Improvement of control performance in the random vibration tester using fuzzy CMAC neural network

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ABSTRACT

These papers introduce control method of random vibration test using fuzzy CMAC (Cerebellar Model Articulation Controller) based on high learning power of CMAC and knowledge-expression of fuzzy.

Proposed method is real time control method in time domain in some other way spectra method based on amplitude frequency response.

These papers establish the random vibration control theory to neglect the change of mathematical models of vibration testers according to its types and sizes, using fuzzy CMAC neural network.

Finally, its accuracy is proved through MATLAB SIMULINK simulation.

Keywords: CMAC neural network, random vibration tester, variable structure control, sliding control, environmental test

Declaration of interest: none

1. Introduction

The random vibration test is to simulate the actual environment of rocket (artificial satellite) during its flight. Now there are mainly two methods for control of random vibration testers. There are two methods such as spectra feedback method and waveform control method. These two methods have advantages and shortcomings respectively.

The spectra method takes advantages that its control algorithm is simple, while its control isn't so accurate, then waveform control method takes advantages in the view of control accuracy but control algorithm is complex.

In the literature [1], the control scheme for random vibration testers by the spectral feedback is proposed, considering that the random signal control by controlling the frequency spectra of excitation signal in real time is more convenient than by servo feedback because input of the vibration testers is random. These methods are also proposed in many literatures such as [2,5].

Also, in many literatures such as [3,4], they mentioned that the waveform must be controlled to improve the control performance of the random vibration testers.

In this paper, waveform control method of random vibration testers is considered.

In recent years, to improve the accuracy of random vibration testers, many researchers have studied waveform control of random vibration signal and realized the anticipated results.

Nowadays, to achieve the waveform control of random vibration signal, many optimal control methods are used. However, these optimal control methods presuppose exact model of random vibration testers or the model without uncertainty so it is difficult for these methods to be applied to the plant with uncertainty.

In order to solve these problem high-learning CMAC is used control domain such as [7, 9, 10]. There are two main problems for linear optimal control- one is to determine the design goal in terms

of performance index and another is for the plant with sharp variance of parameters. To overcome

these problems, there are model tracking control method based on model tracking principle and model reference sliding mode control method operating by sliding mode as the design method consideration of uncertainties in terms of parameters and model.

However, these methods require boundary of uncertainty for the control plant during its design, so in this paper, based on the present research achievements, to achieve unknown control purpose in the design, variable structure control method for the random vibration testers using fuzzy CMAC neural network is proposed and proved.

In Section 2 is mentioned mathematical modeling of random vibration test, in section 3 is mentioned design of random vibration test control system using fuzzy-CMAC.

In section 4 is mentioned simulation result using MATLAB.

2. Mathematical model of the random vibration tester

In the vibration testers, the main plant is an exciter.

To derive the transfer property of the exciter, let us assume as follows.

(1) Attenuation and loss of the machine components are neglected because it is too small.

(2) Effective resistance and inductance of the advance winding take the mean value because of some reasons such as permeability change of iron according to the effect between the advance winding and its surrounding metal construction, and flux density.

(3) The ratio of the exciting force in terms of the currents $\Gamma = Bl$ is constant.

(4) The body of the shake table is fixed so its motion is neglected.

First, consider the advance winding. Assume that the driving winding consists of circuited loop, then it can be described as follows.

Fig. 1. Equivalent circuit of the advance winding with one short-circuited loop

From fig 1, we have the following differential equation
\n
$$
\begin{cases}\nR_1I_1 + L_1 \frac{d}{dt}I_1 - MI_2 + P \frac{dx}{dt}I_2 + U_1 = 0 \\
R_2I_2 + L_2 \frac{d}{dt}I_2 - M \frac{d}{dt}I_1 = 0\n\end{cases}
$$
\n(1)

where R_1, R_2, L_1, L_2, M are resistance, inductance and mutual inductance in terms of stator and rotor respectively and P is counter electromotive force coefficient owing to the motion of vibrator.

Then, consider the moving part.

The simplified structure of moving part equals as shown in fig 2.

The kinetic state of shake table can be expressed using the dynamic model with 1 freedom and concentration parameter. (Fig 2-5)

Fig. 3. Simplified moving part

Form Fig 2, the dynamic model of the moving part follows
\n
$$
m\ddot{x}(t) + c\dot{x}(t) + kx = f_{\equiv} - f_{\equiv} - mg
$$
\n(2)

where $f_{ro}(t)$ is Lorenz force then can be expressed as follows.

$$
f_{\rm ro} = BII \tag{3}
$$

Then, $f_{\text{spring}}(x, \dot{x})$ is elastic force by suspension spring.

