New approach to asserting the Riemann hypothesis (2)

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Abstract: we will establish a relationship between the classic Riemann Zeta function and Gauss's estimate for the prime numbers for the sequence of $x_n = e^{n^s}$ where s is a real s> 1. *We will use* The Dusarat 1999 inequality *to show that* **P(x) ≃ π(x) or P(x)** Gauss estimation of prime numbers P(x) $= \frac{x}{Lnx}$ and π(x) the function that counts prime numbers.The key to this proof is the sequence of $x_n = e^{n^s}$ where s is a real s> 1

I) New relationship between Riemann's Zeta function and Gauss's estimate for prime numbers P(x) $= \frac{x}{Lnx}$

For the following $x_n = e^{n^s}$ where n is a non-zero natural number and s > 1

For Gauss estimation of prime numbers P(x) $= \frac{x}{Lnx}$

$P(e^{n^s}) = \frac{e^{n^s}}{\log(e^{n^s})} = \frac{e^{n^s}}{n^s}$.

$\frac{P(e^{n^s})}{e^{n^s}} = \frac{1}{n^s}$ which gives $\sum_1^{+\infty} \frac{P(e^{n^s})}{n^s} = \sum_1^{+\infty} \frac{1}{n^s} = \xi(s)$ for s > 1.

Conclusion: for P(x) $= \frac{x}{Lnx}$ **so** $\sum_1^{+\infty} \frac{P(e^{n^s})}{e^{n^s}} = \xi(s); s > 1$ **where P is the Gaussian prime number count function and ξ is the classical Riemann function.**

The other meaning also true i.e. if ξ(s) = $\sum_1^{+\infty} \frac{P(e^{n^s})}{e^{n^s}}$ **so P**(x) $= \frac{x}{Lnx}$.

II) Poof of Riemann hypothesis

$\pi(x)$: The function of counting prime numbers

The Dusarat 1999 inequality gives $\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+ \frac{1.2762}{Lnx})$, the reduction is true for $x \geq 599$.In the following we will always take $x \geq e^7$ and P(x) $= \frac{x}{Lnx}$

$\frac{x}{lnx}(1+ \frac{1}{Lnx}) \leq \pi(x) \leq \frac{x}{Lnx}(1+ \frac{1.2762}{Lnx})$ which gives

$\frac{x}{Lnx\, Lnx} \leq \pi(x) - P(x) \leq \frac{1.2762\, x}{Lnx\, Lnx}$ divide by the real x with $x \geq e^7$

$\frac{1}{Lnx\, Lnx} \leq \frac{\pi(x)}{x} - \frac{P(x)}{x} \leq \frac{1.2762}{Lnx\, Lnx}$ replace x by $x_n = e^{n^s}$.with $n \geq 7$ and s > 1.

$\frac{1}{n^{2s}} \leq \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{P(e^{n^s})}{e^{n^s}} \leq \frac{1.2762}{n^{2s}}$ With n ≥ 7and s > 1 let's go to the sum between 7 and +∞

$\sum_7^{+\infty} \frac{1}{n^{2s}} \leq \sum_7^{+\infty} \left(\frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{P(e^{n^s})}{e^{n^s}}\right) \leq \sum_7^{+\infty} \frac{1.2762}{n^{2s}}$

$\xi(2s)- (1+ \frac{1}{2^{2s}}+ \frac{1}{3^{2s}}+ \frac{1}{4^{2s}}+ \frac{1}{5^{2s}}+ \frac{1}{6^{2s}}) \leq \sum_7^{+\infty} \left(\frac{\pi\left(e^{n^s}\right)}{e^{n^s}} - \frac{P(e^{n^s})}{e^{n^s}}\right) \leq 1.2762(\xi (2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right))$

The deference between the two framing terms gives the errors

R(s) = 0.2762($\boldsymbol{\xi (2s) - \left(1 + \frac{1}{2^{2s}} + \frac{1}{3^{2s}} + \frac{1}{4^{2s}} + \frac{1}{5^{2s}} + \frac{1}{6^{2s}}\right)}$**)**

Examples for s=1 a calculation with Géogebra gives R (1) = 0.04

For s large enough R(s) ≃ 0

So : for $\boldsymbol{s > 1}$ $\boldsymbol{\sum_7^{+\infty} \frac{P(e^{n^s})}{e^{n^s}} \simeq \sum_7^{+\infty} \frac{\pi\left(e^{n^s}\right)}{e^{n^s}} \simeq \xi (s) - \left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s}\right)}$

So P(x) ≃ π(x) the following $\boldsymbol{x_n = e^{n^s} \geq 599}$ **where n is a natural number and s > 1**

(According the conclusion I i.e. if ξ**(s) =** $\boldsymbol{\sum_1^{+\infty} \frac{P(e^{n^s})}{e^{n^s}}}$ **so P(x) =**$\boldsymbol{\frac{x}{Ln\, x}}$ **)**

III) Conclusion

Π(x) the function that counts prime numbers

P(x) =$\frac{x}{Lnx}$

P(x) ≃ π(x) for the following $\boldsymbol{x_n = e^{n^s} \geq 599}$ **where n is a natural number and s > 1**

Which affirms the Riemann hypothesis.

References

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