Space Time Potential Theory: A Fundamental Second-Order Approach

Arend Lammertink

Schoolstraat 107, 7471 WV, Goor, The Netherlands. E-mail: lamare@gmail.com

Abstract.
This paper presents a unified second-order model that resolves key limitations of traditional first-order potential theories in both fluid dynamics and electromagnetism. By employing the vector Laplacian and defining a space-time derivative operator, $\frac{d}{dt} = -k\Delta$, we establish a fundamental connection between spatial structure and temporal evolution, providing a more complete and physically consistent framework.
This approach integrates the electric and magnetic fields with force and torque densities, reinterpreting charge, current, and electromagnetic fields in terms of fluid dynamic quantities such as mass density and momentum diffusivity.
Additionally, the model proposes a potential unification of gravitational and electromagnetic forces by expressing the gravitational potential as proportional to the square of the electric field. This redefinition creates a seamless link between the two forces, treating gravitational interactions as a secondary effect of electric field behavior.
Higher-order time derivatives, such as jerk and yank, are introduced to further extend the framework's ability to describe dynamic systems in both fluid and electromagnetic contexts.
The results demonstrate consistency across scales, from quantum phenomena to cosmological dynamics, offering a comprehensive alternative to existing theories while eliminating gauge fixing ambiguities and enhancing mathematical coherence.

1 Introduction

In fluid dynamics and electromagnetism, potential theory plays a central role in describing flow and field interactions. However, potential fields, as traditionally defined, present several unresolved issues that prevent them from being fully comprehensive. One key limitation is that the potential fields in these models are not uniquely defined and rely only on first-order spatial operators, namely the gradient and curl. As a result, these first-order models do not utilize the second spatial derivative, the vector Laplacian, which is a unique and complete operator in three-dimensional space, as established by the fundamental theorem of vector calculus. This introduces both ambiguities and inconsistencies, particularly when comparing fluid dynamics and electromagnetic theory.

The need for a consistent and fundamental approach based on the vector Laplacian is evident in both fields. In fluid dynamics, potential theory is commonly framed using the scalar potential φ and vector potential $\vec{A}$, which are higher-dimensional generalizations of the two-dimensional stream function. In electromagnetism, Maxwell's equations introduce similar scalar and vector potentials to describe electric and magnetic fields. However, both frameworks suffer from inherent limitations in their first-order formulations, leading to a lack of completeness and ambiguity in the boundary conditions and physical interpretation of the fields.

In fluid dynamics, an incompressible fluid flow can be described using the following potential decomposition for the velocity field $\vec{v}$ [1]:

$$\vec{v} = \nabla\varphi + \nabla \times \vec{A}, \tag{1}$$

where:
φ is the scalar potential, satisfying the Laplace equation:

$$\Delta\varphi = 0, \tag{2}$$

and $\vec{A}$ is the solenoidal vector potential $\nabla\cdot\vec{A} = 0$, governed by the Poisson equation:

$$\Delta\vec{A} = -\vec{\omega}, \tag{3}$$

where $\vec{\omega} = \nabla \times \vec{v}$ is the vorticity field.

The issue with this representation lies in the fact that the scalar and vector potentials are not uniquely defined. Their values depend on the flow boundary conditions and the topological properties of the domain. This leads to a degree of arbitrariness in their selection, which introduces uncertainty in solving real-world problems.

In electromagnetism, Maxwell's equations are expressed using the electric field $\vec{E}$ and magnetic field $\vec{B}$. These fields are related to the scalar potential Φ and the vector potential $\vec{A}$:

$$\vec{B} = \nabla \times \vec{A}, \tag{4}$$

$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}. \tag{5}$$

While this formulation successfully describes electromagnetic interactions, the potentials Φ and $\vec{A}$ are subject to gauge freedom, meaning they can be transformed without altering the physical fields. This lack of uniqueness in the potentials mirrors the problem seen in fluid dynamics.

The key limitation of these first-order models is their failure to employ the vector Laplacian, which is the only unique second-order spatial derivative in three dimensions. The Laplacian allows for a more rigorous, second-order treatment of the fields by decomposing them into divergence-free and curl-free components, as outlined in the Helmholtz decomposition.

The fundamental theorem of vector calculus states that any sufficiently smooth vector field can be uniquely decomposed into the sum of a curl-free (irrotational) field and a divergence-free (solenoidal) field. This decomposition is intimately tied to the vector Laplacian and provides a mathematically complete way of describing physical fields, free from the ambiguities present in first-order models.

It is often thought that the Helmholtz decomposition is restricted to static fields, but studies have confirmed its applicability to time-varying fields as well [2], particularly in the context of electromagnetic fields (EM) [3]. The decomposition holds for fields that are sufficiently smooth and decay at large distances, regardless of their time dependency. In the case of time-varying fields, the decomposition allows for a rigorous breakdown of the field into curl-free and divergence-free components, which can provide deeper insights into both the temporal and spatial evolution of the fields.

