Pigeonhole Principle and the Goldbach Conjecture

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Abstract:

In this paper we use the concept of density of a subset of Natural numbers to an stadistical approach to Goldbach Conjecture.

1- Concepts:

In this new approach to the Goldbach conjecture I have used concepts like the density of a subset in other bigger set and the Prime Number Theorem.
We call the density of even numbers as $\frac{1}{2}$, and the density of even number less than a number as $\sigma_{(2n)}=\frac{x}{2}$.
We call the density of prime numbers as the division of the number of primes less than a number between the number itself. $\sigma_{(p)}=\frac{\frac{x}{\ln x}}{x}=\frac{x}{x\ln x}$.

The growth of pairs of prime numbers is stablish by the function $G(p_i+p_j)=\sum_{n=1} n$,
for $(i=j \wedge i\neq j)$
expresing this that we can form pairs like (11),(11,22,12),(11,22,33,12,13,23),
(11,22,33,44,12,13,14,23,24,34),(11,22,33,44,55,12,13,14,15,23,24,25,34,35,45) and so on.

So the density of number primes for number of pairs of number primes to a number, namely the growth of pairs of number primes will be:

$$A=\frac{x}{x\ln x}\cdot\sum_{n=1}^{\frac{x}{\ln x}} n$$

And we should ask now, It is bigger the growth of the pairs of prime numbers than the growth of even numbers respect $\mathbb{N}$?

$$A>\sigma_{(2n)}\Rightarrow\frac{x}{x\ln x}\cdot\sum_{n=1}^{\frac{x}{\ln x}} n>\frac{x}{2}$$

The answer is “yes” for x>79.1818

2-Conclusions:

If we use the pigeonhole principle we can say that all even numbers >78 could be discomposed as two primes since there more density in the pairs of prime number than density of the even numbers. And as we can compute for even numbers $4\leqslant x\leqslant 78$ we can say that is possible that the Goldbach conjecture is true for all even number >4.