Closed form solution to the Hubble tension based on $R_h = ct$ cosmology

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Abstract

Haug and Tatum have recently solved the Hubble tension within a type of $R_h = ct$ cosmology using an intuitive, smart trial-and-error search algorithm. The trial-and-error algorithm demonstrates that one can start with the measured CMB temperature and a rough estimate of H_0. Based on the algorithm, one ends up matching the entire distance ladder of the observed supernovas by finding a value for H_0. However, this is a numerical search procedure, even though it can be completed in a fraction of a second on a standard computer. Here, we will demonstrate that the trial-and-error numerical method is not needed and that the Hubble tension can be resolved using the same Haug and Tatum model through a closed-form solution. This means one simply solves an equation to find the correct H_0 value. This is possible because an exact mathematical relation between H_0 and the CMB temperature has recently been established, in combination with the linearity in an $R_h = ct$ model.

Keywords: Hubble tension close dorm; Hubble constant; Cosmological redshift; z; CMB temperature.

1 The Haug-Tatum cosmological model

The Haug and Tatum [1, 2] cosmological model that we will discuss is unique in that it provides an exact mathematical relation between the CMB temperature, the Hubble constant and the cosmological red-shift. The Haug-Tatum cosmological model has developed over time in multiple stages. It is consistent with the $R_h = ct$ principle, which describes a universe expanding at the speed of light without accelerated expansion. There are several $R_h = ct$-type cosmological models, and these models are still actively discussed in recent literature, see for example [3–6]. Melia [7] has recently demonstrated that $R_h = ct$ cosmology seems more in line with recent observations from the James Webb Space Telescope than the Λ-CDM model. The question of which cosmological model best fits different observed properties of the universe will undoubtedly be an ongoing discussion in the years to come. This paper offers additional evidence in favor of $R_h = ct$ cosmology, as it seems that even with a closed-form solution, we can resolve the Hubble tension within such a cosmological model.

In 2015, Tatum et al. [8] presented the following formula for the Cosmic Microwave Background (CMB) temperature, which was later formally derived based on the Stefan-Boltzmann law [9, 10] by Haug and Wojnow [11, 12]:

$$T_{CMB,0}=\frac{\hbar c^3}{k_b 8\pi G\sqrt{M_c m_p}}=\frac{\hbar c}{k_b 4\pi\sqrt{R_H 2 l_p}} \tag{1}$$

where k_b is the Boltzman constant and $m_p=\sqrt{\frac{\hbar c}{G}}$ is the Planck mass, $l_p=\sqrt{\frac{G\hbar}{c^3}}$ is the Planck length [13, 14], and $R_h=\frac{c}{H_0}$ is the Hubble radius and $M_c=\frac{c^3}{2GH_0}$ is the mass (equivalent) of the critical Friedmann [15] universe.

It has also recently been derived using a geometric mean approach, see [16]. Additionally, Haug and Tatum [1] have demonstrated that to be consistent with the observed relation $T_t=T_0(1+z)$, see [17–19], the predicted redshift must be given by:

$$z=\sqrt{\frac{R_h}{R_t}}-1 \tag{2}$$

Haug and Tatum has also show that the distance relation must be:

$$R_H-R_t=\frac{c}{H_0}\left(1-\frac{1}{(1+z)^2}\right) \tag{3}$$

Solving for H_0 gives:

$$H_0=\frac{c\left(1-\frac{1}{(1+z)^2}\right)}{D} \tag{4}$$

Here, $D=R_H-R_t$ represents the distance between us and the object emitting the observed photons. Haug and Tatum then show that the first term of the Taylor expansion yields:

$$H_0\approx\frac{2zc}{D}. \tag{5}$$

and since $D=2d$, where d is the distance in the Λ-CDM model one get the standard relation $H_0\approx\frac{2zc}{D}=\frac{zc}{d}$ for $z\ll 1$.

Furthermore, Haug and Tatum demonstrate that the predicted redshift must satisfy:

$$z_{pre}=\sqrt{\frac{R_h}{R_t}}-1=\sqrt{\frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0(1+z_{obs,i})k_b 4\pi}\right)^2\frac{1}{2l_p}}}-1. \tag{6}$$

They then use a smart trial-and-error algorithm, such as the Newton-Raphson method or the bisection method, to find the value of H_0 that minimizes the sum of the prediction errors $\sum_{i=1}^{n}\frac{z_{pre,i}-z_{obs,i}}{z_{obs,i}}$. They demonstrate that this approach leads to a single H_0 value that perfectly matches the model with the full observed distance ladder, something that seems to solve the Hubble tension.

