Mass and Spacetime Quantization of the Generalized Dirac Equation based on Octonion-Sedenion Algebra: Towa5ds Derivation of the Fine Structure Constant Beyond the 12th Decimal-Digit Precision

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Abstract

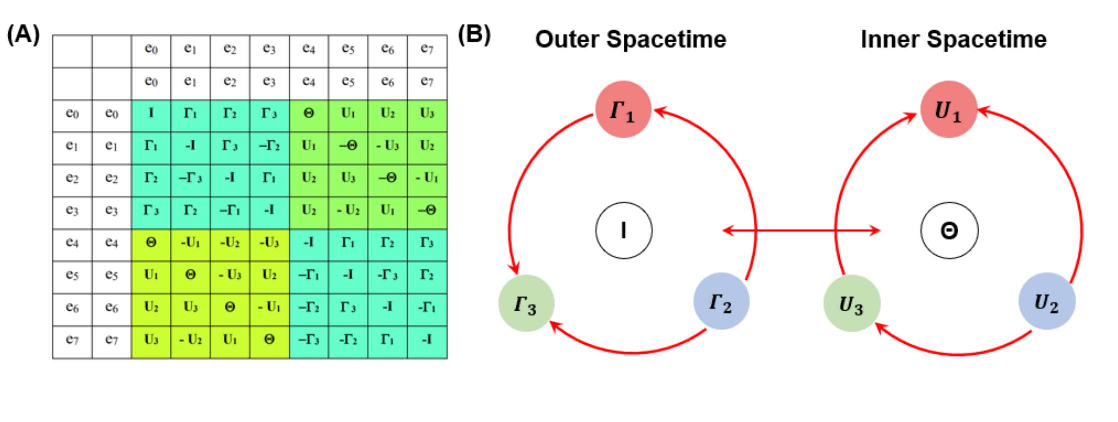
We present an approach to solving the mystery of the fine structure constant (α) using hyper-complex algebra. Extending Einstein's continuous 4D Minkowski space and the Dirac equation, we address internal 4D or 12D spacetime in particles for the octonion and sedenion models. We propose that particle mass originates from internal dynamics, not the Higgs mechanism in Yang-Mills theory. By quantizing mass and internal spacetime, we derive a geometric constant of 137 for the octonion model, and 137.03599920605017 for the sedenion model, precisely matching the experimental value of $1/\alpha = 137.035999206\ (11)$ to within $\sim 10^{-12}$. Our theory also suggests a fundamental mass energy of 0.05 eV, likely related to neutrinos. This work reveals that α is not merely a physical parameter but a dimensionless geometric constant, akin to π or Euler's constant. We found simple empirical mass-ratio formulas linking α to the masses of the electron, Higgs boson, quarks, and Planck mass, elucidating the role of this constant in fundamental forces. This theory revolutionizes the understanding of mass and spacetime quantization, going beyond the Standard Model and opening new paths toward quantum gravity and grand unification.

Keywords: Fine structure constant, Dirac equation, Mass-Spacetime quantization, octonion, sedenion, Standard Model

One of the most profound mysteries in physics is the dimensionless fine structure constant,[1,2] which governs electromagnetic interactions. Unlike quantities with physical units, such as the speed of light, electron mass, or size of the earth, the fine structure constant resembles mathematical constants like π or Euler constant, devoid of physical units. This universal constant, defined by the ratio of an electron's charge to the Planck constant and the speed of light, quantifies the strength of electromagnetic interactions. Its value is crucial: a deviation of just 4% would inhibit the carbon formation in stars, preventing all living creatures. The constant's approximate value of 1/137 has remained one of the mysteries in physics, confounding some of the greatest minds. Pauli once remarked that his first question in the afterlife would be the meaning of this constant.[3] Feynman once called it a magic number that comes to us without understanding.[4] In addition to this enigma, unresolved issues persist in the Standard Model,[5] such as the relationships between particle's masses, Koide's empirical mass-ratio rules,[6] the existence of three generations of leptons and quarks, and whether these particles are truly elementary. In this work, we focus on addressing the electron's mass in a generalized Dirac equation which has been treated traditionally as a free parameter,[7] and propose a theory to point out the geometric origin for mass and the fine structure