To overcome the inconsistencies and incompleteness of first-order models, we propose a new framework based on the space-time derivative operator:

$$\frac{d}{dt} = -k\Delta, \tag{6}$$

where k represents the momentum diffusivity or kinematic viscosity and Δ is the vector Laplacian. This second-order model builds directly on the Helmholtz decomposition, ensuring that the derived scalar and vector potentials are uniquely determined by the second spatial derivative, avoiding the arbitrariness of first-order models.

By using the vector Laplacian as the core operator, this model provides a unified framework that treats both the linear and angular components of the fields consistently. This resolves the boundary condition ambiguities in fluid dynamics potential theory and eliminates the gauge redundancy seen in Maxwell's equations.

Furthermore, it offers a more fundamental interpretation of the connection between space and time, where the second-order space-time derivative operator describes how fields evolve in both space and time, based on geometric principles as well as the fundamental principles of vector calculus.

2 Methods

2.1 Fundamental Space-Time Derivative Operator

In this section, we establish a mathematical connection between space and time, grounded in the principles of vector calculus. Specifically, we derive the space-time derivative operator by applying the vector Laplacian to a fluid velocity field. This operator describes how the spatial structure of the field influences its evolution over time, reflecting how momentum, force, and related quantities propagate in the medium.

It is important to clarify that this model does not suggest any transformation of space into time or a change in the passage of time, as might be inferred from relativistic theories. Instead, the space-time connection in this context refers to the temporal evolution of physical fields based on their spatial distribution, a concept familiar in many areas of physics such as fluid dynamics and wave propagation.

The result reveals a deep relationship between diffusivity, the second spatial derivative (vector Laplacian) and time evolution in fluid dynamics and electromagnetism. The time evolution of the field is governed by the spatial gradients described by the vector Laplacian, without implying any direct transformation between space and time. This framework provides a structured way to model how spatial variations in the medium drive the propagation and change of fields over time.

The vector Laplacian $\Delta\vec{F}$ is the generalization of the scalar Laplacian, applied component-wise to a vector field $\vec{F}$. In Cartesian coordinates, for a vector field $\vec{F} = (F_x, F_y, F_z)$ the vector Laplacian is given by:

$$\Delta\vec{F} = (\Delta F_x, \Delta F_y, \Delta F_z), \tag{7}$$

where Δ is the scalar Laplacian operator $\Delta = \nabla^2$. This operator plays a central role in governing the evolution of physical quantities in space, and in our model, it forms the basis of the second-order space-time derivative that links spatial structure to temporal evolution. It is uniquely defined as the second spatial derivative of a vector field $\vec{F}$, given by:

$$\Delta\vec{F} = \nabla(\nabla\cdot\vec{F}) - \nabla\times(\nabla\times\vec{F}), \tag{8}$$

where the first term represents the linear component (divergence based), and the second term represents the angular component (curl based).

We begin by negating the vector Laplacian and applying it to the velocity field $\vec{v}$, multiplied by the dynamic viscosity of the fluid η:

$$-\Delta\eta\vec{v} = -\nabla(\nabla\cdot\eta\vec{v}) + \nabla\times(\nabla\times\eta\vec{v}), \tag{9}$$

We can write this out to define the following fields:

$$p = \nabla \cdot \eta\vec{v} = \eta\nabla \cdot \vec{v}$$
$$\vec{\tau} = \nabla \times \eta\vec{v} = \eta\nabla \times \vec{v} = \eta\vec{\omega}$$
$$\vec{f_l} = -\nabla p = -\eta\nabla(\nabla \cdot \vec{v}) \tag{10}$$
$$\vec{f_a} = \nabla \times \vec{\tau} = \eta\nabla \times \vec{\omega},$$

where p is the scalar pressure field, $\vec{\tau}$ is the torque density field, $\vec{f_l}$ is the linear force density field, and $\vec{f_a}$ is the angular force density field.

And since the curl of the gradient of any twice-differentiable scalar field is zero ($\nabla \times \nabla p = 0$) and the divergence of the curl of any vector field is zero ($\nabla \cdot \nabla \times \vec{\tau} = 0$) as well, we have obtained a Helmholtz decomposition of the vector field $\eta\vec{v}$ and we can recognize that the linear force density field $\vec{f_l}$ is curl-free and the angular force density field $\vec{f_a}$ is divergence-free.