However, here we simply solve equation (6) for H_0, which yields:

$$H_0=T_0^2\frac{k_b^2 32\pi^2 l_p}{\hbar^2 c}\frac{(1+z_{obs,i})^2}{(1+z_{pre,i})^2} \tag{7}$$

In the case where the predicted redshift $z_{pre,i}$ is exactly equal to the observed redshift $z_{obs,i}$, we must have $\frac{(1+z_{obs,i})^2}{(1+z_{pre,i})^2}=1$. Substituting $\frac{(1+z_{obs,i})^2}{(1+z_{pre,i})^2}=1$ back into equation (7) gives:

$$H_0=T_0^2\frac{k_b^2 32\pi^2 l_p}{\hbar^2 c}=T_0^2\mho \tag{8}$$

The last part, the Greek upsilon: $\mho = \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} = \frac{k_b^2 32\pi^2 \sqrt{G}}{\hbar^{3/2} c^{5/2}}$, is a composite constant made up of well-known constants (which we [20, 21] have coined $\mho$). This is the same formula as given by [20], but here we have just demonstrated that this formula is strictly valid only when the predicted redshift exactly matches the observed redshift, or as we soon will see we can use equation (7) to match the full distance ladder of observed supernova redshifts by simply finding this one H_0 value directly from the current measured CMB temperature.

This means that we only need to know T_0 and this constant to closely match all observed cosmological redshifts. The reason we say "close to perfect" rather than "perfect" is due to small measurement errors in both the measured CMB temperature and in G, and that is the only uncertainty in this method. The Boltzmann constant, the speed of light, and the reduced Planck constant have no uncertainty, as they have been exactly defined since the 2019 NIST CODATA standard.

2 Predictions relative to the observations using the full distance ladder of the Union 2 database

Here, we will see if our model can match all the observed cosmological redshifts by simply determining the H_0 constant from equation (8). However to demonstrate the superiority of equation (8), we will first instead use the predicted value for H_0 by for example Riess et al. [22] of $H_0 = 73.04 \pm 1.04$ km/s/Mpc. We plot the Riess et al. value, accounting for 2 standard deviations (STD), and from this, we get Figure 1. The blue line represents the predicted redshift from $H_0 = 73.04$ km/s/Mpc, while the green lines represent the 2 STD confidence interval, i.e., $\pm 2 \times 1.04$ km/s/Mpc. We can see that even the 95% confidence interval falls outside the observations, meaning that any H_0 value within this interval does not come close to matching the observed redshifts in our cosmological model.

Figure 2 demonstrates the results we get when we instead calculate H_0 based on equation (8) when using the Dhal et al [23] measured CMB value of $T_0 = 2.725007 \pm 0.000024K$. According to our theory, this should provide a perfect match between the observed and predicted values, and as we can see, the observed and predicted values lie on top of each other. The confidence interval is now so narrow that even if we plotted it, it would appear to overlap with the observed values. The predicted $H_0 = 66.8712 \pm \ km/s/Mpc$ when using this measured CMB temperature.

Figure 3 demonstrate the results we get when we calculate H_0 based on equation (8) when the measured CMB value of Fixsen [24]: $T_0 = 2.72548 \pm 0.00057K$, this lead to a basically perfect match between predicted and observed SN Ia redshifts with a predicted $H_0 = 66.8943 \pm 0.0287\ km/s/Mpc$

