

# Lorentz Covariant Gravity Theory with the Subsidiary Dynamic Symmetry of General Coordinate Transformation Covariance

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**Abstract** Physical phenomena, sometimes with the exception of gravity, are usually assumed to be described by Lorentz transformation covariant theories, and the validity of the Lorentz transformation has been empirically verified to very high accuracy. The Einstein equation of gravity theory, however, has an infinite set of metric solutions, an infinite subset of which aren't Lorentz covariant, and one of the latter might be taken as valid, e.g., the Robertson-Walker metric for cosmology. But if all of nongravitational physics is in fact Lorentz covariant, it would almost certainly be physically inconsistent for gravity theory not to be Lorentz covariant as well. The solution ambiguity of the Einstein equation is a consequence of its important symmetry of general coordinate transformation covariance. However the four-vector potential form of electromagnetic theory has an analogous solution ambiguity as a consequence of its important symmetry of gauge transformation invariance, but in that case it is standard practice to break this symmetry by imposing the retarded Lorentz gauge condition, the simplest gauge condition which is Lorentz covariant and causal. Here we show that both gauge transformation invariance in electromagnetic theory and general coordinate transformation covariance in gravity theory arise spontaneously from fully Lorentz covariant initial assumptions. These subsidiary dynamic symmetries crucially affect the structure of the equations of their respective theories, but any solutions they happen to admit which aren't fully Lorentz covariant are ipso facto excluded by the fully Lorentz covariant initial assumptions.

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## 1. Lorentz-covariant extension of electrostatic phenomena to dynamic electromagnetism

Electromagnetism is taken to be the result of a conserved scalar entity  $Q$  called charge. The four-vector density-flux of charge  $Q$ , i.e., the current density  $j^\lambda(x)$ , is the *source* of the four-vector electromagnetic potential  $A_\mu(x)$ . Local charge conservation implies that,

$$\partial j^\lambda(x)/\partial x^\lambda = 0. \quad (1.1)$$

In the static limit  $j^\lambda(x) \rightarrow (c\rho(\mathbf{x}), \mathbf{0})$ , where  $\rho(\mathbf{x})$  is the static charge density, and in that limit  $A_0(\mathbf{x})$  is assumed to be the electrostatic potential that follows from  $\rho(\mathbf{x})$  by Coulomb's Law,

$$-\nabla_{\mathbf{x}}^2 A_0(\mathbf{x}) = 4\pi\rho(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c). \quad (1.2)$$

The nonrelativistic equation of motion of a test body of mass  $m$  and charge  $e$  in the electrostatic potential  $A_0(\mathbf{x})$  is of course,

$$m d^2\mathbf{x}/dt^2 = -e\nabla_{\mathbf{x}}A_0(\mathbf{x}). \quad (1.3a)$$

The Eq. (1.3a) nonrelativistic equation of motion corresponds to the nonrelativistic Lagrangian,

$$L(d\mathbf{x}/dt, A_0(\mathbf{x})) = (m/2)|d\mathbf{x}/dt|^2 - eA_0(\mathbf{x}), \quad (1.3b)$$

which, when the electrostatic potential  $A_0(\mathbf{x})$  vanishes, of course becomes the free nonrelativistic mass- $m$  test-body Lagrangian  $L(d\mathbf{x}/dt) = (m/2)|d\mathbf{x}/dt|^2$ .

