The Empty Set Constructs the Natural Numbers

Jiang Yang

Abstract

In this paper, I construct natural numbers by using empty sets, cardinality of set theory and definite operations. Based on the discussion of kernel numbers dynamic space reasoning in 〔1〕 to 〔4〕, the ruler set is introduced. And the enhanced definition of one-to-one correspondence mapping is called one-to-one correspondence ordinal mapping. And it makes the Continuum Hypothesis(CH) a new conclusion.

0. From Nonexistence to Pass Into Existence

Among integers, 0 is something that doesn't exist.Natural numbers start with 1.n set theory, no element is defined as the empty set, denoted as Ø. In accordance with Definition 2.3 of 〔1〕

Definition 0.0 The cardinality of the empty set is 0, denoted as

$$0 = |\varnothing| \tag{0.0}$$

Definition 0.1 The set consisting of all subsets of the empty set is denoted by,

$$\{\{\varnothing\}\} \tag{0.1}$$

Its cardinality is 1, denoted as

$$1 = |\{\{\varnothing\}\}| \tag{0.2}$$

Definition 0.2 The set of all subsets of $\{\{\varnothing\}\}$ or $\{1\}$, denoted by

$$\{\varnothing, \{\varnothing\}\} = \{\varnothing, \{1\}\} \tag{0.3}$$

Its cardinality is 2, denoted as

$$2 = |\{\varnothing, \{\varnothing\}\}| = |\{\varnothing, \{1\}\}| \tag{0.4}$$

In the same way,

Definition 0.3 The set of all subsets of $\{\emptyset, \{\emptyset\}\}$ or $\{1,2\}$, denoted by

$$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} = \{\emptyset, \{1\}, \{2\}, \{1,2\}\} \tag{0.5}$$

Its cardinality is 4, denoted as

$$\begin{aligned} 4 &= |\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}| \\ &= |\{\emptyset, \{1\}, \{2\}, \{1,2\}\}| \end{aligned} \tag{0.6}$$

Obviously no set with cardinality between 2 and 4 can be obtained by the power set method. Then we know that of the natural numbers

$$3 = 1 + 2$$

In set theory,

$$3 = |\{0\} \cup \{1,2\}|$$

Hence there exist certain operations, such as addition and division of natural numbers, union operations in set theory, and so on。 By choosing one of these operations, we can construct the sets of cardinalities of non-power sets.

Definition 0.4 The set of all subsets of non-power sets, $\{0,1,2\}$, is denoted as,

$$\{\{\emptyset\}, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\} \tag{0.7}$$

Its cardinality is 8, denoted as

$$8 = |\{\{\emptyset\}, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}| \tag{0.8}$$

Definition 0.5 A non-power set cardinality can be constructed by adding 1 operation.

So there are 5, 6, 7 non-power set cardinalities between cardinality 4 and 8.

In addition, in set theory, the one-to-one equivalence relation can be formally defined as,

Definition 0.6 Let A and B be two nonempty sets, if there exists a mapping from A to B

$$f : A \to B \tag{0.9}$$

Make $\forall a \in A, a' \in A$, if $f(a) = f(a')$, then there must be $a = a'$. Also for $\forall b \in B$ have a unique $a \in A$ such that $f(a) = b$,then f is said to be a one-to-one mapping.

This means that each element has a correspondence between the two sets and each element has a unique corresponding element.

1. Power Set Cardinality and Constructing Non-Power Set Cardinality in Natural Numbers

Definition 1.1 The set of powers in the natural numbers is written as

$$2^I = \{\ 2^{I(i)} \mid 2^{\{0,1,2,\cdots,i\}} : i \in \mathbb{N}\} \tag{1.1}$$

The cardinality of each power set is

$$2^i = |\,2^{I(i)}| \quad i \in \mathbb{N} \tag{1.2}$$

Definition 1.2 The non-power sets of natural numbers are denoted

$$\begin{aligned}\neg 2^I = \{\, &\neg 2^{I(i)}(j) \mid \{(2^i+1)\,,(2^i+2)\,,\cdots\ ,(2^i+j)\} : \\ &0 < j < (2^{i+1}-2^i), i \in \mathbb{N}\}\end{aligned} \tag{1.3}$$

The cardinality of each non-power set is

$$2^i + j = |\neg 2^{I(i)}| + j : 0 < j < (2^{i+1}-2^i), i \in \mathbb{N} \tag{1.4}$$