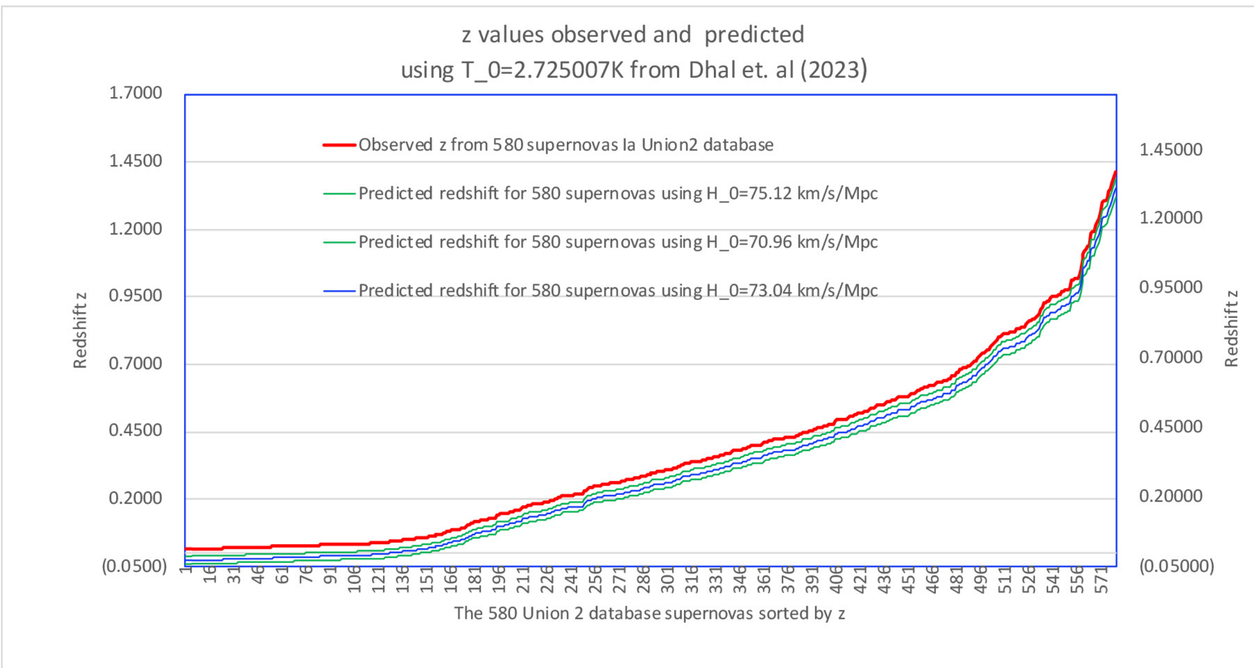

Figure 1: This figure shows observed redshift values from 580 type Ia supernovae, sorted by redshift (blue line). Based on the measured CMB temperature by Dhal et al. (2023) of $2.725007K$, the blue line represents our predictions based on $H_0 = 73.04\ km/s/Mpc$, and the green lines represent the 2 STD confidence interval $\pm 2 \times 1.04\ km/s/Mpc$. We find that the Riess et al. H_0 value cannot match the observed redshifts in this $R_h = ct$ model.