Making use of the Lorentz invariance of differential proper time  $d\tau \stackrel{\text{def}}{=} dt\sqrt{1 - |(d\mathbf{x}/dt)/c|^2}$ , and assuming the Lorentz covariance of the dynamic electromagnetic four-vector potential  $A_\mu(x)$ , we readily extend the nonrelativistic Eq. (1.3b) Lagrangian to a Lorentz-invariant Lagrangian, i.e.,

$$L(dx^\mu/d\tau, A_\nu(x)) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (e/c)A_\nu(x)(dx^\nu/d\tau), \quad (1.4a)$$

where  $\eta_{\mu\nu}$  is the Minkowski metric,

$$\eta_{00} = 1, \eta_{11} = \eta_{22} = \eta_{33} = -1 \text{ and } \eta_{\mu\nu} = 0 \text{ if } \mu \neq \nu. \quad (1.4b)$$

In the nonrelativistic regime where  $|d\mathbf{x}/dt| \ll c$  the Eq. (1.4a) Lorentz-invariant Lagrangian plus the dynamically-inert constant term  $(m/2)c^2$  goes over into the Eq. (1.3b) nonrelativistic Lagrangian. When the electromagnetic four-vector potential  $A_\mu(x)$  vanishes, the Eq. (1.4a) Lorentz-invariant Lagrangian of course becomes the Lorentz-invariant Lagrangian for the free mass- $m$  test body, namely,

$$L(dx^\mu/d\tau) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau). \quad (1.4c)$$

An unexpected feature of the Eq. (1.4a) Lorentz-invariant Lagrangian  $L(dx^\mu/d\tau, A_\nu(x))$  is that if  $A_\nu(x)$  is modified by adding a term of the form  $\partial\chi(x)/\partial x^\nu$  to it, where  $\chi(x)$  is an arbitrary scalar field, the dynamics of the mass- $m$ , charge- $e$  test body is unaffected. That is so because,

$$\begin{aligned} L(dx^\mu/d\tau, A_\nu(x) + \partial\chi(x)/\partial x^\nu) - L(dx^\mu/d\tau, A_\nu(x)) = \\ -(e/c)(\partial\chi(x)/\partial x^\nu)(dx^\nu/d\tau) = d[-(e/c)\chi(x)]/d\tau, \end{aligned} \quad (1.4d)$$

and terms of a Lagrangian which thus are a derivative of an entity with respect to  $\tau$  don't contribute to the dynamics. The property of dynamic electromagnetism that adding a term of the form  $\partial\chi(x)/\partial x^\mu$  to  $A_\mu(x)$  doesn't alter its dynamics is called electromagnetic gauge transformation invariance. Since gauge transformation invariance *is a spontaneous subsidiary effect of making dynamic electromagnetism Lorentz covariant, any further consequences it has ipso facto don't alter dynamic electromagnetism's Lorentz covariance.*

Upon applying the Lagrangian equation of motion  $d[\partial L/\partial(dx^\lambda/d\tau)]/d\tau = \partial L/\partial x^\lambda$  to the Eq. (1.4a)  $L = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (e/c)A_\nu(x)(dx^\nu/d\tau)$ , we obtain the Lorentz covariant dynamical equation,

$$m \eta_{\lambda\nu}(d^2x^\nu/d\tau^2) = (e/c)[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \quad (1.5a)$$

a gauge transformation invariant result. In the normal form for Lorentzian dynamics Eq. (1.5a) reads,

$$m(d^2x^\mu/d\tau^2) = (e/c)\eta^{\mu\lambda}[(\partial A_\nu(x)/\partial x^\lambda) - (\partial A_\lambda(x)/\partial x^\nu)](dx^\nu/d\tau), \quad (1.5b)$$

where  $\eta^{\mu\lambda}$  is the matrix inverse of the Minkowski metric  $\eta_{\lambda\nu}$  defined by Eq. (1.4b); in fact, of course,  $\eta^{\mu\lambda} = \eta_{\mu\lambda}$ . Eq. (1.5b) is the well-known gauge-invariant Lorentz Force Law of dynamic electromagnetism.

