Real Constants Index in Derivatives and Integrals

Juan Elías Millas Vera.
Zaragoza (Spain). May 2024.

0- Abstract:
I taught myself Calculus seriously the last months and I want to share some thoughts about Real constants Calculus with a single variable. I will may be mostly know this results by professionals but, in any case it could be interesting to share my conclusions in a short paper.

1- Introduction:
Derivatives and Integrals have them start in 17th century by Newton and Leibniz, very useful in Physics but still studied as field in Mathematics. In this text I will use Leibniz's notation. As I will show here it is possible to do a logic derivation and a logic integration with index not belong to integers. I said more, it can be applied even to Real Numbers. So, for (c,n,m,a,b) constants belong to Real Numbers and a variable x, then we can define Derivation as:

$$\frac{d^m cx^n}{dx^m} = cnx^{(n-m)} \quad (1)$$

Indefinite Integration as:

$$\int cx^n dx^m = \frac{cx^{(n+m)}}{n+m} \quad (2)$$

And Definite Integration as:

$$\int_a^b cx^n dx^m = \frac{cb^{(n+m)}}{n+m} - \frac{ca^{(n+m)}}{n+m} \quad (3)$$

2- Proof:
To proof that this results are valid we will use Fundamental Theorem of Calculus to go from one result to the main equation (the equation before the transformation)
So, applying integration to the result of (1):

$$\int cnx^{(n-m)} dx^m = \frac{ncx^{(n-m+m)}}{n-m+m} = \frac{ncx^n}{n} = cx^n$$

Which is the equation pre-derivation.
And applying derivation to result of (2):

$$\frac{d^m\left(\frac{cx^{(n+m)}}{n+m}\right)}{dx^m} = \frac{c(n+m)x^{(n+m-m)}}{(n+m)} = cx^n$$

Which is the equation pre-integration.

3-Conclusions:
We have a powerful tool to do m-derivations and m-integrations of a variable index n.

4- References:
[1] Spivak, Michael – Calculus.