Next, we express the total force density $\vec{f}$ as the sum of the linear and angular components:

$$\vec{f} = \vec{f_l} + \vec{f_a} = -\eta\Delta\vec{v}. \tag{11}$$

According to Newton's second law, force density is related to the product of mass density ρ and acceleration $\vec{a}$:

$$\vec{f} = \rho\vec{a}. \tag{12}$$

Substituting this into the expression for $\vec{f}$, we obtain:

$$\rho\vec{a} = -\eta\Delta\vec{v}. \tag{13}$$

Dividing both sides of the equation by the mass density ρ, we arrive at the velocity diffusion equation:

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\eta}{\rho}\Delta\vec{v}. \tag{14}$$

This equation describes how the velocity field evolves over time based on the spatial distribution of velocity. The quantity $k = \frac{\eta}{\rho}$ is the momentum diffusivity or kinematic viscosity, usually denoted by ν in fluid dynamics, which governs how momentum diffuses through the fluid.

Thus, the acceleration $\vec{a}$ can be written as:

$$\vec{a} = \frac{d\vec{v}}{dt} = -k\Delta\vec{v}. \tag{15}$$

This result expresses the time evolution of the velocity field in terms of the second spatial derivative, highlighting the fundamental role of diffusivity in this process.

Finally, by dividing both sides of the equation by $\vec{v}$, we define the space-time derivative operator:

$$\frac{d}{dt} = -k\Delta. \tag{16}$$

This operator establishes a direct relationship between time evolution and the spatial structure of a field, governed by the Laplacian. Since the vector Laplacian is the unique second-order spatial derivative in 3D, and the Helmholtz decomposition uniquely separates the linear (curl-free) and angular (divergence-free) components, this operator represents a fundamental connection between space and time. It captures how the spatial configuration of fields drives their evolution over time, with diffusivity k acting as the proportionality constant.

In classical electromagnetism, the concept of diffusion is typically not associated with field propagation, as electromagnetic waves travel without dissipation in a vacuum. However, in our model, the diffusion equation is introduced not as a mechanism of energy dissipation but rather as a natural outcome of applying the second-order space-time derivative operator to the evolution of field quantities. This diffusion equation captures the way momentum and other field-related quantities spread throughout space.

The vector Laplacian Δ describes how a vector field varies spatially, and when coupled with diffusivity k to form the space-time derivative operator $\frac{d}{dt} = -k\Delta$, this leads to a diffusion equation for quantities such as velocity or force density. Specifically, this represents how the spatial structure of the field evolves over time, analogous to how diffusion equations describe the spread of particles or heat in traditional systems. In this model, the equation does not imply physical dissipation but rather reflects the redistribution of field quantities in space over time, preserving the energy of the system.

By defining this second-order operator, we describe how spatial gradients (captured by the Laplacian) drive the temporal evolution of the fields. The result is a diffusion equation that governs the evolution of the fields without introducing energy dissipation, maintaining consistency with the principles of electromagnetism.

The diffusion equation in this model serves three main purposes:

- Spatial Evolution: The diffusion equation here serves to express how the fields evolve in space, with the vector Laplacian representing spatial variation.
- Momentum Redistribution: In the context of this model, the diffusion equation describes the redistribution of momentum and force densities within the medium.
- Mathematical Coherence: The second-order equation naturally arises from the use of the vector Laplacian and is consistent with the principles of vector calculus, providing a structured way to describe both the linear and angular components of the field.

Therefore, the introduction of the diffusion equation is a mathematical consequence of using the vector Laplacian and space-time derivative operator to describe field evolution, rather than a physical assumption about dissipation. The equation reflects how spatial gradients in the medium drive the propagation of fields, consistent with the behavior of electromagnetic waves in a non-dissipative environment.

2.2 Extending to Higher-Order Time Derivatives

By applying the space-time derivative operator to the acceleration field $\vec{a}$, we can compute the time derivative of acceleration, commonly known as jerk $\vec{j}$:

$$\vec{j} = \frac{d\vec{a}}{dt} = -k\Delta\vec{a}. \tag{17}$$

Jerk describes how acceleration changes over time and is often relevant in systems where sudden changes in motion are involved. In the context of this model, it reflects how the acceleration field evolves due to the spatial distribution of forces and torques within the medium.

We can now also define the vector potential $\vec{A}$ for the acceleration field, which is analogous to the second order version of the vector potential in fluid dynamics $\vec{A}_{fd}$ [1]:

$$\Delta\vec{A}_{fd} = -\nabla \times (\nabla \times \vec{A}_{fd}) = -\vec{\omega}. \tag{18}$$

This can be rewritten to:

$$\nabla \times (\nabla \times \vec{A}_{fd}) = \vec{\omega} = \nabla \times \vec{v}. \tag{19}$$

Or:

$$\nabla \times \vec{A}_{fd} = \vec{v}. \tag{20}$$

By defining the vector potential $\vec{A}$ for the acceleration field rather than the velocity field, we obtain:

$$\nabla \times \vec{A} = \vec{a} = k\nabla \times \nabla \times \vec{v}, \tag{21}$$

which can be simplified to:

$$\vec{A} = k\nabla \times \vec{v} = k\vec{\omega} = \frac{1}{\rho}\vec{\tau}. \tag{22}$$

resulting in a unit of measurement in $[\frac{m^2}{s^2}]$ or velocity squared.