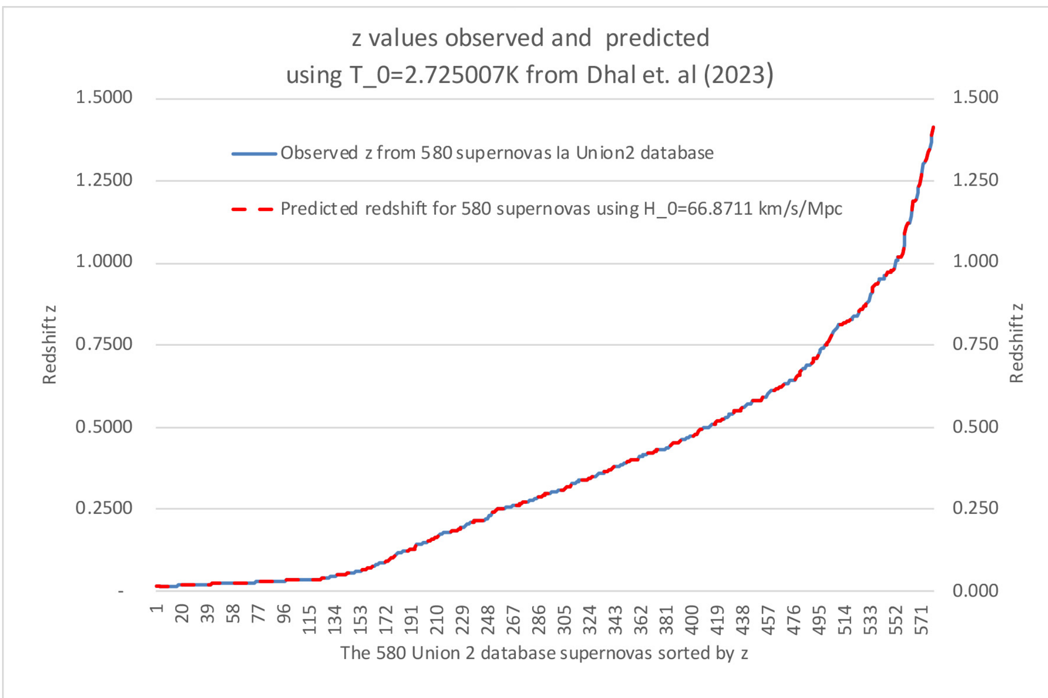

Figure 2: This figure shows observed redshift values from 580 type Ia supernovae, sorted by redshift (blue line). Based upon the measured CMB temperature by Dhal et al (2023) of $2.725007K$, the red line represents our predictions based on $H_0 = 66.8712\ km/s/Mpc$, which we extracted from the data using equation (8).

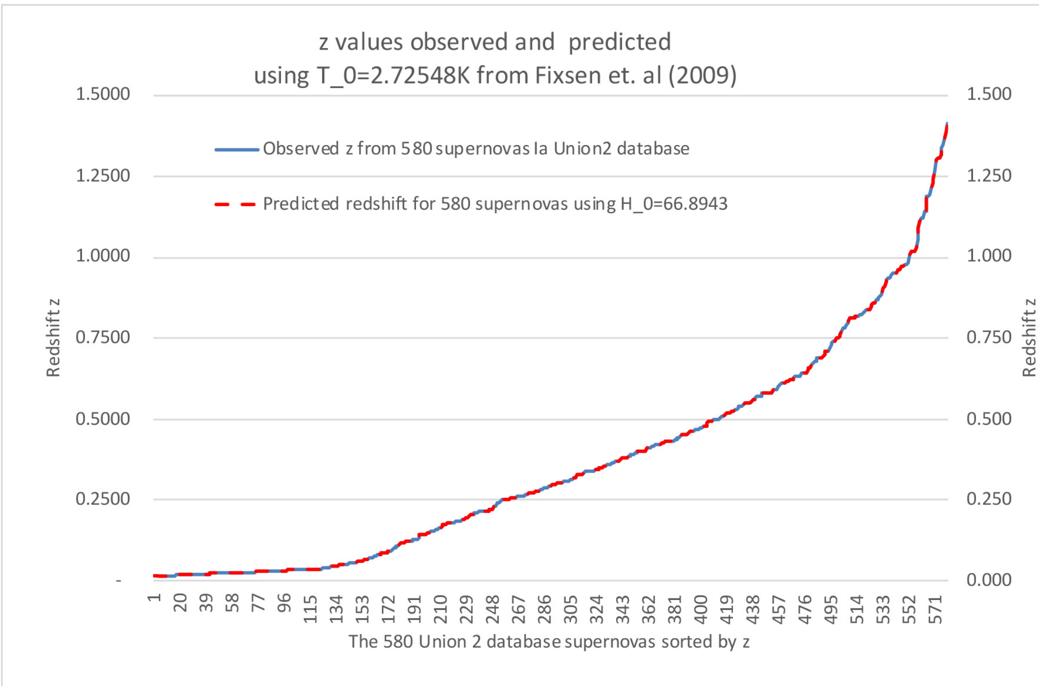

Figure 3: This figure shows observed redshift values from 580 type Ia supernovae, sorted by redshift (blue line). Based upon the measured CMB temperature by Fixen et al (2009) of $2.72548K$, the red line represents our predictions based on $H_0 = 66.8943\ km/s/Mpc$ calculated from equation (8).

3 What about $z = \frac{R_h}{R_t}$ scaling?

Haug and Tatum actually outline two models, one with $z = \sqrt{\frac{R_h}{R_t}}$ scaling and one with $z = \frac{R_h}{R_t}$ scaling. The first one is consistent with $T_t = T_0(1+z)$, and the latter is consistent with $T_t = T_0(1+z)^{\frac{1}{2}}$. Observational studies support $T_t = T_0(1+z)$ over $T_t = T_0(1+z)^{\frac{1}{2}}$, however, one should be careful here and not completely close the door on $T_t = T_0(1+z)^{\frac{1}{2}}$ before carefully going through these studies and checking their assumptions, etc.

