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KOIDE FORMULA AND THE CONNECTION OF ELEMENTARY PARTICLE MASSES WITH THE FINE-STRUCTURE CONSTANT α.

Abstract. *The article explores the relationship between the masses of elementary particles and the fine-structure constant α. In addition to the Koide formula, a new formula is proposed, which is as accurate as the Koide formula. The new formula demonstrates the connection between the masses of four elementary particles (electron, proton, tau lepton, and muon) with the fine-structure constant α:*

$$\frac{\sqrt[3]{m_e m_\mu m_\tau}}{3^2 m_e + 3^1 m_\tau + 3^0 m_p} = \alpha$$

Considering the high precision of the fine-structure constant and the accurate experimental values of the electron, proton, and muon masses, the new formula provides a significantly more accurate value for the mass of the tau lepton compared to the current experimental value:

m_τ/m_e=3477.0298… (1776.7586… MeV)

The obtained value of the tau lepton mass (1776.7586... MeV/c^2) is within the range of the experimental value but does not coincide with the result obtained by the Koide formula (1776.9688... MeV/c^2). The Koide formula and the new formula yield different values for the mass of the tau lepton with high precision in both results. Both mass values are significantly more accurate than the current experimental value. The discrepancy between the results of the two formulas requires explanation.

Keywords*: Nambu formula, Barut formula, Koide formula, spectrum of elementary particle masses, electron, muon, tau lepton, proton, fine-structure constant.*

1. Introduction

Explaining the spectrum of masses of elementary particles remains one of the unresolved challenges in modern physics [1]. The idea of providing an explanation for the spectrum of elementary particle masses based on the connection of masses with the fine-structure constant belongs to Y. Nambu [2]. He proposed an empirical formula that generates the spectrum of masses of elementary particles. This idea was further developed in the work of A. O. Barut [3]. Research aimed at obtaining empirical formulas that generate the masses of elementary particles continues to this day. The focus of the research is on refining and improving empirical mathematical models that generate the spectrum of masses of elementary particles. However, the accuracy of the masses of elementary particles obtained from empirical formulas remains significantly below the precision of their experimental values [4, 5].

In 1983, Koide proposed an empirical formula [6] linking the masses of leptons. The Koide formula significantly surpasses the accuracy of Nambu's and Barut's formulas and similar empirical formulas. The Koide formula is attractive for its high precision and for demonstrating the interconnectedness of the masses of elementary particles. It stands out as the only equation connecting the constants of masses of elementary particles with high precision. The search for equations similar to the Koide formula, which include the masses of other elementary particles, remains a current challenge.

2. Nambu Formula

Y. Nambu proposed the following empirical formula:

$$m = \frac{N \bullet m_e}{2\alpha} \qquad (1)$$

where N is a positive number, m_e is the mass of the electron, and α is the fine-structure constant.

The empirical formula (1) has been studied by many authors [7-21]. Each particle has its own value of N. For example, with $N = 27$, the formula gives a mass value close to the proton's mass. The Nambu formula provides quanta for particle masses: 35, 70, 105, 140 MeV, etc. [9-13, 22]. For some elementary particles, the formula allows obtaining mass values with deviations within 1% [4].

The value of the Nambu formula lies in its very simple empirical equation demonstrating the connection of the fundamental constant of quantum electrodynamics α with the values of elementary particle masses. However, the formula's drawback includes the lack of theoretical justification and the relatively low accuracy of the obtained mass values of elementary particles.

3. Barut Formula

Building on Nambu's idea, Barut in 1979 derived the following empirical relationship for lepton masses [3]:

$$m(N) = m_e\left(1 + \frac{3}{2\alpha}\sum_{k=0}^{N} k^4\right) \qquad (2)$$

where N is a positive number, and m_e is the mass of the electron. Unlike the Nambu formula, Barut's formula represents the difference in masses through the fine-structure constant α. Authors have pointed to the fundamental nature of the difference in masses of pions and muons in the mass distribution of elementary particles [16].

4. α-m_e-Quantization of Mass Spectrum

The relationships between the experimental values of elementary particle masses indicate a quantized mass spectrum. Both the Barut and Nambu formulas generate a quantized mass spectrum of elementary particles. Nambu's and Barut's formulas show that the quantization of elementary particle masses is determined by two fundamental constants: the fine-structure constant α and the mass of the electron m_e. Mac Gregor even introduced the term "α-quantization" [9-13].

The value of the Nambu and Barut formulas lies in being the first to emphasize the existence of a dependence of the mass spectrum on two fundamental constants of quantum electrodynamics—the electron mass m_e and the fine-structure constant α. However, a drawback of both formulas is the lack of theoretical justification and the relatively low accuracy of the obtained mass values of elementary particles. The low accuracy is a consequence of the limited capabilities of empirical formulas. Accuracy within 1% is clearly insufficient for formulas generating the mass spectrum, especially considering the much higher precision of experimental mass values. It is evident that the quantization of the mass spectrum follows a more complex law than the simplified mathematical models presented by the empirical Nambu and Barut formulas. The law governing the mass spectrum cannot be represented by simplified mathematical models.