To move from the abstract concept of jerk to something more physically tangible, we can multiply jerk by the mass density ρ to obtain yank density $\vec{y}$, where yank is the time derivative of force. Yank has only recently been named and studied in biomechanics, where it has been shown to be an important variable in sensorimotor systems [4]:

$$\vec{y} = \rho\vec{j} = -\rho k\Delta\vec{a} = -\eta\Delta\vec{a}. \tag{23}$$

Here, yank density describes how force density changes over time within the medium. It represents the rate at which force is applied or altered in a given volume, providing a higher-order insight into the dynamics of the system.

In this model:

- Force density $\vec{f}$ describes the distribution of forces acting within the medium.
- Yank density $\vec{y}$ describes how this force density changes over time, adding a layer of dynamical detail.

The introduction of yank density is especially useful in time-sensitive and reactive systems where changes in forces need to be rapidly addressed. In the referenced study, yank was found to be crucial for activities like prey capture, postural stability, and escape responses, which all rely on how quickly forces can be adjusted.

The significance of incorporating yank into the second-order framework lies in the fact that it allows the model to describe higher-order time derivatives consistently, in a way that was not previously possible. This means that:

- The model can capture rapid changes in force (as yank is the time derivative of force) and torque (through the angular component), which are essential in dynamic systems.
- The resulting fields, including yank density, are free from the ambiguities that often arise in first-order models, as they are derived using the vector Laplacian, ensuring both mathematical correctness and physical consistency.

The inclusion of yank density further strengthens the model by providing a higher level of detail about how the medium reacts to time-dependent forces, while also aligning with the growing recognition of yank as an important variable in biomechanics and other fields.

2.3 Application to the Medium

The newly defined space-time derivative operator allows for a unified treatment of fluid dynamics and electromagnetism by considering both systems as governed by second-order equations derived from the vector Laplacian. In this section, we use the space-time derivative operator to describe the medium and, through this, we redefine key physical properties such as charge, current, and the electromagnetic fields in terms of fluid dynamic quantities like mass density, viscosity and momentum diffusivity.

To apply the space-time derivative operator to a medium, we first define its physical properties in a manner consistent with the fluid dynamics framework. These properties include the mass density ρ, the dynamic viscosity η, and the diffusivity k, which govern the medium's response to forces, torques, and diffusive processes.

We start by defining the dynamic viscosity η in $[\frac{kg}{m \cdot s}]$ using the inverse of the vacuum permeability μ_0:

$$\eta = \frac{1}{4\pi \times 10^{-7}}. \tag{24}$$

Next, we define the mass density ρ to have the same value as the vacuum permittivity ϵ_0, but with units of $[kg/m^3]$. Using the standard relation between ϵ_0, c(speed of light), and μ_0, we get:

$$\epsilon_0 = \frac{1}{c^2\mu_0}. \quad (25)$$

And substituting $\rho = \epsilon_0$ and $\eta = 1/\mu_0$, we obtain:

$$\rho = \frac{\eta}{c^2}. \quad (26)$$

This establishes the medium's mass density in terms of both viscosity and the speed of light.

The momentum diffusivity or kinematic viscosity k is defined as:

$$k = \frac{\eta}{\rho}. \quad (27)$$

Subtituting $\eta = 1/\mu_0$ and $\rho = \epsilon_0$, we obtain:

$$k = \frac{1}{\mu_0\epsilon_0} = c^2, \quad (28)$$

implying that the value of the diffusivity in this framework corresponds to the square of the speed of light. However, the unit of measurement is $[m^2/s]$, indicating that k governs the relationship between spatial and temporal variations of the fields.

2.3.1 Fundamental Nature of Diffusivity k

The unit $[m^2/s]$ reveals that diffusivity k is more than just a measure of momentum diffusivity in a fluid-like system — it represents a fundamental geometric relationship between space and time. Unlike quantities that involve mass or force, k involves only meters and seconds, which are the basic units of space and time, respectively. This strongly suggests that diffusivity k plays a fundamental role in the propagation of fields through space, analogous to the role of the speed of light c in electromagnetism and special relativity.