The standard scaling $z = \frac{R_h}{R_t}$ leads to

$$z_{pre} = \frac{R_h}{R_t} - 1 = \frac{\frac{c}{H_0}}{\left(\frac{\hbar c}{T_0(1+z_{obs,i})k_b 4\pi}\right)^2 \frac{1}{2l_p}} - 1. \tag{9}$$

and solved for H_0 this gives:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \frac{(1+z_{obs,i})}{(1+z_{pre,i})} \tag{10}$$

The only difference is we have the last term $\frac{(1+z_{obs,i})}{(1+z_{pre,i})}$ rather than $\frac{(1+z_{obs,i})^2}{(1+z_{pre,i})^2}$ as we had in the $z = \sqrt{\frac{R_h}{R_t}}$ scaling model. As a "perfect" match between predictions and observations also now means we have $z_{pre,i} = z_{obs,i}$ and therefore: $\frac{(1+z_{obs,i})}{(1+z_{pre,i})} = 1$, this means both types of scaling lead to exactly the same end result, as in both cases we get:

$$H_0 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \times 1 = T_0^2 \frac{k_b^2 32\pi^2 l_p}{\hbar^2 c} \tag{11}$$

This means both types of scaling give the same H_0 to match all the observed cosmological redshifts that we have tested so far on the Union-2 database. This also means both models are consistent with the same thermodynamical Friedmann equation recently derived [25].

The main difference between the two models is that one has $T_t = T_0(1+z)$, while the other has $T_t = T_0(1+z)^{\frac{1}{2}}$. Additionally, the predicted distance to the redshift differs between the two models. The Hubble tension is resolved in both models. However, for a cosmological model to be robust, it must naturally pass a long series of other tests. Our $z = \sqrt{\frac{R_h}{R_t}}$ model predicts $T_t = T_0(1+z)$, which seems to be consistent with observations, while the $z = \frac{R_h}{R_t}$ model predicts $T_t = T_0(1+z)^{\frac{1}{2}}$, which does not seem to fit observations. However, one must be careful not to draw premature conclusions, as all assumptions made in such observational studies need to be carefully examined.

4 Conclusion

Haug and Tatum have outlined a way to solve the Hubble tension inside $R_h = ct$ cosmology based on new exact relations between the CMB temperature the Hubble constant and redshift, they however use a numerical search algorithm to do so. Even if their method is intuitive and powerful we here demonstrate one can simply solve one of their equations and further based on logic get to the one single H_0 value that make their model matching all observed SN Ia. In other words this leads to a closed form solution of the Hubble tension in side $R_h = ct$ cosmology. We get a $H_0 = 66.8712 \pm 0.0019\ km/s/Mpc$ when relying on the very precise Dhal et al measured CMB value matching leading to matching all the observed SN Ia redshifts across the full distance ladder in the Union2 database. This is the same value

Haug and Tatum got from their numerical search algorithm solution when solving the Hubble tension. It is basically the same solution, one is using numerical search algorithm while the later used closed form solution. The closed form solution is naturally more elegant as no numerical search rutine with many calculations are needed to find the H_0 that matches all the supernovas.

We have also mathematically demonstrated that one gets exactly the same H_0 value to match all supernovae, whether one uses the cosmological redshift scaling of the form $z = \sqrt{\frac{R_h}{R_t}}$ or $z = \frac{R_h}{R_t}$. The difference between these two $R_h = ct$ models is the predicted distance to emitted photons, and that the first model predicts $T_t = T_0(1+z)$, while the other predicts $T_t = T_0(1+z)^{\frac{1}{2}}$, with $T_t = T_0(1+z)$ seeming to best fit observations. We can conclude that Haug and Tatum have likely solved the Hubble tension, and this paper clarifies why, providing deeper detail as well as a closed-form solution to the Hubble tension problem.

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Declarations

Conflict of interest

The authors declare no conflict of interest.

Data availability statements

The supernova Union-2 database that we have used can be found here:
https://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt