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Classification of Pythagorean triples based on differences in the tangents of their triangles

Annotation. The classification of Pythagorean triples is based on differences in the tangents of their triangles in a certain group, the value of which tends to one divided by a prime number and has the form of a fraction. The corresponding tables containing primitive and non-primitive Pythagorean triples are constructed in accordance with the proposed visual classification. Based on the presented classification, the concept of the "parent" Pythagorean triple is introduced – a primitive triple underlying a series (table) of Pythagorean triples derived from it. The first "parent" Pythagorean triple among the set of primitive tangent values of their triangles is defined by us as "119, 120, 169". It is shown that the first "parent" Pythagorean triple on the basis of the increase of the smaller legs of their triangles – 3, 4, 5 – can be considered as the initial structural unit of a two–dimensional planar construction of a figure – a right triangle, and the product of the numbers 3, 4, 5 – equal to 60 - it is advisable to consider, in turn, as a structural unit of a three-dimensional A three–dimensional figure is a parallelogram.

Keywords: classification, Pythagorean triple, structural unit, triangle tangent, "parent" Pythagorean triple.

Introduction. Relatively many formulas have been found for generating Pythagorean triples. At the same time, there is no relatively simple "non-formulaic" way to generate Pythagorean triples. In this regard, we assumed that there are certain signs of Pythagorean triples, on the basis of which their relatively simple, "easy" mathematical generation and the possibility of visual representation of the results of this generation are possible.

The main part. In the analysis of primitive Pythagorean triples, the tangent values of their triangles were determined based on the increase of the smaller legs of their triangles. It is revealed that a number of triples has a certain "step" in the meaning of the tangent. On this basis, we have highlighted in different colors those Pythagorean triples that "fit" into a similar sequence of "steps" (Table 1). Table 1 shows 105 primitive Pythagorean triples up to the values of a four-digit number and the tangent value of their triangles. Based on the tangent values of the triangles of the triples under consideration, periodicity was revealed – the tangent values increase by a certain amount, which allowed us to assume that there is a possibility of creating a classification of Pythagorean triples on this basis. Based on the data in Table 1, we have selected the initial triples for the proposed classification (highlighted in bold italics in Table 1): 3-4-5; 8-15-17; 20-21-29…

Table 1 – Primitive Pythagorean triples based on the increase of the smaller legs of their triangles and the tangent value of their triangles

Pythagorean (primitive) triples	tg	Pythagorean (primitive) triples	tg	Pythagorean (primitive) triples	tg
3, 4, 5	**1,3333**	44, 117, 125	2,659090909	120,209,241	1,741666667
5, 12, 13	2,4	44,483,485	10,97727273	120,391,409	3,258333333
7, 24, 25	3,42857	**48, 55, 73**	**1,145833333**	123,836,845	6,796747967
8, 15, 17	**1,875**	48,575,577	11,97916667	124,957,965	7,717741935
9, 40, 41	4,4444444	51, 140, 149	2,745098039	129,920,929	7,131782946
11, 60, 61	5,45454	52, 165, 173	3,173076923	132,475,493	3,598484848
12, 35, 37	2,916666	52,675,677	12,98076923	133,156,205	1,172932331
13, 84, 85	6,461538462	56,783,785	13,98214286	135,352,377	2,607407407
15, 112, 113	7,46666666	57, 176, 185	3,087719298	136,273,305	2,007352941
16, 63, 65	3,9375	**60, 91, 109**	**1,516666**	**140,171,221**	**1,221428571**
17, 144, 145	8,470588235	60, 221, 229	3,683333333	145,408,433	2,813793103
19, 180, 181	9,473684211	60,899,901	14,98333333	152,345,377	2,269736842
20, 21, 29	**1,05**	**65, 72, 97**	**1,107692308**	155,468,493	3,019354839
20, 99, 101	4,95	68, 285, 293	4,191176471	156,667,685	4,275641026
21, 220, 221	10,47619048	69, 260, 269	3,768115942	160,231,281	1,44375
23, 264, 265	11,47826087	75,308,317	4,106666667	161,240,289	1,49068323
24, 143, 145	5,958333333	76,357,365	4,697368421	165,532,557	3,224242424
25,312,313	12,48	84,187,205	2,226190476	168,425,457	2,529761905
27,364,365	13,48148148	84,437,445	5,202380952	168,775,793	4,613095238
28, 45, 53	1,607142	85,132,157	1,552941176	175,288,337	1,645714286
28, 195, 197	6,964285714	87,416,425	4,781609195	**180,299,349**	**1,661111111**
29,420,421	14,48275862	**88,105,137**	**1,193181818**	184,513,545	2,788043478
31,480,481	15,48387097	92,525,533	5,706521739	185,672,697	3,632432432
32, 255, 257	7,96875	93,476,485	5,11827957	189,340,389	1,798941799
33, 56, 65	**1,6969**	**95,168,193**	**1,768421053**	195,748,773	3,835897436
33,544,545	16,48484848	96,247,265	2,572916667	200,609,641	3,045
35,612,613	17,48571429	100,621,629	6,21	203,396,445	1,950738916
36, 77, 85	2,138888889	**104,153,185**	**1,471153846**	**204,253,325**	**1,240196078**
36,323,325	8,972222222	105,208,233	1,980952381	205,828,853	4,03902439
37,684,685	18,48648649	105,608,617	5,79047619	**207,224,305**	**1,082125604**
39, 80, 89	2,051282051	108,725,733	6,712962963	215,912,937	4,241860465
39,760,761	19,48717949	111,680,689	6,126126126	216,713,745	3,300925926
40,399,401	9,975	115,252,277	2,191304348	217,456,505	2,101382488
41,840,841	20,48780488	116,837,845	7,215517241	220,459,509	2,086363636
43,924,925	21,48837209	**119,120,169**	**1,008403361**	225,272,353	1,208888889

Based on the values of the tangent of Pythagorean triples presented in Table 1, we have constructed two diagrams that visually represent a certain ordering of the tangent values of primitive Pythagorean triples: in the first diagram, in ascending order of the smaller legs of their triangles (Figure 1).

We have identified (classified) groups of triples based on differences in tangents in this group, which can be represented as fractions: 1/1, 1/2, 1/3 ... (tables 2, 3, 4, 5, 6, 7, 8, 9). Based on the presented classification, the concept of the "parent" Pythagorean triple is introduced – primitive the triple that underlies the series (table) of Pythagorean triples derived from it. The first "parent" Pythagorean triple among the many primitive ones is defined by us as "119, 120, 169" based on the increasing values of the tangent of their triangles, and "3, 4, 5" based on the increasing values of the smaller legs of their triangles (see Table 1).

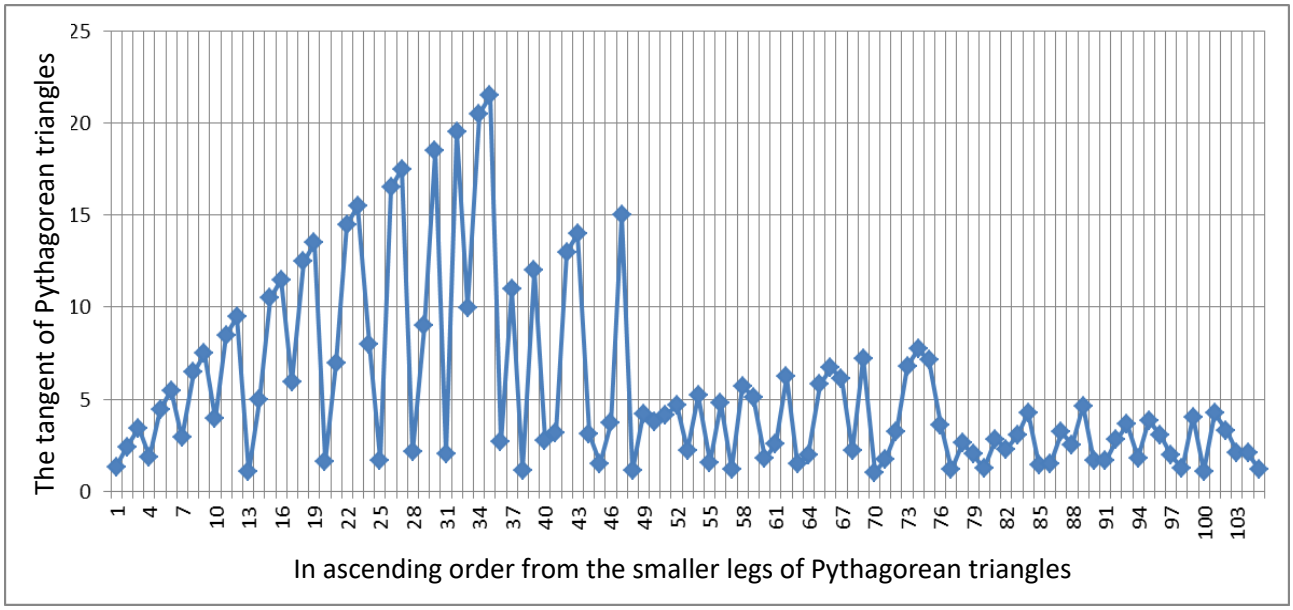

Figure 1 – Diagram of the tangent values of primitive Pythagorean triples in ascending order of the smaller legs of their triangles

The basis for the proposed classification into certain groups are also the following signs:

– the difference between the values of the second and third digits of the Pythagorean triple (equal in the considered groups (columns) of the "parent" Pythagorean triples to the numbers 1, 8, 9, 32, 25, 72, 49, 81...);

– the difference between the values of the first digits of Pythagorean triples in the rows of the first table columns and doubling in adjacent columns, and from table to table – by +2, +6, +8, +10, +16, +14, +24, +18... units;

– double the value of the numbers 1, 8, 9, 32, 25, 72, 49, 81... as a result of summing the "opposite" two Pythagorean triples (minus 1, 2, 4... units in the order of increasing columns of the tables from right to left) (see tables 2, 3, 4, 5, 6, 7, 8, 9).

Some vertical rows of primitive Pythagorean triples are interrupted by non-primitive triples after a strictly defined number of lines (triples). For example, in rows with $\Delta tg \approx 0.333...(1/3)$ (Table 3), $\Delta tg \approx 0.2...(1/5)$ (Table 5), $\Delta tg \approx 0.143...(1/7)$ (Table 7), $\Delta tg \approx 0.111...(1/9)$ (Table 9), the corresponding breaks are observed after 3, 5, 7, 9 Pythagorean triples in a column. In the adjacent column, a similar increase is predictable with an offset (increase) by one row in relation to the previous "discontinuous" group of numbers. These non-primitive Pythagorean triples (interrupting the series of the group) have one common divisor equal to the denominator of the fraction of this series. For example, the non-primitive triple 45, 108, 117 is assigned by us to the group of triples with a fraction 1/3 and has a common divisor of 3.

The ordering of the presented classification in the form of tables is also found in external features. According to the order of the tables presented by us, tables with a "solid" display of numbers in the table (one number above another) alternate with tables where a certain order of alternation of primitive and non-primitive Pythagorean triples is found. At the same time, from table to table with intermittent alternation, this order increases by two units. For example, Table 3 has an interval between primitive and non-primitive triples in 2 rows, table 5 in 4 rows, table 7 in 6 rows, and so on…

Table 2 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈1,0…(1/1))

№	+2	∑+4 (-0)	∑+4 (-0)	+4	∑+8 (-1)	∑+8 (-1)	+8	∑+16 (-2)	∑+16 (-2)	+16	∑+32 (-4)	∑+32 (-4)
		Difference 1 Δtg≈1,0…(1/1)			Difference 2 Δtg≈1,0…			Difference 4 Δtg≈1,0…			Difference 8 Δtg≈1,0…	
1.	**3**	**4**	**5**	8	15	17	20	48	52	48	140	148
2.	5	12	13	12	35	37	28	96	100	64	252	260
3.	7	24	25	16	63	65	36	160	164	80	396	404
4.	9	40	41	20	99	101	44	240	244	96	572	580
5.	11	60	61	24	143	145	52	336	340	112	780	788
6.	13	84	85	28	195	197	60	448	452	128	1020	1028
7.	15	112	113	32	255	257	68	576	580	144	1292	1300
8.	17	144	145	36	323	325	76	720	724	…	…	…
9.	19	180	181	40	399	401	…	…	…	…	…	…
10.	21	220	221	44	483	485	…	…	…	…	…	…
11.	23	264	265	48	575	577	…	…	…	…	…	…
12.	25	312	313	52	675	677	…	…	…	…	…	…
13.	27	364	365	56	783	785	…	…	…	…	…	…
14.	29	420	421	60	899	901	…	…	…	…	…	…
15.	31	480	481	64	1023	1025	…	…	…	…	…	…
16.	33	544	545	68	1155	1157	…	…	…	…	…	…
17.	35	612	613	72	1295	1297	…	…	…	…	…	…
18.	37	684	685	76	1443	1446	…	…	…	…	…	…
19.	39	760	761	80	1599	1601	…	…	…	…	…	…
20.	41	840	841	84	1763	1765	…	…	…	…	…	…
21.	43	924	925	88	1935	1937	…	…	…	…	…	…

Table 3 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,333…(1/3))

№	+6	∑+4 (-0)	∑+4 (-0)	+12	∑+8 (-1)	∑+8 (-1)	+24	∑+16 (-2)	∑+16 (-2)	+48	∑+32 (-4)	∑+32 (-4)
		Difference 9 Δtg≈0,333…(1/3)			Difference 18 Δtg≈0,333…			Difference 36 Δtg≈0,333…			Difference 72 Δtg≈0,333…	
1.	–	–	–	48	55	73	108	144	180	240	364	436
2.	27	36	45	60	91	109	132	224	260	288	540	612
3.	**33**	**56**	**65**	72	135	153	156	320	356	336	748	820
4.	39	80	89	84	187	205	180	432	468	384	988	1060
5.	45	108	117	96	247	265	204	560	596	432	1260	1332
6.	51	140	149	108	315	333	228	704	740	480	1564	1636
7.	57	176	185	120	391	409	252	864	900	528	1900	1972
8.	63	216	225	132	475	493	276	1040	1076	…	…	…
9.	69	260	269	144	567	585	…	…	…	…	…	…
10.	75	308	317	156	667	685	…	…	…	…	…	…
11.	81	360	369	168	775	793	…	…	…	…	…	…
12.	87	416	425	180	891	909	…	…	…	…	…	…
13.	93	476	485	192	1015	1033	…	…	…	…	…	…
14.	99	540	549	204	1147	1165	…	…	…	…	…	…
15.	105	608	617	216	1287	1305	…	…	…	…	…	…
16.	111	680	689	228	1435	1453	…	…	…	…	…	…
17.	117	756	765	240	1591	1609	…	…	…	…	…	…
18.	123	836	845	252	1755	1773	…	…	…	…	…	…
19.	129	920	929	264	1927	1945	…	…	…	…	…	…
20.	135	1008	1017	276	2107	2125	…	…	…	…	…	…
21.	141	1100	1109	288	2295	2313	…	…	…	…	…	…

Table 4 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,5…(1/2))

№	+8	∑+8 (-0)	∑+8 (-0)	+16	∑+16 (-2)	∑+16 (-2)	+32	∑+32 (-4)	∑+32 (-4)	+64	∑+64 (-8)	∑+64 (-8)
		Difference 8 Δtg≈0,5…(1/2)			Difference 16 Δtg≈0,5…			Difference 32 Δtg≈0,5…			Difference 64 Δtg≈0,5…	
1.	**20**	**21**	**29**	48	64	80	112	180	212	256	480	544
2.	28	45	53	64	120	136	144	308	340	320	768	832
3.	36	77	85	80	192	208	176	468	500	384	1120	1184
4.	44	117	125	96	280	296	208	660	692	448	1536	1600
5.	52	165	173	112	384	400	240	884	916	512	2016	2080
6.	60	221	229	128	504	520	272	1140	1172	576	2560	2624
7.	68	285	293	144	640	656	304	1428	1460	640	3168	3232
8.	76	357	365	160	792	808	336	1748	1780	704	…	…
9.	84	437	445	176	960	976	…	…	…	…	…	…
10.	92	525	533	192	1144	1160	…	…	…	…	…	…
11.	100	621	629	…	…	…	…	…	…	…	…	…
12.	108	725	733	…	…	…	…	…	…	…	…	…
13.	116	837	845	…	…	…	…	…	…	…	…	…
14.	124	957	965	…	…	…	…	…	…	…	…	…
15.	132	1085	1093	…	…	…	…	…	…	…	…	…
16.	140	1221	1229	…	…	…	…	…	…	…	…	…
17.	148	1365	1373	…	…	…	…	…	…	…	…	…
18.	156	1517	1525	…	…	…	…	…	…	…	…	…
19.	164	1677	1685	…	…	…	…	…	…	…	…	…
20.	172	1845	1853	…	…	…	…	…	…	…	…	…
21.	180	2021	2029	…	…	…	…	…	…	…	…	…

Table 5 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,2…(1/5))

№	+10	∑+4 (-0)	∑+4 (-0)	+20	∑+8 (-1)	∑+8 (-1)	+40	∑+16 (-2)	∑+16 (-2)	+80	∑+32 (-4)	∑+32 (-4)
		Difference 25 Δtg≈0,2…(1/5)			Difference 50 Δtg≈0,2…			Difference 100 Δtg≈0,2…			Difference 200 Δtg≈0,2…	
1.	**65**	**72**	**97**	140	171	221	300	400	500	640	924	1124
2.	75	100	125	160	231	281	340	528	628	720	1196	1936
3.	**85**	**132**	**157**	180	299	349	380	672	772	800	1500	1700
4.	**95**	**168**	**193**	200	375	425	420	832	932	880	1836	2036
5.	105	208	233	220	459	509	460	1008	1108	960	2204	2404
6.	115	252	277	240	551	601	500	1200	1300	1040	2604	2804
7.	125	300	325	260	651	701	540	1408	1508	1120	3036	3236
8.	135	352	377	280	759	809	580	1632	1732	…	…	…
9.	145	408	433	300	875	925	…	…	…	…	…	…
10.	155	468	493	320	999	1049	…	…	…	…	…	…
11.	165	532	557	340	1131	1181	…	…	…	…	…	…
12.	175	600	625	360	1271	1321	…	…	…	…	…	…
13.	185	672	697	380	1419	1469	…	…	…	…	…	…
14.	195	748	773	400	1575	1625	…	…	…	…	…	…
15.	205	828	853	420	1739	1789	…	…	…	…	…	…
16.	215	912	937	440	1911	1961	…	…	…	…	…	…
17.	225	1000	1025	460	2091	2141	…	…	…	…	…	…
18.	235	1092	1117	480	2279	2329	…	…	…	…	…	…
19.	245	1188	1213	500	2475	2525	…	…	…	…	…	…
20.	255	1288	1313	520	2679	2729	…	…	…	…	…	…
21.	265	1392	1417	540	2891	2941	…	…	…	…	…	…

Table 6 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,25…(1/4))

№	+16	∑+8 (-0)	∑+8 (-0)	+32	∑+16 (-2)	∑+16 (-2)	+64	∑+32 (-4)	∑+32 (-4)	+128	∑+64 (-8)	∑+64 (-8)
		Difference 32 Δtg≈0,25…(1/4)			Difference 64 Δtg≈0,25…			Difference 128 Δtg≈0,25…			Difference 256 Δtg≈0,25…	
1.	**88**	**105**	**137**	192	256	320	416	612	740	896	1440	1696
2.	104	153	185	224	360	424	480	842	964	1024	1920	2176
3.	120	209	241	256	480	544	544	1092	1220	1152	2464	2720
4.	136	273	305	288	616	680	608	1380	1508	1280	3072	3328
5.	152	345	377	320	768	832	672	1700	1828	1408	3744	4000
6.	168	425	457	352	936	1000	736	2052	2180	1536	4480	4736
7.	184	513	545	384	1120	1184	800	2436	2564	1664	5280	5536
8.	200	609	641	416	1320	1384	864	2852	2980	…	…	…
9.	216	713	745	…	…	…	…	…	…	…	…	…
10.	232	825	857	…	…	…	…	…	…	…	…	…
11.	248	945	977	…	…	…	…	…	…	…	…	…
12.	264	1073	1105	…	…	…	…	…	…	…	…	…
13.	280	1209	1241	…	…	…	…	…	…	…	…	…
14.	296	1353	1385	…	…	…	…	…	…	…	…	…
15.	312	1505	1537	…	…	…	…	…	…	…	…	…
16.	328	1665	1697	…	…	…	…	…	…	…	…	…
17.	344	1833	1865	…	…	…	…	…	…	…	…	…
18.	360	2009	2041	…	…	…	…	…	…	…	…	…
19.	376	2193	2225	…	…	…	…	…	…	…	…	…
20.	392	2385	2417	…	…	…	…	…	…	…	…	…
21.	408	2585	2617	…	…	…	…	…	…	…	…	…

Table 7 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,143…(1/7))

№	+14	∑+4 (-0)	∑+4 (-0)	+28	∑+8 (-1)	∑+8 (-1)	+56	∑+16 (-2)	∑+16 (-2)	+112	∑+32 (-4)	∑+32 (-4)
		Difference 49 Δtg≈0,143…(1/7)			Difference 98 Δtg≈0,143…			Difference 196 Δtg≈0,143…			Difference 392 Δtg≈0,143…	
1.	**119**	**120**	**169**	252	275	373	532	624	820	1120	1404	1796
2.	133	156	205	280	351	449	588	784	980	1232	1740	2132
3.	147	196	245	308	435	533	644	960	1156	1344	2108	2500
4.	161	240	289	336	527	625	700	1152	1348	1456	2508	2900
5.	175	288	377	364	627	725	756	1360	1556	1568	2940	3332
6.	189	340	389	392	735	833	812	1584	1780	1680	3404	3796
7.	203	396	445	420	851	949	868	1824	2020	1792	3900	4292
8.	217	456	505	448	975	1073	924	2080	2276	…	…	…
9.	231	520	569	476	1107	1205	…	…	…	…	…	…
10.	245	588	637	504	1247	1345	…	…	…	…	…	…
11.	259	660	709	532	1395	1493	…	…	…	…	…	…
12.	273	736	785	560	1551	1649	…	…	…	…	…	…
13.	287	816	865	588	1715	1813	…	…	…	…	…	…
14.	301	900	949	616	1887	1985	…	…	…	…	…	…
15.	315	988	1037	644	2067	2165	…	…	…	…	…	…
16.	329	1080	1129	672	2255	2353	…	…	…	…	…	…
17.	343	1176	1225	700	2451	2549	…	…	…	…	…	…
18.	357	1276	1325	728	2655	2753	…	…	…	…	…	…
19.	371	1380	1429	756	2867	2965	…	…	…	…	…	…
20.	385	1488	1537	784	3087	3185	…	…	…	…	…	…
21.	399	1600	1649	812	3315	3413	…	…	…	…	…	…