We propose a generalized Dirac equation in which mass is not just a free parameter but is treated as an operator involving four octonion operators, rather than a point-like entity. The traditional Dirac equation[10] assumes an electron without structure, described by four anti-commutative gamma matrices. However, our model employs the octonion algebra to describe the electron's internal dynamics, contributing to its rest mass. The octonion algebra, with its eight base elements, includes two quaternion[11] sub-algebras: one describing the external 4D Minkowski space,[12] where Dirac's equation describes, the other describing the internal degrees of freedom, leading to an effective rest mass arising from internal dynamics and intrinsic mass. Here we consider a generalized Dirac equation where the mass is no longer a number but represents an operator involving four octonion operators. Unlike the Dirac equation which assumes a point-like electron and employs four anti-commutative gamma matrices to describe four degrees of freedom in spacetime, our model employs octonion algebra with four operators to describe the internal structure, and such internal dynamics contributes to the effective rest mass of the electron.

Our model considers eight-dimensional octonion algebra for a particle with an internal structure, among the eight operators four operators for the exterior degrees of freedom in the external 4D spacetime, while the other four anti-commutative operators describe the particle's interior dynamics involving internal 4D spacetime. The octonion algebra, one of the algebras among the hyper-complex algebra,[13] consists of eight base elements, and their multiplication rules are given in Fig. 1.



		e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
		e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	$\mathbf{I}$	Γ_1	Γ_2	Γ_3	Θ	U_1	U_2	U_3
e_1	e_1	Γ_1	$-\mathbf{I}$	Γ_3	$-\Gamma_2$	U_1	$-\Theta$	$-U_3$	U_2
e_2	e_2	Γ_2	$-\Gamma_3$	$-\mathbf{I}$	Γ_1	U_2	U_3	$-\Theta$	$-U_1$
e_3	e_3	Γ_3	Γ_2	$-\Gamma_1$	$-\mathbf{I}$	U_2	$-U_2$	U_1	$-\Theta$
e_4	e_4	Θ	$-U_1$	$-U_2$	$-U_3$	$-\mathbf{I}$	Γ_1	Γ_2	Γ_3
e_5	e_5	U_1	Θ	$-U_3$	U_2	$-\Gamma_1$	$-\mathbf{I}$	$-\Gamma_3$	Γ_2
e_6	e_6	U_2	U_3	Θ	$-U_1$	$-\Gamma_2$	Γ_3	$-\mathbf{I}$	$-\Gamma_1$
e_7	e_7	U_3	$-U_2$	U_1	Θ	$-\Gamma_3$	$-\Gamma_2$	Γ_1	$-\mathbf{I}$

Fig. 1. The octonion algebra and its structures. (**A**) The multiplication table of eight base elements of the octonion algebra. Other than the unity element, all the remaining seven elements are anti-commutative. The table is color-coded to show the octonion algebra contains two quaternion algebras. (**B**) The four-element quaternion set $\{\mathbf{I}, \mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3\}$ describe the exterior 4D spacetime, while the other set $\{\mathbf{\Theta}, \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$ describe the internal spacetime.

One can notice some interesting mathematical aspects of octonions by closely examining the multiplication table. It contains a quaternion set and a quaternion-like set of operators $\{\mathbf{\Theta}, \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$. Their multiplication follows Cayley-Dickson's construction rule,[24] unlike the associative. According to this octonion model, the generalized Dirac equation contains two parts, an outer spacetime and an inner spacetime for the internal structural dynamics that contribute to the effective rest mass for the electron as viewed from the outer Minkowski spacetimes.