We next search for the extension of the static-limit Coulomb's Law expression  $-\nabla_{\mathbf{x}}^2 A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$  given by Eq. (1.2) which is Lorentz covariant and gauge invariant. In this regard we note that the Lorentz-covariant entity  $\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta)$  reduces to  $-\nabla_{\mathbf{x}}^2 A_0(\mathbf{x})$  in the static limit, and that its complementary Lorentz-covariant entity  $\eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta)$  vanishes in the static limit. Thus the Lorentz-covariant extension of  $-\nabla_{\mathbf{x}}^2 A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$  is,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) + K \eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta) = (4\pi/c) \eta_{\mu\nu} j^\nu(x), \quad (1.6a)$$

where  $K$  is an arbitrary constant. To make Eq. (1.6a) gauge transformation invariant, however,  $K$  must be chosen to be  $-1$ . Thus the Lorentz-covariant, gauge transformation invariant extension of the Coulomb's Law expression  $-\nabla_{\mathbf{x}}^2 A_0(\mathbf{x}) = 4\pi(j^0(\mathbf{x})/c)$  is the dynamic electromagnetic field equation,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) - \eta^{\alpha\beta}(\partial^2 A_\alpha(x)/\partial x^\mu \partial x^\beta) = (4\pi/c) \eta_{\mu\nu} j^\nu(x). \quad (1.6b)$$

We now multiply Eq. (1.6b) through by the operator  $(\partial/\partial x^\lambda)\eta^{\lambda\mu}$  and sum over the index  $\mu$ . Since when the expression  $\eta^{\lambda\mu}\eta_{\mu\nu}$  is summed over the the index  $\mu$ , the result is  $\delta_\nu^\lambda$ , and since  $\eta^{\lambda\mu} = \eta^{\mu\lambda}$ , the upshot of doing this is to turn Eq. (1.6b) into,

$$\eta^{\alpha\beta}\eta^{\mu\lambda}(\partial^3 A_\mu(x)/\partial x^\lambda \partial x^\alpha \partial x^\beta) - \eta^{\lambda\mu}\eta^{\alpha\beta}(\partial^3 A_\alpha(x)/\partial x^\beta \partial x^\lambda \partial x^\mu) = (4\pi/c) \delta_\nu^\lambda (\partial j^\nu(x)/\partial x^\lambda), \quad (1.6c)$$

which implies that,

$$0 = \partial j^\lambda(x)/\partial x^\lambda, \quad (1.6d)$$

so *the gauge transformation invariant* Eq. (1.6b) dynamic electromagnetic equation for  $A_\mu(x)$  *compels the local charge conservation of* Eq. (1.1). The Eq. (1.6d) result is *a particular instance* of the fact that imposing the subsidiary dynamic symmetry of gauge transformation invariance *produces locally conserved currents*; e.g., the structure of the coupling of the electromagnetic four-potential  $A_\mu(x)$  to a charged particle's quantum wave function  $\psi(x)$  is subject to the imposition of gauge transformation invariance in order to *ensure* the existence of a locally conserved current constructed from  $\psi(x)$ .

However, the very fact that Eq. (1.6b) is gauge transformation invariant implies that it has an infinite number of solutions: given any solution for  $A_\mu(x)$  *and any scalar function*  $\chi(x)$  *whatsoever*,  $A_\mu(x) + \partial\chi(x)/\partial x^\mu$  *is also a solution that is by no means guaranteed to be Lorentz covariant when, for example,  $\chi(x)$  is independent of  $x^0$* . But since gauge transformation invariance *is a spontaneous subsidiary consequence of making dynamic electromagnetism fully Lorentz covariant, any further consequences that gauge transformation invariance itself happens to have ipso facto don't alter the fully Lorentz covariant nature of dynamic electromagnetism*. Therefore the gauge transformation invariance of Eq. (1.6b) *is necessarily broken in a way which ensures that  $A_\mu(x)$  is Lorentz covariant*. The *simplest* way to ensure that  $A_\mu(x)$  is Lorentz covariant is to impose *the Lorentz condition*,

$$\eta^{\alpha\beta}(\partial A_\alpha(x)/\partial x^\beta) = 0, \quad (1.7a)$$

on  $A_\mu(x)$ , which *reduces* Eq. (1.6b) to,

$$\eta^{\alpha\beta}(\partial^2 A_\mu(x)/\partial x^\alpha \partial x^\beta) = (4\pi/c) \eta_{\mu\nu} j^\nu(x). \quad (1.7b)$$

