A Brief Proof that Pi is Irrational

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Abstract

Assuming π is rational both multiples of π and $\pi/2$ exist as a factorial. But this implies both $\sin(j!)$ and $\cos(j!)$ are zero contradicting the sum of their squares is one.

Easy Case

Suppose for a contradiction $\pi = 22/7$. Then $11!$ will be a multiple of $\{22, 2, 7\}$. Let

$$\frac{7}{22} 11! = n_1.$$

This is an integer and yields $\pi n_1 = 11!$, giving $\sin n_1 \pi = \sin(11!) = 0$.

Let

$$\frac{2 \cdot 7}{22} 11! = n_2.$$

This too is an integer and yields

$$\frac{\pi}{2} n_2 = 11!,$$

giving $\cos(n_2 \pi/2) = \cos(11!) = 0$.

But using the Pythagorean identity $\cos^2(11!) + \sin^2(11!) = 1$, not 0, a contradiction.

General Case

Assume $\pi = p/q$, then there exists a $j!$ that has factors of $\{p, q, 2\}$ in it. Let

$$n_1 = j! \frac{q}{p}$$

and

$$n_2 = j!\frac{2q}{p}.$$

Both are integers and

$$n_1\frac{p}{q} = j! \text{ and } n_2\frac{p}{2q} = j!$$

gives

$$\sin(j!) = \cos(j!) = 0,$$

contradicting $\cos^2(j!) + \sin^2(j!) = 1$.

References

[1] I. Niven, A simple proof that π is irrational, *Bull. Amer. Math. Soc.* **53** (1947) 509.

[2] ———, *Irrational Numbers*, Carus Mathematical Monographs, no. 11, Mathematical Association of America, Washington, DC, 1985.