We consider the generalized Dirac equation as given by

$$
\begin{gathered}
\mathbf{H} \equiv \sum_{k=1}^{3} p_k \boldsymbol{\Gamma}_{\mathrm{k}} + \Omega\boldsymbol{\Theta} + \sum_{k=1}^{3} P_k \mathbf{U}_k \\
\{\boldsymbol{\Gamma}_{\mathrm{i}}, \boldsymbol{\Gamma}_{\mathrm{j}}\} = \{\mathbf{U}_{\mathrm{i}}, \mathbf{U}_{\mathrm{j}}\} = -\delta_{\mathrm{ij}}\mathbf{I}., \{\boldsymbol{\Gamma}_{\mathrm{i}}, \mathbf{U}_{\mathrm{j}}\} = 0
\end{gathered} \tag{1A}
$$

where we define $\mathbf{H}|\boldsymbol{\Psi}\rangle \equiv -iE|\boldsymbol{\Psi}\rangle$. Based on $E^2|\boldsymbol{\Psi}\rangle = -\mathbf{H}^{2|\boldsymbol{\Psi}\rangle}$, from Eq. (1A) and the rules from the multiplication table in Fig. 1, one obtains

$$
\begin{gathered}
E^2 = c^2p^2 + {m_0}^2c^4 \\
{m_0}^2c^4 = \hbar^2\Omega^2 + c^2\boldsymbol{P}^2 \\
\boldsymbol{p}^2 = \sum_{k=1}^{3} {p_k}^2, \boldsymbol{P}^2 = \sum_{k=1}^{3} {P_k}^2
\end{gathered} \tag{1B}
$$

The above equation is the generalized Einstein's mass-energy relation for a particle with internal dynamics, and the effective rest mass m_0 is contributed by two sources, an internal intrinsic rest mass energy $\hbar\Omega$in the center of mass frame, and a kinetic energy $c\boldsymbol{P}$ from the internal dynamics. The simpler case of Eq. (1A) with $\mathbf{H} = \sum_k p_k\boldsymbol{\Gamma}_k$ corresponds to a massless fermion, and $\mathbf{H} = \sum_k p_k\boldsymbol{\Gamma}_{\mathrm{k}} + m_0\boldsymbol{\Theta}$ corresponds to a point-like fermion with a rest mass m_0.

In Eq. (1) we generalize Dirac's equation in 4D spacetime to a higher dimension with both exterior and interior spacetime showing that the effective mass can be acquired through the internal kinetic energy of the internal dynamics, without invoking the Higgs mechanism.[14] Such a mechanism was proposed to solve the dilemma of massless gauge boson in the Yang-Mills theory.[15]

Our generalized Dirac equation shows that the electron mass is not a free parameter. Instead, by treating the mass as an operator in the 5$^{\text{th}}$ dimension, we introduce mass quantization within the octonion framework. The internal structure is modeled with four octonion base elements, and the mass eigenvalue is associated with a discrete, integer multiple of a fundamental unit corresponding to the electron's mass. We derive a value for the fine structure quantization, not from physical interactions.

$$
\begin{gathered}
\mathbf{H} \equiv \mathbf{p} + \mathbf{m}, \mathbf{m} = \mathbf{M} + \mathbf{P} \\
\mathbf{p} = \textstyle\sum_{k=1}^{3} \mathrm{p_k}\boldsymbol{\Gamma}_{\mathrm{k}}, \mathbf{P} = \sum_{k=1}^{3} \mathrm{P_k}\mathbf{U}_{\mathrm{k}}, \mathbf{M} = \Omega\boldsymbol{\Theta}
\end{gathered} \tag{2}
$$

Because of the use of anti-commutative octonion operators for these momentum and mass operators in the 4D outer and inner spacetime, these operators and the constituent components for the momentum along each of the three spatial axes are all anti-commute with each other. To achieve the mass quantized, we need to associate tits eigenvalue to be discrete and can be described as an integer multiple of a fundamental unit. All these operators have discrete eigenvalues as an integer multiple. For the electron, the basic mass unit must be the mass of a single electron, which is the most stable and lightest among known particles.