This elastic force is determined linearly by Hooke's law but the spring on the vibration testers is hard so it's considered as linear one

$$
f_{\text{spring}} = kx \tag{4}
$$

where *k* is elastic coefficient.

From the above equations, we have the following differential equation model.

$$
\begin{cases}\nR_1I_1 + L_1I_1 - MI_2 + P\dot{x}(t)I_2 + U_1 = 0 \\
R_2I_2 + L_2I_2 - MI_1 = 0 \\
m\ddot{x}(t) + c\dot{x}(t) + kx = f_{\text{ro}} - f_{\text{spring}} - mg \\
f_{\text{spring}} = BI_2l \\
f_{\text{g}}(x, \dot{x}) = kx(t)\n\end{cases} \tag{5}
$$

In the above equation, considering that R_1, R_2 are so small, eq. (5) can be rewritten as follows

$$
\dot{X} = f(X) + gu \tag{6}
$$

where $X = \begin{bmatrix} x & \dot{x} & I_2 \end{bmatrix}$ is state variable and g is gain in terms of control input u. So in the case without testing plant, the system can be considered 3-order nonlinear system.

3. Controller design of random vibration testers using CMAC neural network

3.1. Design of the model tracking variable structure controller

In this section, consider the proposed controller design of model reference variable structure controller.

First of all, consider the structure of general model reference tracking control law.

Linear stationary plant is as follows.

$$
\dot{x}(t) = Ax(t) + Bu(t)
$$
\n(7)

The required reference model is as follows

$$
\dot{w}(t) = A_m w(t) + B_m r(t) \tag{8}
$$

where $x \in R^n$ and $w \in R^n$ are state vectors of the plant respectively, $u \in R^m$ is control vector, $r \in R^r$ is input vector, and A, B, A_m, B_m are matrices with suitable dimensions.

Assume that (A, B) is controllable and all eigenvalues of system matrix for the reference model have negative real parts.

Tracking error *e* is difference between state of the plant and one of the model, it is as follows.

$$
e(t) = x(t) - w(t) \tag{9}
$$

In model reference method, this error must be converged to zero asymptotically.

Differentiating error equation (9) in terms of time, it is as follows.

$$
\dot{e}(t) = \dot{x}(t) - \dot{w}(t) \tag{10}
$$

Then, dynamics of model tracking error system can be determined from eq (7) and eq (8).
 $\dot{e}(t) = Ax(t) - A_m w(t) + Bu(t) - B_m r(t)$ (11)

$$
\dot{e}(t) = Ax(t) - A_m w(t) + Bu(t) - B_m r(t)
$$
\n(11)

Adding and subtracting term $\mathbf{A}\mathbf{w}(t)$ in Eq (11), it is as follows.

$$
\dot{e}(t) = Ae(t) + (A - A_m)w(t) + Bu(t) - B_m(t)
$$
\n(12)

So it's clear that perfect model tracking system can be determined in terms of given plant and model. If error is zero after arbitrary time *t*, we have the following error equation from Eq (7) and Eq (8).

$$
\dot{e}(t) = (A_m - BK)e(t) \tag{13}
$$

If (Am, B) is controllable pair, we can arrange the eigenvalues of the closed system matrix at the arbitrary locations, like this, we can regulate the convergence time of error.

Detailed control law is as follows.

$$
u(t) = u_1(t) + u_2(t)
$$
 (14)

Separating into each element, it is as follows

$$
u_1(t) = -Ke(t) \tag{15}
$$

$$
u_2(t) = Fx(t) + Gr(t)
$$
\n⁽¹⁶⁾

where F and G are matrices satisfying the model match condition.

Based on the above, consider the variable structure controller design of model tracking control system with sliding mode.

In general, performance of the system required in the design is defined by the model and the controller is designed to minimize the difference between model state and control system state.

Here, the part of the control law- Eq (9) must be determined for the closed system to be robust in terms of both parameter variances from the plant and other external disturbance.

Consider the stationary multi variable control plant expressed by Eq (1).

Here the output of the plant must track the output of model Eq (8).

Dynamics of model tracking system can be obtained from Eq (11) and adding and subtracting the term A_m , it is as follows.

$$
\dot{e}(t) = Ae(t) + (A - A_m)w(t) + Bu(t) - B_m(t)
$$
\n(17)

The design goal is to determine the coefficient matrix of sliding hyper plane *S* and its discrete control law for state error so as to reach sliding hyper plane.