According to the Definition 1.3 of [2], ∞ is regarded as a kernel number of the dynamic space of natural number, i.e

$$A^k = \infty \tag{1.5}$$

The shell family is

$$S(2^I) = \left\{2^{I(i)} : i \in \mathbb{N}\right\} \tag{1.6}$$

The medium between shells is

$$M_i = \left\{ \neg 2^{I(i)}(j) \,|\, \{(2^i + 1), (2^i + 2), \cdots, (2^i + j)\} \right.$$
$$\left. 0 < j < (2^{i+1} - 2^i) \right\} \qquad i \in \mathbb{N} \tag{1.7}$$

And the whole medium set

$$M(\neg 2^I) = \left\{ M_i \,|\, i \in \mathbb{N} \right\} \tag{1.8}$$

According to Definition 1.6 of [3], the kernel number dynamic space can be obtained

$$QUAT(2^I) = Quaternary\left\{ S(2^I), M(\neg 2^I), \infty = A^k, lim \right\} \tag{1.9}$$

According to [4] if the dynamic limit is $lim(\aleph_1)$, the kernel number dynamic space can be obtained

$$QUAT(2^I) = Quaternary\left\{ S(2^I), M(\neg 2^I), \infty, lim(\aleph_1) \right\} \tag{1.10}$$

Corollary 1.1: In medium, M_i , there are ordered natural numbers.

Proof: According to [4] the medium, M_i, can be composed of natural numbers, rational numbers, irrational numbers, and elfin numbers. If the corresponding shell pair,

$$(\aleph_0, 2^{\aleph_0}) \tag{1.11}$$

Its intermediate medium is

$$M_{\aleph_0} \tag{1.12}$$

There must be two natural numbers n_1, n_2

$$n_1 \neq n_2 : n_1 \in M_{\aleph_0}, n_2 \in M_{\aleph_0}, n_1 \in \mathbb{N}, n_2 \in \mathbb{N} \tag{1.13}$$

And there must be $n_1 < n_2$ or $n_1 > n_2$ 。

Testify to the end.

Corollary 1.2 There is a non-power set cardinality between shell pairs $(\aleph_0, 2^{\aleph_0})$.

Proof : Suppose the two natural numbers n_1, n_2 of the Corollary 1.1 have

$$n_1 < n_2 \tag{1.14}$$

According to the definition of shell $S(2^I)$ and medium $M_i\ (i \in N)$, n_1, n_2 can be obtained that both are cardinalities of non-power set, and there are

$$\aleph_0 < n_1 < n_2 < 2^{\aleph_0} \tag{1.15}$$

Testify to the end.

Corollary 1.3 The Continuum Hypothesis (CH) is not true in the kernel number dynamic space $QUAT(2^I)$.

The following modifications are made to the Definition 0.6,

Definition 1.3 Let B be nonempty set, if there exists a mapping from $\mathbb{N}$ to B

$$f : A \to B \tag{1.16}$$

Make $\forall n_1 \in \mathbb{N}, n_1' \in \mathbb{N} \wedge \forall n_2 \in \mathbb{N}, n_2' \in \mathbb{N}$, if $f(n_1) = f(n_1'), f(n_2) = f(n_2')$, then there must be $n_1 = n_1', n_2 = n_2'$. And if it's $n_1 < n_2$, it's $f(n_1) < f(n_2)$. Also for $\forall b \in B$ have a unique $n \in \mathbb{N}$ such that $f(n) = b$, then f is said to be an one-to-one ordinal mapping or cardinality mapping.

This means that each element between the two sets has an ordinal correspondence, and each element has a unique correspondence.

In Definition 1.3, the set of natural numbers $\mathbb{N}$ becomes the ruler set. Any set that establishes a one-to-one ordinal (cardinality) mapping with $\mathbb{N}$ has both an one-to-one correspondence and must have an ordinal relationship with $\mathbb{N}$.

In Definition 1.2, the constructed non-power set $\neg 2^I$ uses the addition operation, so the division operation can extend the non-power set $\neg 2^I$ to rational numbers.