According to Eq. (2), $\mathbf{m} = \mathbf{M} + \mathbf{P}$, $\mathbf{M} = \Omega\mathbf{\Theta}$ and $\mathbf{P} = \mathrm{P}_1\mathbf{U}_1 + \mathrm{P}_2 + \mathbf{U}_3 + \mathrm{P}_3\mathbf{U}_3$for the interior spacetime, these two equations become $\mathbf{m}^2 = \mathbf{M}^2 + \mathbf{P}^2$ and $\mathbf{P}^2 = {\mathrm{P}_1}^2 + {\mathrm{P}_2}^2 + {\mathrm{P}_3}^2$. The quantization of the mass-energy and three momentum components along each axis is equivalent to the quantization of the interior time and space. In terms of the fundamental units of mass and momentum, these two equations can be expressed in simple equations involving purely integers, i.e., $a_0 = \sqrt{{a_4}^2 + {a_5}^2}$for $\mathbf{m}^2 = \mathbf{M}^2 + \mathbf{P}^2$ and $a_5 = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}for\boldsymbol{P}^2 = {\boldsymbol{P}_1}^2 + {\boldsymbol{P}_2}^2 + {\boldsymbol{P}_3}^2$, where $\{a_0, a_1, a_2, a_3, a_4, a_5\}$ are the eigenvalues for $\{\mathbf{m}, \mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{M}, \mathbf{P}\}$ respectively. Because the eigenvalues represent the fundamental mode of$\boldsymbol{m}$, so these two integers n_0, n_5 must be prime numbers. Based on Eq. (2) one obtains

$$exp(\mathbf{m}\tau) = cos(\omega\tau) + (\mathbf{m}/\omega)\, sin(\omega\tau)\,,$$
$$\omega \equiv \sqrt{\Omega^2 + {P_1}^2 + {P_2}^2 + {P_3}^2} \tag{3}$$

The above result indicates the effective rest mass of the electron contains kinetic energy from its internal dynamics and an intrinsic mass Ω. For$exp(\boldsymbol{m}t)\,|\Psi\rangle = 0$, one has$\boldsymbol{m}|\Psi\rangle = 0$, which represents the wave equation for the particle at a rest frame of the external spacetime so that $\boldsymbol{H} = \mathbf{m} = \mathbf{M} + \mathbf{P}_1 + \mathbf{P}_2 + \mathbf{P}_3$. Type equation here. The oscillation $\Omega = \Omega_0\sqrt{{a_4}^2 + {a_1}^2 + {a_2}^2 + {a_3}^2}$, where Ω_0 is the fundamental frequency. Based on the principle of mass quantization and the 4D internal spacetime, the component integers must satisfy the constraints listed in Table 1.

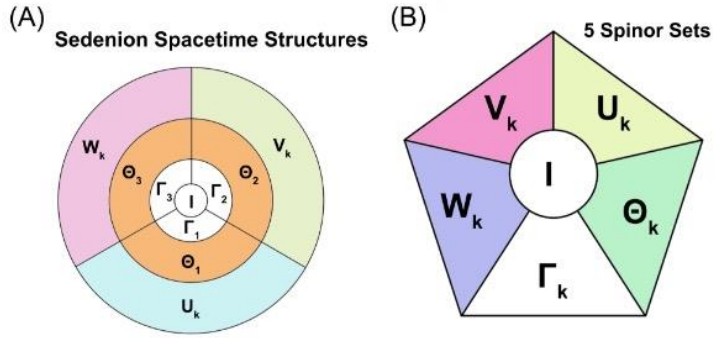
Table 1. **Constraints on integer equations for quantized rest mass, internal momentum, and intrinsic mass, according to the octonion model**

$\mathbf{m} = \mathbf{M} + \mathbf{P}$	$\mathbf{P} = P_1\mathbf{U}_1 + P_2\mathbf{U}2 + P_3\mathbf{U}_3$	$\mathbf{M} = \Omega_0\mathbf{\Theta}$	$a_0, a_5: prime$ $a_1, a_2, a_3, a_4: integers > 1$