Now, let us define the switching function in terms of error as follows.

$$
l(e) = Se(t) \tag{18}
$$

So, the following hyper plane can be defined in the error vector space as follows.

$$
Se(t) = \{e \in R^n : Se = 0\}
$$
 (19)

When the closed system has its sliding motion, the error state satisfies the following equation. $Se(t) = 0$ (20)

Differentiating this equation in terms of time and substituting Eq (19), it is as follows.

$$
S\dot{e}(t) = S(Ae(t) + (A - A_m)x(t) + Bu(t) - B_m r(t))
$$
\n(21)

If the matrix product *SB* is nonsingular, the equivalent control law during sliding motion can be determined from Eq (21).

$$
u_{eq} = -(SB)^{-1} S(Ae(t) + (A - A_m)x(t) - B_m r(t))
$$
\n(22)

Substituting this equivalent signal into model tracking error system Eq (17), it is as follows.

$$
\dot{e}(t) = (I - B(SB)^{-1}S)(Ae(t) + (A - A_m)x(t) - B_m r(t))
$$
\n(23)

Assume that matrix (A, B, A_m, B_m) satisfies the model match condition.

Comparing the dynamics of Eq (23) with the one of the reduced order sliding motion by the sliding invariance condition, it can be shown that its dynamic motions are equal.

Therefore, considering that *x* and *r* are disturbance exerted on error system dynamics, the model match condition ensures that the dynamics of the variable structure error system with the sliding motion is insensitive to the disturbance.

Finally, Eq (23) can be written as follows.

$$
\dot{e}(t) = (I - B(SB)^{-1}S)Ae(t) = A_{eq}e(t)
$$
\n(24)

The error system with sliding motion is nearly equal to n-m order and the response is determined by n-m nonzero eigenvalues of Eq (24).

To attenuate error fast enough during the sliding motion, n-m eigenvalues must be arranged far away from imaginary axis in the left half plane of complex number sufficiently.

On the other hand, if the matrix pair (Am, B) is given, the sliding hyperplane matrix *S* can be found. If a suitable hyperplane matrix *S* and nonlinear control structure using unit vector are chosen, we can determine controller parameters in order to lead the error vector to zero space of the matrix S and keep it in the subspace S of the sliding surface.

Also, m eigenvalues associated with the parameter matrix Φ for controller parameters determine the velocity of the error state to reach into sliding surface.

 $P_2 \in R^{m \times m}$ is a symmetric matrix which satisfies Lyapunov equation

$$
P_2\Phi + \Phi^T P_2 = -I \tag{25}
$$

Then, the part of error feedback u_1 in Eq (14) is defined as $u_1 + u_n$. Here, each part is as follows.

$$
u_t = -(SB)^{-1}(SA_m - \Phi S)e(t)
$$
 (26)

$$
u_{t} = -(SB)^{-1}(SA_{m} - \Phi S)e(t)
$$
\n
$$
u_{n} = -\rho(t, e)(SB)^{-1} \frac{P_{2} s(t)}{\|P_{2} s(t)\|}, s(t) \neq 0
$$
\n(27)

Finally, the model tracking variable structure control law is as follows.
 $u(t) = u_1(t) + u_n(t) + u_2(t)$

$$
u(t) = u_1(t) + u_n(t) + u_2(t)
$$
\n(28)

The scalar function $\rho(t, e)$ multiplied before unit vector component is only related to the amount of ertainty and it must be defined by the amount of uncertainty as follows
 $\rho(t, x) \ge \frac{\|S\| \left[\|M\| (k_1 \|x(t)\| + k_2) + k_3 \|u_t(t)\$ uncertainty and it must be defined by the amount of uncertainty as follows plied before unit vector component is
the amount of uncertainty as follows
 $(k_1 || x(t) || + k_2) + k_3 || u_1(t) || + \alpha(t, x)$ *e*) multiplied before unit vector component is
efined by the amount of uncertainty as follows
 $S\left\| \begin{bmatrix} ||M||(k_1||x(t)||+k_2)+k_3||u_1(t)||+\alpha(t,x)|| \end{bmatrix} \right\|$

it must be defined by the amount of uncertainty as follows
\n
$$
\rho(t,x) \ge \frac{\|S\| \left[\left\| M \right\| (k_1 \left\| x(t) \right\| + k_2) + k_3 \left\| u_1(t) \right\| + \alpha(t,x) \right] + \gamma_2}{(1 - k_3 k(\Delta) \left\| B_2^{-1} \right\|)} \tag{29}
$$

where γ_2 is design parameter.