In classical fluid dynamics, kinematic viscosity k describes how momentum spreads through a medium, but in this framework, it extends beyond describing physical fluid motion. By equating k with c^2, we highlight its fundamental connection to wave propagation and field dynamics. Since c is the speed at which electromagnetic waves propagate through vacuum, and $k = c^2$ in this context, we see that the diffusion described here is analogous to the propagation of information or disturbances through a medium, not the dissipation of energy.

The appearance of only spatial ($[m^2]$) and temporal ($[s]$) units in k indicates that this constant captures the intrinsic scaling of space and time at the infinitesimal level. The infinitesimal relationship between spatial diffusion and time evolution is governed purely by this constant, making it a scaling factor for how physical quantities like velocity, force, or charge density evolve in time as they spread through space. The fact that this constant is equal to the square of the speed of light suggests that any process described by this model—whether in fluid dynamics, electromagnetism, or other field theories—evolves at a rate governed by the speed of light.

Thus, k not only controls the dynamics of momentum in a fluid but also describes how physical disturbances propagate in time and space in a more universal sense. This points to the idea that k plays a role similar to that of c in relativistic theories, bridging the gap between space, time, and field propagation, although this does not imply that diffusivity k should be considered as an absolute universal constant since inversely proportional to mass density ρ.

2.4 Elementary charge and mass

With the space-time derivative operator applied to describe fundamental properties of the medium, we can now extend this approach to redefine elementary physical quantities such as charge, by relating them to mass flow and other dynamic properties. First, we work out the unit of measurement for charge in Coulomb by equating the units of measurement of ρ in $[\frac{kg}{m^3}]$ and ϵ_0 in $[\frac{C^2}{N-m^2}]$ or $[\frac{C^2 \cdot s^2}{kg \cdot m^3}]$:

$$\frac{C^2 \cdot s^2}{kg \cdot m^3} = \frac{kg}{m^3} \Rightarrow C^2 \cdot s^2 = kg^2 \Rightarrow C = \frac{kg}{s} \quad (29)$$

Thus, we reinterpret charge in Coulombs as having units of $[\frac{kg}{s}]$, aligning the concept of charge with mass flow in our framework.

Elementary charge e retains its standard value from the SI system but is now expressed in terms of $[kg/s]$:

$$e = 1.602176634 \times 10^{-19}. \quad (30)$$

Having redefined charge in terms of mass and time, we now turn our attention to elementary mass, leveraging observations from quantized vortices in rotating superfluids. This allows us to introduce a new understanding of mass within the context of the diffusivity of the medium. Using the quantization of circulation $\kappa_o = h/m$ [5], we define elementary mass as:

$$m = \frac{h}{k}, \quad (31)$$

with h Planck's constant. This provides a mass of approximately 7.372×10^{-51} kg, yielding interesting properties such as a Compton frequency of 1 Hz, which challenges the mass-energy equivalence principle.

2.5 Vacuum charge density

Within this framework, charge density is treated as mass flux density, reinterpreting the traditional notion of charge in terms of the flow of mass per unit volume and time. This approach allows us to view charge as a dynamic property of the medium, tied directly to the physical characteristics of the system, such as viscosity and mass density.

We define the vacuum charge density ρ_{q0} as:

$$\rho_{q0} = e\frac{\eta}{h}, \quad (32)$$

where:

- e is the elementary charge,
- η is the dynamic viscosity of the medium, and
- h is Planck's constant.

This yields units of $[\frac{\text{kg}}{\text{m}^3 \cdot \text{s}}]$, aligning the vacuum charge density with a mass flux-based interpretation of charge.

In this formulation, vacuum charge density represents the inherent charge distribution within the medium, even in the absence of external particles or currents. This definition reflects a deeper connection between mass and charge, where charge can be understood as the movement or flow of mass in time. The relationship $\rho_{q0} = e\frac{\eta}{h}$ suggests that the charge density in the vacuum is proportional to the dynamic properties of the medium, specifically its viscosity, and inversely related to Planck's constant.

This proportionality emphasizes the idea that charge is not a standalone entity but is intrinsically linked to the dynamics of the medium. In this view, the vacuum itself carries a background charge density, which could explain subtle electromagnetic effects even in empty space. This charge density contributes to the broader electromagnetic behavior of the medium, reinforcing the connection between electromagnetic phenomena and fluid dynamics principles.

With the concept of vacuum charge density established, we can now redefine the electromagnetic fields in terms of this mass flux-based interpretation of charge. The vacuum charge density provides a natural foundation for expressing both the electric and magnetic fields as emergent properties of the medium. In the following sections, we will explore how these redefined fields integrate seamlessly with the overall framework of second-order dynamics, where force and torque densities govern the behavior of the system.

This redefinition not only grounds the idea of charge in more fundamental properties of the medium but also paves the way for a unified treatment of gravitational and electromagnetic forces, as both can now be derived from a common understanding of mass flux and charge density within the medium.

2.6 Electric and Magnetic Fields

Using the definitions of the medium, we now extend these concepts to redefine the electric and magnetic fields in fluid dynamical terms.

Coulomb's law traditionally defines the relationship between charge q and electric force $\vec{F}$. In our model, we observe that the electric field $\vec{E}$ carries units of $[m/s]$ corresponding to velocity. Thus, we define the electric field as:

$$\vec{E} = \frac{1}{\rho_{q0}}\vec{f_l}, \quad (33)$$

where $\vec{f_l}$ is the linear force density. This redefinition aligns Coulomb's law with:

$$\vec{F} = q\vec{E} = \frac{q}{\rho_{q0}}\vec{f_l}. \quad (34)$$

Ampere's law in its original form (without the displacement current) is given by:

$$\nabla \times \vec{B} = \mu_0\vec{J}. \quad (35)$$

Substituting $\eta = 1/\mu_0$, we redefine the current density $\vec{J}$ as:

$$\vec{J} = \eta\nabla \times \vec{B}. \quad (36)$$

However, by relating the magnetic field $\vec{B}$ to the torque density $\vec{\tau}$, we define $\vec{B}$ as:

$$\vec{B} = \frac{1}{e}\vec{\tau}, \quad (37)$$

with units of $[\frac{1}{m-s}]$, and the magnetizing field $\vec{H}$ as:

$$\vec{H} = \eta\vec{B} = \frac{\eta}{e}\vec{\tau}. \quad (38)$$

This results in current density $\vec{J}$ being expressed as:

$$\vec{J} = \eta\nabla \times \vec{B} = \nabla \times \vec{H} = \frac{\eta}{e}\nabla \times \vec{\tau}. \quad (39)$$

Finally, the Lorentz force is given by:

$$\vec{F} = q\left(\frac{1}{\rho_{q0}}\vec{f_l} + \frac{1}{e}\vec{v} \times \vec{\tau}\right). \quad (40)$$

This consistent formulation integrates Coulomb's law, Ampere's law, and the Lorentz force into a unified framework where:

- The electric field $\vec{E}$ is defined in terms of the linear force density $\vec{f_l}$.
- The magnetic field $\vec{B}$ is defined in terms of the torque density $\vec{\tau}$.
- The units of measurement align properly with the definitions and physical interpretations in the fluid dynamics-inspired model.

2.7 Wave equation and operator

To derive the wave equation in the context of the new model, we can start by considering the space-time derivative operator:

$$\frac{d}{dt} = -k\Delta. \quad (41)$$

So the second space-time derivative operator becomes:

$$\frac{d^2}{dt^2} = k^2\Delta^2. \quad (42)$$

We can substitue this in the the d'Alembert or wave operator:

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta, \tag{43}$$

to obtain the second order spatial wave operator:

$$\Box = \frac{k^2}{c^2}\Delta^2 - \Delta. \tag{44}$$

We can do the same for the wave equation:

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) u(\mathbf{r}, t) = 0, \tag{45}$$

to obtain the second order spatial wave equation:

$$\left(\Delta - \frac{k^2}{c^2}\Delta^2\right) u(\mathbf{r}, t) = 0. \tag{46}$$

And by substituting:

$$\Delta\vec{v} = -\frac{\vec{a}}{k} \tag{47}$$

and:

$$\frac{k^2}{c^2}\Delta^2\vec{v} = \frac{\vec{j}}{c^2}, \tag{48}$$

with $\vec{j}$ the jerk, this can be simplified to:

$$\frac{\vec{j}}{c^2} + \frac{\vec{a}}{k} = 0. \tag{49}$$

Using separation of variables in spherical coordinates, this equation can be broken down into radial and angular parts, typically involving spherical harmonics for the angular component and spherical Bessel functions for the radial part. This approach will allow solving the second order spatial wave equation for specific boundary conditions and eigenvalues related to the physical properties of the medium. The general solution will be a combination of these functions:

$$\vec{F}(r, \theta, \phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \left[A_{lm} j_l(kr) + B_{lm} y_l(kr)\right] Y_{lm}(\theta, \phi), \tag{50}$$

where:

- $j_l(kr)$ are the spherical Bessel functions.
- $y_l(kr)$ are the spherical Neumann functions.
- $Y_{lm}(\theta, \phi)$ are the spherical harmonics.
- A_{lm} and B_{lm} are coefficients determined by boundary conditions.

These solutions are consistent with those found in quantum mechanics, but without the need for probabilistic interpretations. And given that these solutions are valid at both large and small scales, these are sufficient to explain what we now call nuclear forces, so the hypothesis of the existence of distinct nuclear forces (strong and weak) is no longer necessary.