$a_0 = {a_4{}^2}^2 + a_5{}^2$ $a_0, a_5: prime$	$Triplet: \{a_1, a_2, a_3\}$ $a_5{}^2 = a_1{}^2 + a_2{}^2 + a_3{}^2$	M^2 $= a_4{}^2 \Omega_0{}^2$	$\lvert mod(R/a_0, \pi)\rvert < 10^{-2}$ $R = \prod_{k=1}^{4} a_k$ $a_0 = \sum_{k=1}^{4} a_k^2$

According to the criteria listed in the table we have obtained the first set of prime numbers with $a_0 = 137, a_5 = 11, a_1 = 2, n_2 = 6, a_3 = 9,$ which satisfies Eq. (3) ad the prime number constraints on $a_0, a_5$for them to represent a fundamental mode. The operator $\boldsymbol{\Lambda}$ defined as $\mathbf{\Lambda} \equiv (4\mathbf{\Theta} + 2\mathbf{U}_1 + 6\mathbf{U}_2 + 9\mathrm{U}_3)/\sqrt{137}$ is a magic matrix, behaving as an imaginay matrix with $\mathbf{\Lambda}^2 = (4\mathbf{\Theta}_5 + 2\mathbf{U}_1 + 6\mathbf{U}_2 + 9\mathrm{U}\ \ )^2/137 = -\mathbf{I}$, like an imaginary number $i^2 = -1$. This interesting property makes the magic number 137 ubiquitous in quantum physics as the fine structure constant. Therefore, we believe the mystery of this dimensionless fine structure constant is not a physical parameter which is typically associated with length, time, weight, etc. It is an interesting pint that the product of $a_0 = 137, a_5 = 11, a_1 = 2, a_2 = 6, a_3 = 9$ equals 432, and $432/137\pi - 1 = mod(432/137, \pi) = 0.00336 \approx 0$, with about 0.35% deviation. This value of 432 is the product of the eigenvalue matrix spanned by four operators $\{\mathbf{\Theta}, \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$ for the inner spacetime, this indicates the 4 by 4 matrix behaves almost like an imaginary number except that it is a 4x4 matrix here. With this fact, and $\boldsymbol{m}^2 = \Omega_0{}^2(4\mathbf{\Theta} + 2\mathbf{U}_1 + 6\mathbf{U}_2 + 9\mathrm{U}_3)^2 = 137\Omega_0{}^2$, we obtain the relation between the electron rest mass and the fundamental frequency as $m_0{}^2/\Omega_0{}^2 = 137 = (e^2/\hbar c)^{-1}$. This result links the electron's mass is related to the fundamental frequency. According to this octonion model, the mass quantization leads precisely to $1/\alpha$ = 137.

To improve the accuracy, we need to expand the octonion model to the sedenion model, which offers a much richer structure for a particle's internal structure, for constructing SU(5) generators,[19] which contains $\mathbf{su(3)} \oplus \mathbf{su(2)} \oplus \mathbf{u(1)}$ algebra, which is the foundation for the Standard Model. The sedenion algebra introduces 12 operators to describe spacetime, accommodating three generations of particles. We expand the 8-element octonion algebra model to the 16-element sedention algebra model. The sedenion multiplication table is given in Fig. 2.



(C)