As shown in Eq (29), if the information for uncertainty of the plant is given, we can calculate minimum boundary of $\rho(t, x)$ to keep the sliding motion using it.

But when the uncertainty boundary of disturbance is not clear in the design, strong assumption for disturbance amount increases the control effect thus the amplitude of the high frequency near the sliding hyper plane becomes bigger, and otherwise weak assumption can't ensure the quality of the reach process as parameter $\rho(t, x)$ then it can't satisfy the condition $s\dot{s} < 0$ by the effect of disturbance.

Thus, in order to be satisfied reach condition to sliding hyper plane under the disturbance, it's useful to change parameter $\rho(t, x)$ according to s and s in order to satisfy the condition $s\dot{s} < 0$.

Therefore, in the next part, a method that changes $\rho(t, x)$ according to s and s using fuzzy CMAC neural network.

3.2. Design of the variable structure controller by fuzzy CMAC neural network

Here, it is considered how to design the controller if the amount of disturbance is not defined in design as proposed before.

Let us define $V = \frac{1}{2} s^2$ as Lyapunov function, then $\dot{V} = s\dot{s}$.

The fuzzy control rule can ensure asymptotic stability with designing the switched control input to satisfy the reach condition $\dot{s}s$ < 0 based on Lyapunove condition \dot{V} < 0 and the condition for stable surface $s = \dot{s} = 0$.

The fuzzy control rule is based on the following sliding control principle for the standardized phase plane.

Condition 1: The switching control input must be negative on the sliding surface but positive outside the sliding surface.

Condition 2: With the increment of distance between actual state and sliding surface, the absolute value of the switching control input must be increased.

Based on the above conditions, the fuzzy control rule changing the switching control input adaptively is as follows.

Rule 1: IF $S = P$ AND $DS = P$ THEN $U_{fs} = NH$ Rule 2: IF $S = P$ AND $DS = Z$ THEN $U_{fs} = NB$ Rule 3: IF $S = P$ AND $DS = N$ THEN $U_{fs} = NM$ Rule 4: IF $S = Z$ AND $DS = P$ THEN $U_{fs} = NS$ Rule 5: IF $S = Z$ AND $DS = Z$ THEN $U_{fs} = ZE$ Rule 6: IF $S = Z$ AND $DS = N$ THEN $U_{fs} = PS$ Rule 7: IF $S = N$ AND $DS = P$ THEN $U_{fs} = PM$ Rule 8: IF $S = N$ AND $DS = Z$ THEN $U_{fs} = PB$ Rule 9: IF $S = N$ AND $DS = N$ THEN $U_{fs} = NH$

Rule 1 is derived from the condition that the biggest switching control input is required for the sliding mode condition if the sliding variable *s* is on the sliding surface and differentiation of *s* is positive.

Rule 5 is derived from the condition that the switching control input must be zero if the sliding variable *s* is on the sliding surface and differentiation of *s* is zero.

Other fuzzy rules can be defined in the same way. The standardization coefficients N_S , N_{DS} and N_K are defined by trial and error according to the practical control problems. S and DS use the Gaussian functions.

Unlike the traditional CMAC neural network, the fuzzy CMAC neural network uses fuzzy logic.

The difference between the fuzzy CMAC neural network and the traditional neural network is that the membership function guides associative cells. In other words, functions in the output field are defined in the real field (0, 1].

Fig 4 shows the example of two-dimension fuzzy CMAC

Fig 4. The structure of two-dimension fuzzy CMAC where $n_g = 9$. The mapping method is the same as one in the traditional CMAC. The output of fuzzy CMAC neural network is as follows

$$
r = \sum_{i=0}^{n_g - 1} w_i \mu_i
$$
 (30)

where *j* is the corresponding storage address for *i* and is defined as follows.

$$
\mu_i = \mu_{1,i} \mu_{2,i} \tag{31}
$$

where $\mu_{k,i}$ is the *i*th membership value of k^{th} input in the middle layer *M*. Cost function is as follows.

$$
J=\frac{1}{2}s^2
$$

The weight adjustment law is designed as follows.

$$
w_{j+1} = w_j - \beta s \mu_i(x) \tag{32}
$$

-The design scheme of the variable structure controller based on the proposed fuzzy CMAC neural network

Step 1: initial step

All storages of fuzzy CMAC neural network are initialized.