Although the operator  $\eta^{\alpha\beta}(\partial^2/\partial x^\alpha \partial x^\beta)$  *doesn't have a unique inverse, only its retarded inverse is physically appropriate to the causality of  $A_\mu(x)$  with respect to  $j^\nu(x)$* . That together with Eq. (1.7b) implies that,

$$A_\mu(x) = (4\pi/c) [\eta^{\gamma\delta}(\partial^2/\partial x^\gamma \partial x^\delta)]_{\text{ret}}^{-1} (\eta_{\mu\nu} j^\nu(x)). \quad (1.7c)$$

The Eq. (1.7c) *causal and Lorentz covariant result for the dynamic electromagnetic potential  $A_\mu(x)$  is the consequence of breaking the gauge transformation invariance of* Eq. (1.6b) *by applying the retarded Lorentz gauge condition*<sup>[1]</sup> *to it*. The Eq. (1.7c) *result for  $A_\mu(x)$  can readily be shown to be consistent with the* Eq. (1.7a) *Lorentz condition because  $j^\lambda(x)$  satisfies the local charge conservation condition  $\partial j^\lambda(x)/\partial x^\lambda = 0$* .

## 2. Lorentz-covariant extension of static Newtonian gravity to dynamic metric gravity

Gravity is taken to be the result of conserved energy-momentum, a four-vector  $P^\mu$ . The second-rank symmetric-tensor density-flux of energy-momentum  $P^\mu$ , i.e., the energy-momentum tensor  $T^{\mu\nu}(x)$ , is the *source* of the second-rank symmetric-tensor gravitational potential  $\phi_{\mu\nu}(x)$ . Gravity *itself*, however, *also contributes to the total conserved energy-momentum, a nonlinear effect that can be neglected in the weak-gravity*

static limit, where it is assumed that the static energy density  $T^{00}(\mathbf{x})$  divided by  $c^2$  is an effective mass density which acts as the source of the gravitational potential component  $\phi_{00}(\mathbf{x})$  in accord with the static differential Newtonian Law of Gravity,

$$\nabla_{\mathbf{x}}^2 \phi_{00}(\mathbf{x}) = 4\pi G (T^{00}(\mathbf{x})/c^2). \quad (2.1)$$

The nonrelativistic equation of motion of a test body of mass  $m$  in the static Newtonian gravitational potential  $\phi_{00}(\mathbf{x})$  is of course,

$$m d^2 \mathbf{x}/dt^2 = -m \nabla_{\mathbf{x}} \phi_{00}(\mathbf{x}). \quad (2.2a)$$

The Eq. (2.2a) nonrelativistic equation of motion corresponds to the nonrelativistic Lagrangian,

$$L(d\mathbf{x}/dt, \phi_{00}(\mathbf{x})) = (m/2)|d\mathbf{x}/dt|^2 - m \phi_{00}(\mathbf{x}), \quad (2.2b)$$

which, exactly as the Eq. (1.3b) nonrelativistic Lagrangian leads to its corresponding Eq. (1.4a) Lorentz-invariant Lagrangian, leads to the following corresponding Lorentz-invariant Lagrangian,

$$L(dx^\mu/d\tau, \phi_{\mu\nu}(x)) = -(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau) - (m/c^2)\phi_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau), \quad (2.3a)$$

which can be reexpressed in the astonishingly simple dynamic gravitational metric form,

$$L(dx^\mu/d\tau, g_{\mu\nu}(x)) = -(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau), \quad (2.3b)$$

where,

$$g_{\mu\nu}(x) \stackrel{\text{def}}{=} \eta_{\mu\nu} + (2/c^2) \phi_{\mu\nu}(x). \quad (2.3c)$$