	$\mathbf{I}$	U_1	U_2	U_3	V_1	V_2	V_3	W_1	W_2	W_3	Γ_1	Γ_2	Γ_3	Θ_1	Θ_2	Θ_3
$\mathbf{I}$	$\mathbf{I}$	U_1	U_2	U_3	V_1	V_2	V_3	W_1	W_2	W_3	Γ_1	Γ_2	Γ_3	Θ_1	Θ_2	Θ_3
U_1	U_1	$-I$	$-U_3$	U_2	$-\Theta_3$	$-iW_3$	iW_2	Θ_2	iV_3	$-iV_2$	$-i\Theta_1$	$-U_3$	U_2	$-i\Gamma_1$	$-W_1$	V_1
U_2	U_2	U_3	$-I$	$-U_1$	iW_3	$-\Theta_3$	$-iW_1$	$-iV_3$	Θ_2	iV_1	U_3	$-i\Theta_1$	$-U_1$	$-i\Gamma_2$	$-W_2$	V_2
U_3	U_3	$-U_2$	U_1	$-I$	$-iW_2$	iW_1	$-\Theta_3$	iV_2	$-iV_1$	Θ_2	$-U_2$	U_1	$-i\Theta_1$	$-i\Gamma_3$	$-W_3$	V_3
V_1	V_1	Θ_3	iW_3	$-iW_2$	$-I$	$-V_3$	$V\Gamma_2$	$-\Theta_1$	$-iU_3$	iU_2	$-i\Theta_2$	$-V_3$	V_2	W_1	$-i\Gamma_1$	$-U_1$
V_2	V_2	$-iW_3$	Θ_3	iW_1	V_3	$-I$	$-V_1$	iU_3	$-\Theta_1$	$-iU_1$	V_3	$-i\Theta_2$	$-V_1$	W_2	$-i\Gamma_2$	$-U_2$
V_3	V_3	iW_2	$-iW_1$	Θ_3	$-V_2$	V_1	$-I$	$-iU_2$	iU_1	$-\Theta_1$	$-V_2$	V_1	$-i\Theta_2$	W_3	$-i\Gamma_3$	$-U_3$
W_1	W_1	$-\Theta_2$	$-iV_3$	iV_2	Θ_1	iU_3	$-iU_2$	$-I$	$-W_3$	W_2	$-i\Theta_3$	$-W_3$	W_2	$-V_1$	U_1	$-i\Gamma_1$
W_2	W_2	iV_3	$-\Theta_2$	$-iV_1$	$-iU_3$	Θ_1	iU_1	W_3	$-I$	$-W_1$	W_3	$-i\Theta_3$	$-W_1$	$-V_2$	U_2	$-i\Gamma_2$
W_3	W_3	$-iV_2$	iV_1	$-\Theta_2$	iU_2	$-iU_1$	Θ_1	$-W_2$	W_1	$-I$	$-W_2$	W_1	$-i\Theta_3$	$-V_3$	U_3	$-i\Gamma_3$
Γ_1	Γ_1	$-i\Theta_1$	$-U_3$	U_2	$-i\Theta_2$	$-V_3$	V_2	$-i\Theta_3$	$-W_3$	W_2	$-I$	$-\Gamma_3$	Γ_2	iU_1	iV_1	iW_1
Γ_2	Γ_2	U_3	$-i\Theta_1$	$-U_1$	V_3	$-i\Theta_2$	$-V_1$	W_3	$-i\Theta_3$	$-W_1$	Γ_3	$-I$	$-\Gamma_1$	iU_1	iV_2	iW_2
Γ_3	Γ_3	$-U_2$	U_1	$-i\Theta_1$	$-V_2$	V_1	$-i\Theta_3$	$-W_2$	W_1	$-i\Theta_3$	$-\Gamma_2$	Γ_1	$-I$	iU_2	iV_3	iW_3
Θ_1	Θ_1	$-i\Gamma_1$	$-i\Gamma_2$	$-i\Gamma_3$	W_1	W_2	W_3	$-V_1$	$-V_2$	$-V_3$	iU_1	iU_2	iU_3	$-I$	$-\Theta_3$	Θ_2
Θ_2	Θ_2	$-W_1$	$-W_2$	$-W_3$	$-i\Gamma_1$	$-i\Gamma_2$	$-i\Gamma_3$	U_1	U_2	U_3	iV_1	iV_2	iV_3	Θ_3	$-I$	$-\Theta_1$
Θ_3	Θ_3	V_1	V_2	V_3	$-U_1$	$-U_2$	$-U_3$	$-i\Gamma_1$	$-i\Gamma_2$	$-i\Gamma_3$	iW_1	iW_2	iW_3	$-\Theta_2$	Θ_1	$-I$