Step 2: mapping from X space to A space

As inputs of the fuzzy CMAC neural network, quantized *s* and its differentiation and membership values for 9 rules are calculated.

Step 3: output Using Eq (29), the output of the fuzzy CMAC neural network is calculated and sent out. Step 4: learning

At the end of each control period, quantized *s* and its differentiation are used as input variables and the switching function s is used for learning of the fuzzy CMAC neural network by Eq (31).

Step 5: go back to step 2.

4. Simulation example

The proposed control scheme is applied to the random vibration testers.

Table 1. The characteristic values of the random vibration tester

The transfer function model of the random vibration tester is given as follows

$$
G(S) = \frac{KS^2}{(S + \omega_1)(S^2 + 2\xi\omega_2S + \omega_2^2)}
$$
(33)

where $\omega_1 = 14 \pi (rad / s)$, $\omega_2 = 1400 \pi (rad / s)$, $\xi = 0.15$, $K = 6000$.

The reference model of the random vibration tester is as follows

$$
G_m(S) = \frac{KS^2}{(S + \omega_{m1})(S^2 + 2\xi\omega_{m2}S + \omega_{m2}^2)}
$$
(34)

where $\omega_{m1} = 14 \pi (rad/s), \omega_{m2} = 3600 \pi (rad/s), \xi = 1, K = 6000$.

Changing the transfer function into the state space model, we have as follows

$$
\dot{x}(t) = Ax(t) + Bu(t)
$$

\n
$$
y = Cx(t)
$$
\n(35)

where

$$
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ K \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
$$

In the same way, changing the transfer function of the reference model into the state space model, it is as follows

$$
\dot{x}(t) = A_m x(t) + B_m u(t)
$$

\n
$$
y = C_m x(t)
$$
\n(36)

where

$$
A_{m} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{m0} & -a_{m1} & -a_{m2} \end{bmatrix}, \quad B_{m} = \begin{bmatrix} 0 \\ 0 \\ K_{m} \end{bmatrix}, \quad C_{m} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
$$

Then, arranging both poles at $(-50, 0)$ so that two eigenvalues of A_{eq} can be located far away from the imaginary axis, we can get $S = [2500 100 1]$.

Using Eq (16), (27)-(30), the control law can be determined as follows
\n
$$
u(t) = u_1(t) + u_n(t) + u_2(t) = [9.38e5 \quad 2.15e4 \quad 3.76] \cdot \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} +
$$
\n[-7.96e5 -1.83e4 -3.55] $\cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ + 2r - $\rho(x, t) sign(s)$ (37)

The fuzzy membership function sets as Gaussian membership function.

To prove the robustness and disturbance rejection of the sliding controller, we changed the plant parameters in the boundary of 30% and the random signal that its mean value is zero and the variance is 1, is exerted on the output of the plant as disturbance.

Then $\rho(x,t)$ is determined by the proposed fuzzy CMAC neural network.

Matlab Simulink configuration in Fig 5 shows in more detail.

Fig 5. Synthesis Simulink configuration of the proposed control scheme

In a few words, the random signal and sine signal are used as the testing signal and the simulation result with the random signal is of importance because it is a random vibration tester.

The one next to the input signal is a filter, which is designed suitable for the mission of random vibration tester with the bandwidth of 5-2000Hz.

Fig 6. The sliding controller Simulink configuration by the fuzzy CMAC neural network

Fig 7. Spectral and tracking curve ($\omega_2 = 1300 \pi (rad/s)$, $\xi = 0.7$, $K = 6000$) The error characteristics according to the testing product are as follows The results by the Zikhonov method are presented in [11].
Table 3 Simulink results for various Simulink results for various cases

As shown through the simulation, novel proposed control scheme has bigger errors in the learning process of the CMAC neural network but smaller one after learning than the traditional control scheme. In addition, the simulation results show that the proposed method has a lower frequency domain error than the Zikhonov method presented in [11].

These papers establish the random vibration control theory to neglect the change of mathematical models of vibration testers according to its types and sizes, using fuzzy CMAC (Cerebellar Model Articulation Controller) neural network.

5. CONCLUTION

In the paper, we have done the mathematical modeling of the random vibration test system and framed the control algorithm using the fuzzy CMAC neural network based on it. Finally, using MATLAB SIMULINK, we have constructed Simulink configuration and simulated so proved the effectiveness of the proposed method.

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