The Eq. (2.3b) Lorentz-invariant Lagrangian  $-(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$  for a test body in the Lorentz-covariant dynamic gravitational metric  $g_{\mu\nu}(x)$  merely swaps the Minkowski metric  $\eta_{\mu\nu}$  in the Eq. (1.4c) Lorentz-invariant Lagrangian  $-(m/2)\eta_{\mu\nu}(dx^\mu/d\tau)(dx^\nu/d\tau)$  of a free test body for the Lorentz-covariant dynamic gravitational metric  $g_{\mu\nu}(x)$ . Thus gravity is very simply and precisely characterized as a Lorentz-covariant distortion of the Minkowski metric of space-time.

The Eq. (2.3b) Lagrangian  $-(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$ , in addition to being Lorentz-invariant has the form of an invariant under general coordinate transformations when it is assumed that  $g_{\mu\nu}(x)$  transforms as a covariant second-rank symmetric tensor under general coordinate transformations. Thus, just as gauge transformation invariance is a spontaneous subsidiary consequence of making dynamic electromagnetism Lorentz covariant, general coordinate transformation covariance is a spontaneous subsidiary consequence of making dynamic gravitation Lorentz covariant.

We note that Eq. (2.3c) implies that  $\phi_{\mu\nu}(x) = (c^2/2)(g_{\mu\nu}(x) - \eta_{\mu\nu})$ , which permits the Eq. (2.1) static differential Newtonian Law of Gravity to be reexpressed entirely in terms of  $g_{00}(\mathbf{x})$  and  $T_{00}(\mathbf{x})$ ,

$$\nabla_{\mathbf{x}}^2 g_{00}(\mathbf{x}) = (8\pi/c^4) G T^{00}(\mathbf{x}). \quad (2.4)$$

Applying the Lagrangian equation of motion  $d[\partial L/\partial(dx^\lambda/d\tau)]/d\tau = \partial L/\partial x^\lambda$  to the Eq. (2.3b) gravitational metric Lagrangian  $L = -(m/2)g_{\mu\nu}(x)(dx^\mu/d\tau)(dx^\nu/d\tau)$  yields the following Lorentz covariant result,

$$m g_{\lambda\nu}(x) (d^2 x^\nu/d\tau^2) = -(m/2) [\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda] (dx^\mu/d\tau) (dx^\nu/d\tau). \quad (2.5a)$$

Because of the matrix metric factor  $g_{\lambda\nu}(x)$  on its left side, Eq. (2.5a) isn't in the normal form for Lorentzian dynamics. However, if for all  $x$  the metric  $g_{\lambda\nu}(x)$  has the matrix inverse  $g^{\kappa\lambda}(x)$  such that, when summed over the index  $\lambda$ , the product  $g^{\kappa\lambda}(x)g_{\lambda\nu}(x)$  yields  $\delta_\nu^\kappa$ , then Eq. (2.5a) can be put into the following normal form for Lorentzian dynamics,

$$m (d^2 x^\kappa/d\tau^2) = -(m/2) g^{\kappa\lambda}(x) [\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda] (dx^\mu/d\tau) (dx^\nu/d\tau), \quad (2.5b)$$

which is the gravitational geodesic equation for the motion of a test body in the metric  $g_{\mu\nu}(x)$ . The Eq. (2.5b) gravitational geodesic equation's usual presentation<sup>[2]</sup> is,

$$d^2 x^\kappa/d\tau^2 + \Gamma_{\mu\nu}^\kappa(x) (dx^\mu/d\tau) (dx^\nu/d\tau) = 0, \quad (2.5c)$$

where the affine connection  $\Gamma_{\mu\nu}^\kappa(x)$ <sup>[3]</sup> is defined as,

$$\Gamma_{\mu\nu}^\kappa(x) \stackrel{\text{def}}{=} (1/2) g^{\kappa\lambda}(x) [\partial g_{\lambda\mu}(x)/\partial x^\nu + \partial g_{\lambda\nu}(x)/\partial x^\mu - \partial g_{\mu\nu}(x)/\partial x^\lambda]. \quad (2.5d)$$

We've noted that when the metric  $g_{\mu\nu}(x)$  transforms as a second-rank symmetric covariant tensor under general coordinate transformations, the Eq. (2.3b) Lagrangian which yields the gravitational geodesic equation *is a general invariant*. Therefore it isn't at all surprising that under those circumstances the gravitational geodesic equation *itself transforms as a generally contravariant vector*<sup>[4]</sup>.