5. Koide Formula

Experimental values of masses for certain elementary particles vary significantly in precision. For instance, highly precise values are known for constants such as the fine-structure constant (0.007297 352 5693(11)), muon-electron mass ratio (206.768 2830(46)), and proton-electron mass ratio (1836.152 673 43 (11)). Against such precise constant values, the tau-electron mass ratio stands out with a less accurate value of 3477.23(23).

In 1983, Koide proposed the following formula relating the masses of leptons [6]:

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \qquad (3)$$

where: m_e - mass of the electron, m_μ - mass of the muon, m_τ - mass of the tau lepton.

The formula provides a highly accurate value for the mass of the tau lepton, which remains significantly more precise than the current experimental value to this day:

$$m_\tau = 1776.968884 \pm 0.000065\ MeV/c^2 \qquad (4)$$

The Koide formula stands as the only precise formula linking the masses of leptons. However, when using masses of other elementary particles, the Koide formula does not yield accurate results [23-26]. The advantage of the Koide formula lies in consolidating constants of masses into a single formula, where the values are known with high precision, alongside a less precise constant—the mass of the tau lepton. The Koide formula allows 'pulling up' the accuracy of the less precise constant to the level of more accurate constants. The Koide formula lacks a theoretical explanation [27-30].

6. Formula Linking the Masses of Elementary Particles with the Fine-Structure Constant α

The formulas of Nambu and Barut are mathematical models that reflect regularities discovered in the mass spectrum of elementary particles. The Koide formula illustrates the connection between specific mass constants within the mass spectrum. The appeal of Nambu and Barut's formulas lies in their utilization of the fine-structure constant. The Koide formula stands out for its high precision. Below is a formula that combines the merits of Nambu, Barut, and Koide.

In addition to the Koide formula (3), a new formula is proposed by Koide (Fig. 1), which is no less accurate than the Koide formula. The new formula uses five constants, four of which are known with high precision. The formula links the masses of four elementary particles (electron, proton, tau lepton, and muon) with the fine-structure constant α.

$$\frac{\sqrt[3]{m_e m_\mu m_\tau}}{3^2 m_e + 3^1 m_\tau + 3^0 m_p} = \alpha$$

Fig. 1. Formula linking the masses of four elementary particles with the fine-structure constant α. m_e - mass of the electron, m_μ - mass of the muon, m_τ - mass of the tau lepton, m_p - mass of the proton, α - fine-structure constant.

The formula (Fig. 1) bears some structural resemblance to the Koide formula. However, instead of the number 2/3, it incorporates the dimensionless fundamental physical constant — the fine-structure constant α. Considering the high precision of the fine-structure constant α and the accurate experimental values of the masses of the electron, proton, and muon, the formula provides a significantly more accurate value for the mass of the tau lepton compared to the current experimental value:

$$\frac{m_\tau}{m_e} = 3477.0298... \quad (1776.7586...MeV) \qquad (5)$$

The obtained value of the tau lepton mass (1776.7586…MeV/c^2) falls within its experimental range. However, the value obtained by the formula (Fig. 1) for the tau lepton mass (5) does not coincide with the value obtained by the Koide formula (1776.9688… MeV/c^2). The Koide formula and the new formula yield different values for the mass of the tau lepton with very high precision in both mass values. This is an unexpected result. Both values for the tau lepton mass are significantly more accurate than the current experimental value. The reasons for the discrepancy between the results of the two formulas require explanation.

7. Conclusion

The different calculated values of the tau lepton mass from the Koide formula and the new formula indicate that both formulas cannot be simultaneously correct. This situation could be clarified by either a more accurate experimental value of the tau lepton mass, theoretical justification of the formulas, or a theory of the mass spectrum of elementary particles from first principles. Obtaining a more precise experimental value for the mass of the tau lepton is a challenging task [31]. Therefore, there is a pressing need for the development of a theory of the mass spectrum of elementary particles [1].

8. Conclusions

1. In addition to the Koide formula, a formula is proposed that links the masses of elementary particles with the fine-structure constant:

$$\frac{\sqrt[3]{m_e m_\mu m_\tau}}{3^2 m_e + 3^1 m_\tau + 3^0 m_p} = \alpha$$

2. The new formula provides a value for the tau lepton mass that is significantly more accurate than the current experimental value.

$$\frac{m_\tau}{m_e} = 3477.0298... \quad (1776.7586...MeV)$$

3. The value of the tau lepton mass obtained from the new formula does not coincide with the value obtained from the Koide formula. This is an unexpected result that requires an investigation into the cause of the discrepancy.

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