, the *flow charts* of figs. 1, 2 and 3 illustrate the sequential stages of condensate formation under the flow of parameter r.

Following [8], the first section of the diagram (fig. 1) reflects the formation of electroweak boson condensates W^+W^- and Z:

$$\boxed{U(1) \Rightarrow \begin{pmatrix} v = (HH) \\ 0 \end{pmatrix}_L \Rightarrow \begin{pmatrix} W^+W^- \\ Z \end{pmatrix}_L}$$

Fig. 1: Bifurcations of the electroweak sector.

The second section of the diagram (fig. 2) represents the formation of gluon condensates and Dark Matter, where the latter is considered a large-scale condensate of continuous dimensions dubbed *Cantor Dust* [9].

$$\begin{pmatrix} W^{+}W^{-} \\ Z \end{pmatrix}_{L} \Rightarrow \begin{pmatrix} (gg)_{r} \\ (gg)_{b} \\ (gg)_{\bar{r}} \\ (gg)_{\bar{b}} \end{pmatrix} \Rightarrow (Dark\ Matter)$$

Fig. 2: Bifurcations of gluons and Dark Matter.

Finally, the last section of the diagram shows the formation of neutrino and charged fermion condensates out of Dark Matter (fig. 3).

$$(Dark\ Matter) \Rightarrow \begin{pmatrix} (\nu,\bar{\nu})_{\tau} \\ (\nu,\bar{\nu})_{\mu} \\ (\nu,\bar{\nu})_{e} \end{pmatrix}_{L} \Rightarrow \begin{pmatrix} (\tau^{-},\nu_{\tau})_{L} \\ (\tau^{-})_{R} \\ (\mu^{-},\nu_{\mu})_{L} \\ (\mu^{-})_{R} \\ (e^{-},\nu_{e})_{L} \\ (e^{-})_{R} \end{pmatrix} \Rightarrow \begin{pmatrix} (t\bar{t})_{L,R} \\ (b\bar{b})_{L,R} \\ (s\bar{s})_{L,R} \\ (c\bar{c})_{L,R} \\ (d\bar{d})_{L,R} \\ (u\bar{u})_{L,R} \end{pmatrix}_{(\alpha\bar{\alpha})}$$

Fig. 3: Bifurcations of Dark Matter and fermions.

6. On the chiral nature of dimensional condensates

In Quantum Field Theory, invariance under the CPT symmetry states that physics remains unchanged upon a combined transformation of charge conjugation, parity inversion and time reversal. In the context of our work and with reference to assumption A1) above, charge conjugation (C) substitutes a particle with its antiparticle, parity inversion (P) swaps the "L" and "R" chirality, and time reversal (T) swaps the order of entries in the dimensional condensate pair.

With reference to A1) and fig. 1, we first consider the formation of the Higgs condensate pair out of the vacuum state $v=(HH)$. The CPT operation gives

$$(H,H)_L \xrightarrow[CPT]{} (H,H)_R \tag{12}$$

Applying CPT to the electroweak sector leads to an outcome similar to (12), namely,

$$(W^+W^-)_L \xrightarrow[C]{} (W^-W^+)_L \xrightarrow[P]{} (W^-W^+)_R \xrightarrow[T]{} (W^+W^-)_R \tag{13}$$

and

$$(Z)_L \xrightarrow[C]{} (Z)_L \xrightarrow[P]{} (Z)_R \xrightarrow[T]{} (Z)_R \tag{14}$$

Next, we look at the fermion sector. If neutrinos are of Dirac type, applying the CPT operator to the neutrino section in fig. 3 leads to,

$$(\nu_e, \overline{\nu_e})_L \xrightarrow[C]{} (\overline{\nu_e}, \nu_e)_L \xrightarrow[P]{} (\overline{\nu_e}, \nu_e)_R \xrightarrow[T]{} (\nu_e, \overline{\nu_e})_R \tag{15}$$

If neutrinos are of Majorana type, (15) reduces to

$$(\nu_e, \nu_e)_L \xrightarrow[CPT]{} (\nu_e, \nu_e)_R \tag{16}$$

It follows from (12) - (16) that CPT symmetry *overlaps* "L" and "R" states of Higgs, electroweak bosons and lepton condensates into a single chiral state, a condition akin to *chiral symmetry breaking*.