2.8 Gravitational Potential as the Square of the Electric Field

In this section, we explore a potential unification of the gravitational field and the electromagnetic field by deriving the gravitational force from the properties of the electric field. This approach builds on the idea that a force proportional to the gradient of the square of the electric field exists, as demonstrated in classical electrodynamics [6]. By examining the dimensional consistency and the physical behavior of these fields, we propose that the gravitational force may be represented within this second-order fluid dynamics framework.

In classical electrodynamics, the behavior of a dielectric in an electric field reveals that the force acting on the dielectric is proportional to the gradient of the square of the electric field. This can be understood intuitively: as the electric field polarizes the dielectric, the resulting force arises from the induced polarization charges, which themselves are proportional to the electric field. The force, therefore, scales with the square of the field, and its gradient dictates the strength and direction of the force.

In classical mechanics, the gravitational potential V at a point in space is defined as the gravitational potential energy U per unit mass m:

$$V = \frac{U}{m}, \tag{51}$$

where U is the gravitational potential energy at that point. The gravitational potential is a scalar field that represents the amount of work required to move a mass from a reference point (usually taken to be infinitely far away) to the location of interest. This is analogous to the electric potential in electrostatics, where charge plays the role of mass.

In Newtonian mechanics, the gravitational potential V is related to the gravitational field $\vec{g}$ through the relation:

$$\vec{g} = -\nabla V. \tag{52}$$

However, in our model, we propose a redefinition of the gravitational potential based on the behavior of the electric field $\vec{E}$. Instead of treating the gravitational potential as a function of mass alone, we define it in terms of the square of the electric field:

$$V_g = \frac{1}{2}\vec{E} \cdot \vec{E}. \tag{53}$$

This new gravitational potential V_g is proportional to the square of the magnitude of the electric field, drawing on the concept that the electric field drives both the electromagnetic and gravitational interactions. Here, the gravitational field $\vec{g}$ is expressed as the gradient of this gravitational potential:

$$\vec{g} = -\nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\right). \tag{54}$$

In this framework, the gravitational force on a mass m is given by the gradient of the square of the electric field, much like how a gravitational potential is traditionally linked to the gravitational force:

$$\vec{F}_g = m\vec{g} = -m\nabla\left(\frac{1}{2}\vec{E}\cdot\vec{E}\right). \tag{55}$$

This analogy allows us to unify the gravitational and electric potentials within a single framework, where the electric field is the source for both the gravitational potential and the electric potential.

From a mathematical perspective, the gravitational potential as defined here shares similarities with the Newtonian potential, which is the fundamental solution of the Laplace equation. The Newtonian potential is defined as an operator that acts as the inverse of the Laplacian:

$$\Delta V = -\rho, \tag{56}$$

where ρ is the mass density that generates the potential V. In the context of our model, we maintain the analogy by relating the electric field to the source of the gravitational potential, which emerges naturally from the spatial structure of the electric field.

The gravitational potential in this new formulation thus becomes a second-order scalar field, driven by the square of the electric field, offering a more integrated approach to understanding how gravitational and electromagnetic forces are connected.

3 Conclusions

This paper presents a unified and more complete framework for describing the dynamics of both fluid systems and electromagnetic fields by employing the vector Laplacian and the second order space-time derivative operator. Both fluid dynamics and electromagnetism have long relied on first-order potential theories, which are inherently incomplete due to their reliance on non-unique potentials and first-order spatial operators. By shifting to a second-order framework based on the vector Laplacian, we resolve key ambiguities and provide a mathematically consistent description of physical fields that evolve over space and time.

The space-time derivative operator $\frac{d}{dt} = -k\Delta$, derived from well-established principles, reveals a fundamental relationship between space and time:

- Vector Laplacian: The second-order spatial derivative that uniquely describes the behavior of vector fields in 3D, ensuring that the decomposition into curl-free and divergence-free components is unambiguous.
- Helmholtz Decomposition: The vector Laplacian naturally leads to a decomposition of any vector field into divergence-free and curl-free components, which forms the basis of our model.
- Diffusivity: As a measure of how momentum diffuses in a fluid, momentum diffusivity k provides a physical connection between the space-time derivative operator and the evolution of fluid or electromagnetic fields.

This operator highlights how spatial structures evolve over time in the presence of diffusivity, establishing a rigorous basis for describing the dynamics of a medium, whether it is a classical fluid or an electromagnetic system. This connection offers a deeper understanding of how fields evolve and interact in physical systems, with broad implications for fluid dynamics, electromagnetism, and beyond.

Using this formalization, we redefine electromagnetic fields in terms of fluid dynamical quantities, creating a unified model that ties together charge, mass, and force densities in a cohesive framework. In particular:

- The electric field $\vec{E}$ is related to the linear force density.
- The magnetic field $\vec{B}$ is related to the torque density.