Fig. 2. The of 16 sedenion operators. (A) The schematic representation for three sets of 8-element octonion algebra as a subset of the 16-element sedenion algebra. The sedenion algebra contains three octonion algebra sets, which share the common quaternion algebra, $\{\mathbf{I},\ \boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \boldsymbol{\Gamma}_3\}$ in the white domain for the outer spacetime. (B) The 5-spinor sets the structure of the sedenion algebra. The 4D inner spacetime is further split into triplets. (C) Among these five sets only $\{\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \boldsymbol{\Gamma}_3\}$ represents the outer space, while $\{\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$, $\{\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3\}$, and $\{\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3\}$ represent the inner space, and $\{\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \boldsymbol{\Theta}_3\}$ represent the internal times.

The sedenion algebra can be constructed from a pair of octonions using the Cayley-Dickson construction scheme, which has been used to build octonions from quaternions, and quaternions from complex numbers, like complex numbers from real numbers. Using the similar development strategy given earlier for the octonion algebra, we consider

$$\boldsymbol{m} = \boldsymbol{M} + \boldsymbol{P}_0 + \boldsymbol{P}$$

$$\boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2 + \boldsymbol{P}_3,,$$
$$\boldsymbol{M} = \sum_{k=1}^{3} \Omega_k \boldsymbol{\Theta}_k \,, \boldsymbol{P}_0 = \lambda_0 \boldsymbol{\Lambda}_0,$$
$$\boldsymbol{P}_1 = \sum_{k=1}^{3} \lambda_k \boldsymbol{\Lambda}_k \, \boldsymbol{P}_2 = \sum_{k=4}^{6} \lambda_{k3} \boldsymbol{\Lambda}_k \quad \boldsymbol{P}_3 = \sum_{k=7}^{8} \lambda_k \boldsymbol{\Lambda}_k \qquad (4A)$$

where $\{\boldsymbol{\Theta}_j, \boldsymbol{\Lambda}_k\} = 0, \{\boldsymbol{\Theta}_j, \boldsymbol{\Theta}_k\} = \{\boldsymbol{\Lambda}_j, \boldsymbol{\Lambda}_k\} = -\delta_{jk}$. here are twelve degrees of freedom in the internal spacetime, represented by $\{\boldsymbol{\Theta}_1, \boldsymbol{\Theta}_2, \boldsymbol{\Theta}_3\}$,$\{\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3\}\{\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{V}_3\}$ and $\{\boldsymbol{W}_1, \boldsymbol{W}_2, \boldsymbol{W}_3\}$. One can use the last three sets of operators to create a set of 8+1 $\boldsymbol{\Lambda}_k, k = 1,2\ldots,8$ and $\boldsymbol{\Lambda}_0$, operators which are anti-commutative and non-associative, therefore, they cannot be represented by matrices, such as Gell-Mann's generator matrices for SU(3). From the above equation, we obtain

$$exp(\boldsymbol{m}\tau) = cos(\omega\tau) + (\boldsymbol{m}/\omega)\, sin(\omega\tau)$$
$$\omega^2 \equiv \sum_{k=1}^{3} {\Omega_k}^2 + \sum_{k=0}^{8} {\lambda_k}^2 \qquad (4B)$$

which indicates the effective rest mass of the electron is contributed by internal kinetic energy $\sum_k \quad {\lambda_k}^2$ and intrinsic mass $\sum_k \quad {\Omega_k}^2$. Those eight $\boldsymbol{\Lambda}_k$ operators in Eq. (4A) represent **su**(3) algebra, resembling Gell-Mann's 8 lambda matrices, however, sedenions are non-associative and anti-commutative.

During the search process for the integer set solution, we first need to identify a Pythagorean prime which can be decomposed into a sum of a prime P square and an even integer Q square. Secondly, this prime component needs to be decomposed into triple integer squares. Then, each of the first two component integer squares needs to be further decomposed into a triplet, and the third integer square needs to be decomposed into a doublet so that there will be a total of 8 integer squares to satisfy SU(3) symmetry. Then, the Q integer square needs to be decomposed into a triplet. Any candidate that fails to pass these tests will be disqualified. After an extensive CPU-time demanding search for the right integer set that satisfies the constraints, we have found a Pythagorean prime, ζ =13703599920605017, and relevant decomposed components as listed in Table 2.