Just as in electromagnetic theory the Eq. (1.2) static Coulomb's Law is parlayed into the Eq. (1.6b) gauge transformation invariant equation for the electromagnetic four-vector potential  $A_\mu(x)$ , so in gravity theory the Eq. (2.4) weak-gravity static differential Newtonian Law of Gravity is parlayed into the general coordinate transformation covariant Einstein equation for the metric  $g_{\mu\nu}(x)$ . Since the Einstein equation must imply the Eq. (2.4) differential Newtonian Law of Gravity in the weak-gravity static limit, its left side necessarily involves second derivatives with respect to space-time. Also, its left side is a second-rank symmetric tensor in order to match that property of its energy-momentum source. These requirements, along with general coordinate transformation covariance, pretty much pin down the left side of the Einstein equation as a linear combination of the product of the curvature scalar with the metric and the Ricci curvature tensor, both of which *are nonlinear in the metric*. The final details of the Einstein equation arise from the vanishing of the generally covariant divergence of its left side in order to match that property of its energy-momentum source, and of course also from the requirement that the Einstein equation implies Eq. (2.4) in the weak-gravity static limit.

Because the Einstein equation is generally covariant, every general coordinate transformation of any of its metric solutions is also a metric solution, so it appears to present a daunting solution ambiguity. However, we have seen that its general covariance is a spontaneous subsidiary effect of the search for a Lorentz covariant theory of gravity, just as the gauge transformation invariance of four-vector potential electromagnetism is a spontaneous subsidiary effect of the search for a Lorentz covariant four-vector potential theory of electromagnetism. Therefore we ipso facto only need to consider Lorentz covariant solutions of the Einstein equation. Furthermore, the Riemann and Ricci curvature tensors and the curvature scalar are all constructed from the affine connection and its first derivatives with respect to space-time, but the affine connection *isn't well defined unless the metric has a matrix inverse for all values of  $x$*  (see Eq. (2.5d)). Thus *we must require that  $\det(g_{\mu\nu}(x)) \neq 0$  for all values of  $x$* . It turns out that this requirement *can be combined with an affirmation of the Lorentz covariance of the metric  $g_{\mu\nu}(x)$  in an astonishingly simple way by requiring that  $\det(g_{\mu\nu}(x)) = k$  for all values of  $x$ , where  $k$  is a fixed nonzero constant*. But because  $\det(\eta_{\mu\nu}) = -1$ , the *only* value that the fixed nonzero constant  $k$  can have is  $-1$ .

In 1915 Einstein *also* arrived at this coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$ , but via a *temporary foray* into the idea that physics is covariant under linear coordinate transformations of unit determinant<sup>[5]</sup> *in place of* his 1913 "general relativity" *idée fixe* that physics is covariant under general coordinate transformations. *Because of his very different mode of arrival at the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$* , Einstein apparently was *oblivious* to the twin facts *that it crucially guarantees the existence of the matrix inverse of the metric  $g_{\mu\nu}(x)$  and is consistent with the Lorentz covariance of  $g_{\mu\nu}(x)$* . Regardless, Einstein's application of the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$  in his landmark November 18, 1915 paper *produced the correct values for both Mercury's remnant perihelion shift and the deflection of starlight by the sun's gravity*<sup>[5]</sup>, but *before* he adopted the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$ , Einstein had spent approximately two years struggling with results for Mercury's perihelion shift *which were substantially too small*<sup>[5]</sup>. Furthermore, the *correct* result for the deflection of starlight by the sun's gravity which results from application of the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$  *contradicts the result previously firmly predicted by Einstein's highly-touted Principle of Equivalence*.