By the same token, demanding CPT invariance of charged lepton condensates in fig. 3 gives:

$$(e^-, \nu_e)_L \xrightarrow[C]{} (e^+, 0)_L \xrightarrow[P]{} (e^+, 0)_R \xrightarrow[T]{} (0, e^+)_R \tag{17}$$

At least in principle, (17) explains why enforcing CPT invariance in the bifurcation diagram distinguishes left-handed $SU(2)$ doublets from their corresponding right-handed $SU(2)$ singlets.

7. Concluding remarks

These are the main takeaway points of our analysis:

1) The flavor composition of SM follows from the universal transition to chaos of low-dimensional maps like (10) via succesive bifurcations.

2) Chiral symmetry breaking in SM follows from the enforcing CPT symmetry on all dimensional condensates of the bifurcation diagram.

3) Chiral symmetry breaking starts in the electroweak and neutrino sectors and carries over to the charged fermion sector.

In closing, we note that - as fractional dynamics is likely to become relevant above the electroweak scale – the intrinsic chiral nature of SM may be attributed to the non-commutativity of the parity operator with the Hamiltonian [10].

<u>APPENDIX: On transcritical bifurcations</u>

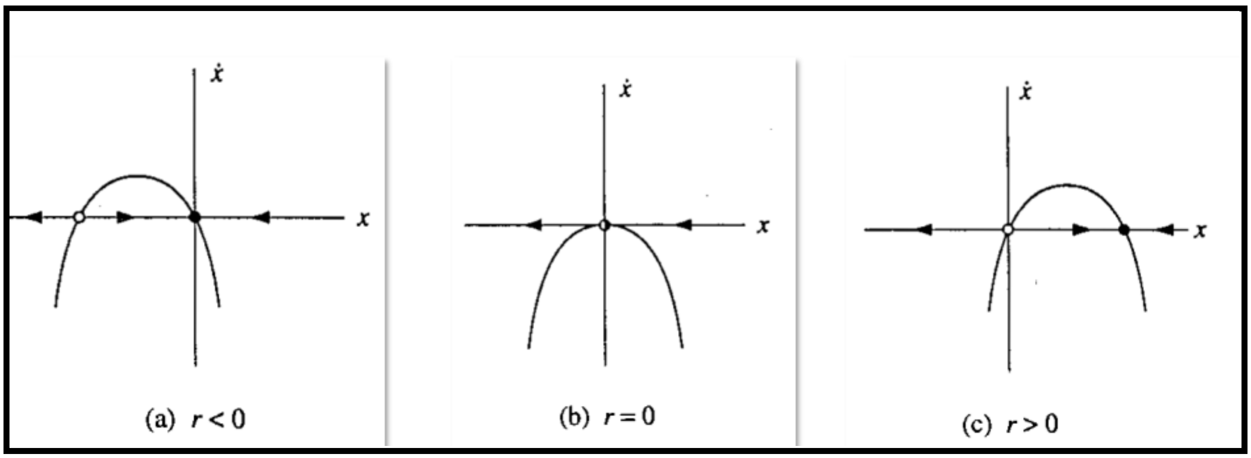

Fig 1: Plot of a typical transcritical bifurcation [7]

Transcritical bifurcations (TB) occur when two fixed points swap stability at the bifurcation point, as direct result of parameter variation. The normal form of TB is $\dot{x} = rx - x^2$, which is identical to (10) for $r_1 = r, r_2 = 1, r_3 = 0$.

For $r < 0$, there is an *unstable fixed point* at $x^* = r$ and a stable fixed point at origin, $x^* = 0$. As r ramps up, the unstable fixed point approaches the origin and coalesces with it at $r = 0$. When $r > 0$, the origin becomes unstable and $x^* = r$ turns into a *stable fixed point*.

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