Table 8 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,166…(1/6))

№	+24	∑+8 (-0)	∑+8 (-0)	+48	∑+16 (-2)	∑+16 (-2)	+96	∑+32 (-4)	∑+32 (-4)	+192	∑+64 (-8)	∑+64 (-8)
		Difference 72 Δtg≈0,166…(1/6)			Difference 144 Δtg≈0,166…			Difference 288 Δtg≈0,166…			Difference 576 Δtg≈0,166…	
1.	**204**	**253**	**325**	432	576	720	912	1300	1588	1920	2912	3488
2.	228	325	397	480	728	872	1008	1620	1908	2112	3584	4160
3.	252	405	477	528	896	1040	1104	1972	2260	2304	4320	4896
4.	276	493	565	576	1080	1224	1200	2356	2644	2496	5120	5696
5.	300	589	661	624	1280	1424	1296	2772	3060	2688	5984	6560
6.	324	693	765	672	1496	1640	1392	3220	3508	2880	6912	7488
7.	348	805	877	720	1728	1872	1488	3700	3988	…	…	…
8.	372	925	997	768	1976	2120	…	…	…	…	…	…
9.	396	1053	1125	…	…	…	…	…	…	…	…	…
10.	420	1189	1261	…	…	…	…	…	…	…	…	…
11.	444	1333	1405	…	…	…	…	…	…	…	…	…
12.	468	1485	1557	…	…	…	…	…	…	…	…	…
13.	492	1645	1717	…	…	…	…	…	…	…	…	…
14.	516	1813	1885	…	…	…	…	…	…	…	…	…
15.	540	1989	2061	…	…	…	…	…	…	…	…	…
16.	564	2173	2245	…	…	…	…	…	…	…	…	…
17.	588	2365	2437	…	…	…	…	…	…	…	…	…
18.	612	2565	2637	…	…	…	…	…	…	…	…	…
19.	636	2773	2845	…	…	…	…	…	…	…	…	…
20.	660	2989	3061	…	…	…	…	…	…	…	…	…
21.	684	3213	3285	…	…	…	…	…	…	…	…	…

Table 9 – Representation of Pythagorean triples based on differences in the tangent of their triangles (Δtg≈0,111…(1/9))

№	+18	∑+4 (-0)	∑+4 (-0)	+36	∑+8 (-1)	∑+8 (-1)	+72	∑+16 (-2)	∑+16 (-2)	+144	∑+32 (-4)	∑+32 (-4)
		Difference 81 Δtg≈0,111…(1/9)			Difference 162 Δtg≈0,111…			Difference 324 Δtg≈0,111…			Difference 648 Δtg≈0,111…	
1.	**207**	**224**	**305**	432	495	657	900	1088	1412	1872	2380	3028
2.	225	272	353	468	595	757	972	1296	1620	2016	2812	3460
3.	243	324	405	504	703	865	1044	1520	1844	2160	3276	3924
4.	261	380	461	540	819	981	1116	1760	2084	2304	3772	4420
5.	279	440	521	576	943	1105	1188	2016	2340	2448	4300	4948
6.	297	504	585	612	1075	1237	1260	2288	2612	2592	4860	5508
7.	315	572	653	648	1215	1377	1332	2576	2900	…	…	…
8.	333	644	725	684	1363	1525	…	…	…	…	…	…
9.	351	720	801	720	1519	1681	…	…	…	…	…	…
10.	369	800	881	756	1683	1845	…	…	…	…	…	…
11.	387	884	965	792	1855	2017	…	…	…	…	…	…
12.	405	972	1053	828	2035	2197	…	…	…	…	…	…
13.	423	1064	1145	864	2223	2385	…	…	…	…	…	…
14.	441	1160	1241	900	2419	2581	…	…	…	…	…	…
15.	459	1260	1341	936	2623	2785	…	…	…	…	…	…
16.	477	1364	1445	972	2835	2997	…	…	…	…	…	…
17.	495	1472	1553	1008	3055	3217	…	…	…	…	…	…
18.	513	1584	1665	1044	3283	3445	…	…	…	…	…	…
19.	531	1700	1781	1080	3519	3681	…	…	…	…	…	…
20.	549	1820	1901	1116	3763	3925	…	…	…	…	…	…
21.	567	1944	2025	1152	4015	4177	…	…	…	…	…	…

Based on the results of the classification of Pythagorean triples and on the basis of the created system of interconnected tables, we present the "parent" Pythagorean triples in ascending order of the tangent values of their triangles in Table 10. Pythagorean triples are also sorted in descending order of the values of the differences in the tangents of triangles (Δtg≈) according to the results of the analysis of the tangent values located next to each other in the Tables.They are presented in table 11.