Table 2. **Pythagorean prime and 12 integer components, representing 12 $\mathbf{su}(3) \oplus \mathbf{su}(2) \oplus \mathbf{u}(1)$ algebra according to the sedenion model**

ζ = 13703599920605017			
G = 89945381 $\mathrm{G}^2 = \sum_{k=1}^{8} {\lambda_k}^2$		F = 119889 $F^2 = \theta^2 + {\lambda_0}^2$ $\theta^2 = \sum_{k=1}^{3} {\theta_k}^2 = 36036^2$ ${\lambda_0}^2 = 114345^2$	
λ_1 =128	λ_5 =1604	θ_1 = 276	λ_0 =114345
λ_2 =160	λ_6 =18888	θ_2 =888	
λ_3 =640	λ_7 =8455391	θ_3 =36024	
λ_4 =24	λ_8 =89547069		

The above decomposition yields eight integer squares, corresponding to $\boldsymbol{su}(\mathbf{3}) \oplus \boldsymbol{su}(\mathbf{2}) \oplus \boldsymbol{u}(\mathbf{1})$ algebra of the Standard Model. By scaling to 137 to be related to the electron's mass, we obtain $1/\alpha = 137.03599920605017$, with 15 decimal digits.

One can construct Gell-Mann's SU(3) generators fusing sedenions requires a mixing between $\{\mathbf{I}, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3\}$ an $\{\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$, it breaks the boundary between the inner and outer spacetime. Therefore, unlike the leptons, quarks, and gluons are spatially confined to the core.[19] Despite some resemblances of $\mathbf{su}(\mathbf{3}) \oplus \mathbf{su}(\mathbf{2}) \oplus \mathbf{u}(\mathbf{1})$ (all together 12 degrees of freedom) algebra to the Standard Model's symmetry group, the sedenion algebra is not associative, unlike the direct product of matrices used in the Standard Model. Therefore, the symmetry group and the corresponding Lie algebra of the Standard Model represent the broken symmetry of the sedenion algebra. For SU(3) generators, which require a mixing between sedenion operators belonging to inner and outer spacetime, gluons and quarks are confined internally, and cannot propagate outside the inner core[19]

In summary, we present a new paradigm that expands Einstein's concept of continuous 4D spacetime, and the assumption in the Standard Model involving the Dirac equation in the description of leptons and quarks as structureless point-line objects. We developed an extended Dirac equation using octonion and sedenion algebra, introducing internal spacetime for elementary particles. This structure, governed by octonion and sedenion algebra, contributes to particle mass without requiring the Higgs mechanism. We derived the fine structure constant as a geometric property of spacetime through mass quantization. Via Pythagorean rules extended for higher dimensional space imposed by the specific constants, our theoretical calculation of the geometric constant achieves an unprecedentedly accurate prediction with a deviation of $\sim 10^{-12}$. This suggests the fine structure constant is not just a traditional physical constant but emerges from spacetime geometry. This dimensionless geometric constant governs electromagnetism, it also plays an intricate role in strong, weak, and gravitational interactions. We have obtained empirical rules for the mass ratio to the electron of $(\pi/\alpha)^2 4/3$ for Higgs boson, $(\pi/\alpha)^2\, 4^2/(5\sqrt{3})$ for top quark, and $(\pi/\alpha)^8\, (9/2)^2$ for the Planck mass with ~ 1% discrepancy. Such simple relations imply the role of π/α to the mass ratios of the Higgs boson, top quark, and Planck mass via a simple formula. These links imply the possibility of quantum gravity theory and the grand unification of all four forces in nature, we elucidate the physical contents of the hyper-complex algebra, offering a framework of spacetime lattice for descriptions of quantum physics of elementary particles, and opening an avenue beyond the Standard Model[21] toward quantum gravity[22-24] and grand unification.[25,26]

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