Definition 1.4: Non-power sets of rational numbers are denoted as,

$$\neg 2^Q = \{Q_i \mid 2^i < q_i < 2^{i+1} :\ q_i \in Q_i\ ,\ i \in \mathbb{N}\} \tag{1.17}$$

The cardinality of each non-power set $\neg 2^Q$ is

$$|\, q_i \,| \in Q_i\ ,\ i \in \mathbb{N} \tag{1.18}$$

The medium between the corresponding shell family $S(2^I)$ is

$$M_i = \left\{Q_i \mid Q_i \in \neg 2^Q \right\} \quad i \in \mathbb{N} \tag{1.19}$$

And the whole medium set

$$M(\neg 2^Q) = \left\{M_i | i \in \mathbb{N}\right\} \tag{1.20}$$

According to Definition 1.6 of [3], the kernel number dynamic space can be obtained

$$QUAT(2^Q) = Quaternary\left\{S(2^I), M(\neg 2^Q), \infty = A^k, lim\right\} \tag{1.21}$$

Corollary 1.4 In the kernel dynamic space $QUAT(2^Q)$, there are rational number cardinality between shell pairs $(\aleph_0 , 2^{\aleph_0})$.

Corollary 1.5 In the kernel number dynamic space $QUAT(2^Q)$, the set, $\mathbb{Q}$,of rational numbers becomes the ruler set. In any set that has an one-to-one correspondence with $\mathbb{Q}$, an one-to-one correspondence ordinal map can be established.

In the same way, we can discuss real numbers and real ruler sets.

2. Order and Cardinality of Elfin Number.

According to the definition of elfin number 2.5 in [5], there are the following inferences.

Definition 2.1 If the kernel number dynamic space $QUAT$ exists two different kernels $A^1 \in QUAT, A^2 \in QUAT$ has

$$|A^1| < |A^2| \tag{2.1}$$

Then the elfin set, E_1 ,of number cloud of A^1 , and the elfin set, E_2 ,of number cloud of A^2 have,

$$|e_1| < |e_2| \quad e_1 \in E_1, e_2 \in E_2 \tag{2.2}$$

Definition 2.2 If there exists a kernel $A^k \in QUAT$ in the kernel number dynamical space $QUAT$, whose set of number cloud of elfin numbers is E_k , then there are

$$|e_1| \approx |e_2| \quad \forall e_1 \in E_k, \forall e_2 \in E_k \tag{2.3}$$

Corollary 2.1 In the kernel number dynamic space $QUAT$, the set composed of real numbers and elfin numbers becomes the ruler set. In any set to which an one-to-one correspondence can be established, an one-to-one correspondence cardinality map can also be established. It is called the number of real elfin and is denoted as.

$$\mathbb{E} = \mathbb{R} \cup E \tag{2.4}$$

Proof : $\forall r \in \mathbb{R}, r$ is a kernel number of the kernel number dynamic space $QUAT$. According to the shell-medium properties of kernel number r, the elfin number in the number cloud of r is ordered. If E_r is an elfin set in a numbers cloud of the r, then there are.

$$\forall e \in E_r \Rightarrow |e| < r \vee |e| > r \tag{2.5}$$

Combined with definition 2.1, the $\mathbb{E}$ is a rulers set.

Testify to the end.

Corollary 2.1 can be derived

Corollary **2.2** In the kernel number dynamic space $QUAT$, there exists cardinalities between shell pairs $(\aleph_i, 2^{\aleph_i})\ i \in \mathbb{N}$.

Proof: Because the elfin number is ordered, and the medium, $M(\neg 2^{\aleph_i})$, of the shell pairs, $(\aleph_i, 2^{\aleph_i})\ i \in \mathbb{N}$, there is

$$M(\neg 2^{\aleph_i}) \subset E$$

hence

$$\forall m \in M(\neg 2^{\aleph_i}) \Rightarrow \aleph_i < |m| < 2^{\aleph_i} \ : i \in \mathbb{N} \tag{2.6}$$

Testify to the end.

Corollary 2.2 can be obtained that the Non-Generalized Continuum Hypothesis(NGCH) in the kernel number dynamic space, $QUAT$, is valid.

References

[1] Jiang Yang, On $Limit$ of Mathematical Analysis and Continuum Hypothesis. viXra:2410.0163 page 15

[2] Jiang Yang, On $Limit$ of Mathematical Analysis and Continuum Hypothesis. viXra:2410.0163 page 3

[3] Jiang Yang, On $Limit$ of Mathematical Analysis and Continuum Hypothesis. viXra:2410.0163 page 4

[4] Jiang Yang, On $Limit$ of Mathematical Analysis and Continuum Hypothesis. viXra:2410.0163 page 5-13

[5] Jiang Yang, On $Limit$ of Mathematical Analysis and Continuum Hypothesis. viXra:2410.0163 page 13-21

Jiang Yang
Independent Researcher
Brentwood, CA 94513
E-mail: jiang.yang600@gmail.com or jiang_yang600@sina.com