Table 10 – The "parent" Pythagorean triples in ascending order of the tangent values of their triangles

№	The "parent" Pythagorean triples	tg
1.	119,120,169	1,008403361
2.	20, 21, 29	1,05
3.	207,224,305	1,082125604
4.	65, 72, 97	1,107692308
5.	88,105,137	1,193181818
6.	204,253,325	1,240196078
7.	3, 4, 5	1,3333
8.	33, 56, 65	1,6969

Table 11 – The "parent" Pythagorean triples in descending order of the values (Δtg≈) of the tangent of their triangles

№	The "parent" Pythagorean triples	Δtg≈
1.	3, 4, 5	1
2.	20, 21, 29	0,5
3.	33, 56, 65	0,33333
4.	88,105,137	0,25
5.	65, 72, 97	0,2
6.	204,253,325	0,16666
7.	119,120,169	0,142857
8.	207,224,305	0,111111

It should be noted the main advantages of the proposed classification over other classifications:

– the visibility of the presented data on Pythagorean triples;

– the ability to quickly search or select the necessary three (triangle) by the value of its tangent;

– the possibility of multiplying the Pythagorean triple according to tabular data;

– the interconnected "structure" of tabular data allows you to almost accurately determine the presence of a primitive and non-primitive triple, its numerical values.

Thus, a peculiar "structure" of Pythagorean triples was created. And every structure has, as a necessary attribute, its own structural unit. It is known that (among the many elementary properties of primitive Pythagorean triples) there are the following properties:

– exactly one of the numbers *a* and *b* is divisible by 3[3];

– exactly one of the numbers *a* and *b* is divisible by 4 [3];

– exactly one of the numbers *a*, *b* and *c* is divisible by 5 [3];

– the maximum number that always divides the product of *a*b*c*, is equal to sixty [4].

In this regard, we propose to consider the primitive Pythagorean triple 3, 4, 5 as the initial structural unit of a two–dimensional planar construction of a figure - a right triangle (Figure 2, A). This "structure" has the initial components (3, 4, 5) of other Pythagorean triples. The derivative of this structural unit – the product of the numbers 3, 4, 5 equal to 60 – should be considered, in turn, as a structural unit of a three–dimensional three-dimensional figure - parallelogram (Figure 2, B).