This allows for a consistent formulation of Coulomb's law, Ampere's law, and the Lorentz force within the same framework, eliminating the need for gauge fixing and ensuring that all fields are uniquely defined.

One of the key advantages of this new approach is its potential to unify the gravitational and electromagnetic forces. By considering the gravitational force as proportional to the gradient of the square of the electric field, we provide a pathway to integrate both forces into a single framework. This unification suggests that both gravitational and electromagnetic phenomena emerge from the same underlying medium, governed by the diffusivity and vector Laplacian.

In this model, gravity is no longer treated as a separate, fundamental force but rather as a secondary effect linked to the gradient of the electric field squared. This challenges the traditional view of gravity and electromagnetism as distinct forces and opens the door for further exploration of how these forces might be integrated into a fluid dynamics-based model of the medium.

We have also shown how the dynamics of the medium can be described using the fundamental relationship between space and time derived from the vector Laplacian. By computing the time derivative of the velocity field, we obtain two acceleration fields representing the linear and angular components of the system. These fields correspond to the electric and magnetic fields in electromagnetism, but are derived using a purely fluid dynamical approach.

Moreover, the second-order time derivatives of these fields allow us to define higher-order fields, such as yank density, the time derivative of force density, and the angular counterpart, which describes the time derivative of torque density. This method provides a straightforward way to compute higher-order time derivatives in 3D, offering a more complete description of the dynamics of the medium than was previously possible.

A significant advantage of this method is that it provides a mathematically rigorous and consistent description of physical fields. The use of the vector Laplacian ensures that all potential fields are uniquely defined, eliminating the need for arbitrary gauge fixing. Furthermore, the formulation relies on only three fundamental units of measurement: meters ($[m]$), seconds ($[s]$), and kilograms ($[kg]$). This simplicity makes it easier to relate the units of measurement used in fluid dynamics to those used in electromagnetism, ensuring dimensional consistency throughout the framework.

Finally, the model allows for the derivation of a second-order spatial wave equation that has no explicit time dependence. The solutions to this wave equation are consistent with those found in quantum mechanics, but without requiring probabilistic interpretations. The model is valid on all scales, from the quantum to the cosmological, offering a potential bridge between classical and quantum physics.

The logic of this medium-based model demands that there is only one fundamental force of nature. Both nuclear and gravitational forces can be understood as different manifestations of this single force, described by the following equation for force density:

$$\vec{f} = \rho\vec{a} = -\rho k\Delta\vec{v} = -\eta\Delta\vec{v}. \tag{57}$$

In addition to force, we have found a fundamental field for yank density, the time derivative of force density:

$$\vec{y} = \rho\vec{j} = -\rho k\Delta\vec{a} = -\eta\Delta\vec{a}. \tag{58}$$

These two fields, along with the second-order spatial wave equation, describe the complete dynamics of the medium, providing a unified and mathematically consistent model that resolves many of the ambiguities in existing theories. This model holds the potential to unify the fundamental forces of nature into a single coherent framework based on fluid dynamics principles, offering new insights into the behavior of mass, space, and time.

Submitted on Month Day, Year / Accepted on Month Day, Year

References

1. Richardson, S., & Cornish, A. Solution of three-dimensional incompressible flow problems. *Journal of Fluid Mechanics*, 1977, 82, 309-319. https://api.semanticscholar.org/CorpusID:13539940
2. Oleinik, V.P. The Helmholtz Theorem and Superluminal Signals. *arXiv: Quantum Physics*, 2003. https://arxiv.org/abs/quant-ph/0311124
3. Xiong, X.Y., Sha, W.E. & Jun Jiang, L. Helmholtz decomposition based on integral equation method for electromagnetic analysis. *Microwave and Optical Technology Letters*, 2014, 56. https://api.semanticscholar.org/CorpusID:109897294
4. Lin, D.C., McGowan, C.P., Blum, K.P., Ting, L.H. Yank: the time derivative of force is an important biomechanical variable in sensorimotor systems. *Journal of Experimental Biology*, 2019, 222(18). https://doi.org/10.1242/jeb.180414
5. Lounasmaa, O.V. and Thuneberg, E. Vortices in rotating superfluid ^{3}He. *Proceedings of the National Academy of Sciences*, 1999, 14, 7760-7767. https://www.pnas.org/doi/10.1073/pnas.96.14.7760
6. Feynman, R.P. and Leighton, R.B. and Sands, M. The Feynman Lectures on Physics, Vol. II: The New Millennium Edition: Mainly Electromagnetism and Matter. Basic Books, 2011, 10-8. https://www.feynmanlectures.caltech.edu/II_10.html#Ch10-F8