*In spite of these hard facts*, Einstein in November 1915 *renewed his adherence to his 1913 "general relativity" *idée fixe* and incredibly absolved the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$  of having any particular significance whatsoever!* Consequently the coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$ , which is *central* to the tremendous achievements of Einstein's landmark November 18, 1915 paper, *isn't mentioned at all in gravity textbooks!* Einstein's bizarrely precipitous *abandonment* of his spectacularly successful November 18, 1915 coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$  *paved the way for Alexandre Friedmann's promotion, starting in 1922, of his alternate coordinate condition  $g_{00}(x) = 1$  for all  $x$ , which flatly rules out  $g_{\mu\nu}(x)$  being Lorentz covariant and totally eliminates the extremely well-established phenomenon of gravitational time dilation because that effect is given by*<sup>[6]</sup>,

$$[(\text{the tick rate of a clock at } x_2)/(\text{the tick rate of a clock at } x_1)] = \sqrt{g_{00}(x_2)/g_{00}(x_1)}. \quad (2.6)$$

In spite of these unphysical characteristics, Friedmann's coordinate condition  $g_{00}(x) = 1$  for all  $x$  was, *without a rational justification for doing so*, incorporated into the standard Robertson-Walker metric form which is almost universally used in cosmology, i.e.,

$$(c d\tau)^2 = (c dt)^2 - (R(t))^2\{(1/(1 - kr^2))(dr)^2 + r^2[(d\theta)^2 + (\sin \theta d\phi)^2]\}. \quad (2.7a)$$

It is realized that metrics of the form of Eq. (2.7a) impose *Newtonian gravity* on cosmological models<sup>[7]</sup>, a consequence of which is a Big Bang singularity wherein  $R(t)$  was equal to zero at a past finite time  $t$ <sup>[8]</sup>. At that past finite time  $t$  when  $R(t)$  was equal to zero, the matrix inverse of the Eq. (2.7a) Robertson-Walker metric form *was obviously undefined*, so at that past finite time  $t$  the affine connection (the gravitational field) *was undefined, as were the curvature tensors!*

The albatross of *Newtonian gravity* which Friedmann's coordinate condition  $g_{00}(x) = 1$  for all  $x$  imposes on cosmological models is, however, *completely unnecessary*. For example, *in the special case that  $k = 0$* , the Newtonian-gravity Robertson-Walker metric which satisfies Friedmann's coordinate condition  $g_{00}(x) = 1$  for all  $x$ , i.e.,

$$(c d\tau)^2 = (c dt)^2 - (R(t))^2\{(dr)^2 + r^2[(d\theta)^2 + (\sin \theta d\phi)^2]\}, \quad (2.7b)$$

*is very easily coordinate-transformed to the metric,*

$$(c d\tau)^2 = (1/S(t))^6 (c dt)^2 - (S(t))^2\{(dr)^2 + r^2[(d\theta)^2 + (\sin \theta d\phi)^2]\}, \quad (2.7c)$$

which clearly satisfies *the Lorentz-covariant coordinate condition  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$  that is non-Newtonian and obviously doesn't permit a Big Bang singularity.*

The very simplest expanding-dust-sphere cosmological model has recently been studied in detail both in Friedmann coordinates, where  $g_{00}(x) = 1$  for all  $x$ , and in Einstein coordinates, where  $\det(g_{\mu\nu}(x)) = -1$  for all  $x$ <sup>[9]</sup>. The deceleration of cosmic expansion in Friedmann coordinates is changed in Einstein coordinates to its acceleration, and the Big Bang in Friedmann coordinates is swapped in Einstein coordinates for a peak in that inflation.

## REFERENCES

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