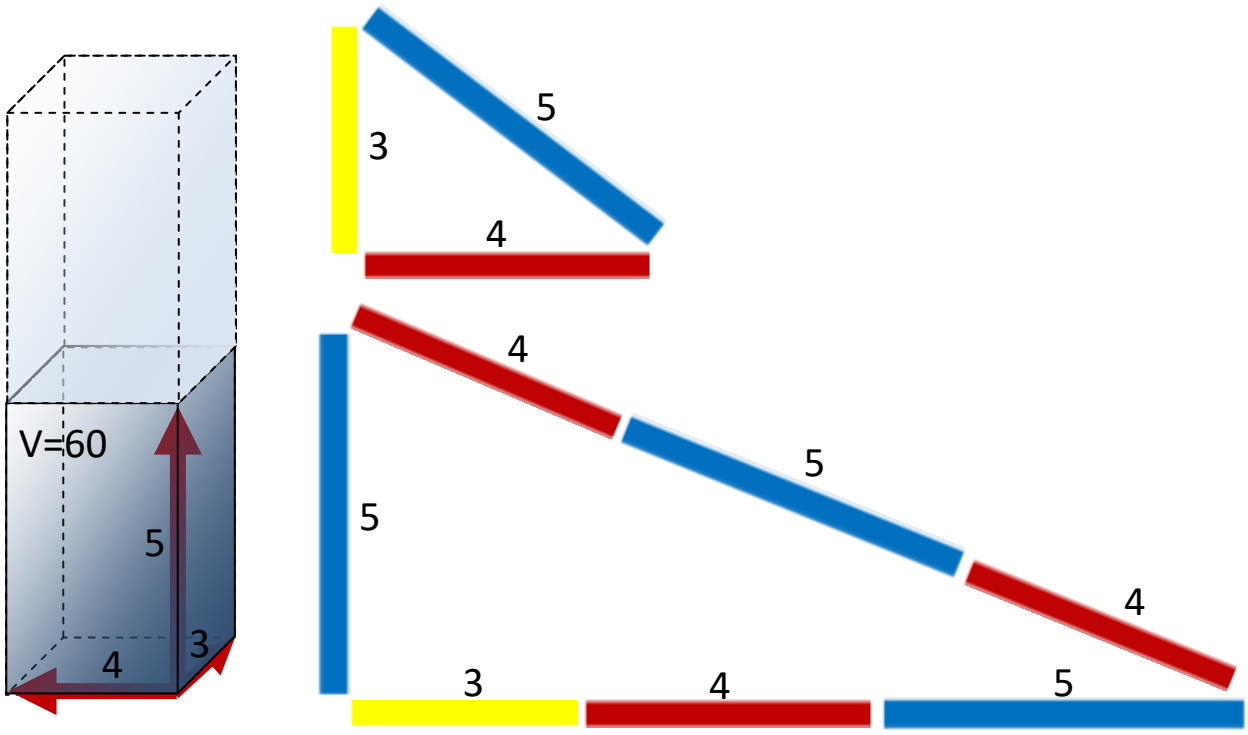

Figure 2 – Structural units of two–dimensional planar construction of a figure – a right triangle (A) and a three-dimensional three-dimensional figure – a parallelogram (B)

The product of the numbers of Pythagorean triples is a multiple of 60, and the maximum number that divides the product of the numbers of Pythagorean triples is also 60 [4]. In this regard, it is assumed that the Pythagorean triple can be characterized by the number of structural units included in it – the number of numbers 3, 4, 5 making up each number of the Pythagorean triple separately (Figure 3) or the number of units multiples of the product of the numbers 3, 4, 5 divided by 60. Thus, the Pythagorean triple has unique mathematical properties: it contains simultaneously units of length and volume (3, 4, 5 and 60), through which not only two–dimensional right triangles with integer values of the legs can be constructed, but also three-dimensional parallelepipeds with integer values of area and volume (see Figure 2).

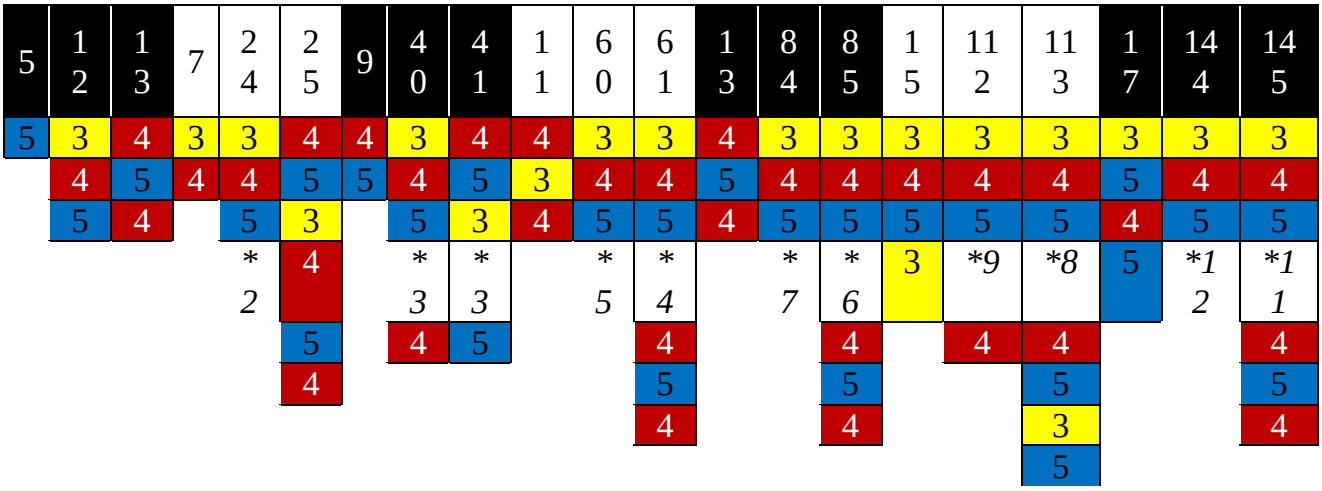

Figure 3 – Structural units of a number of primitive Pythagorean triples

Conclusion.

1. The classification of Pythagorean triples based on differences in tangents in a certain group, the value of which tends to one divided by a prime number and has the form of a fraction.

2. Tables containing primitive and non-primitive Pythagorean triples are constructed in accordance with the proposed visual classification.

3. Based on the presented classification, the concept of the "parent" Pythagorean triple is introduced – a primitive triple underlying a series (table) of Pythagorean triples derived from it. The first "parent" Pythagorean triple among the set of primitive tangent values of their triangles is defined by us as "119, 120, 169".

4. It is shown that the first "parent" Pythagorean triple on the basis of the increase of the smaller legs of their triangles – 3, 4, 5 – can be considered as the initial structural unit of a two–dimensional planar construction of a figure – a right triangle, and the product of the numbers 3, 4, 5 – equal to 60 - it is advisable to consider, in turn, as a structural A unit of a three–dimensional three-dimensional figure is a parallelogram.

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