COLLATZ'S CONJECTURE

A PROOF

by

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INTRODUCTION : -

$Consider\ the\ following\ equation\ for\ positive\ integer$

$value\ of\ 'n',\ f(n,1) = \frac{n}{2}\ if\ n\ is\ even\ and\ 3n+1\ if\ odd.$

$$let\ f(n,r) = f[f(n,r-1),1]\ for\ n \geq 2$$

$Let\ the\ set\ A(n)\ = \{f(n,1), - - - - - -, f(n,r), - - - -,\ \}$

$Let\ the\ set\ B(n) = \{f(n,1), - - - -, f(n,r), - -, 1,4,2,1,4,2,1 -\}$

$As\ per\ Collatz's\ Conjecture\ A(n) = B(n).$

In this proof all the numbers involved are '0' & positive integers

And hence, no further mention of it will be made.

$It\ is\ obvious\ that\ it\ has\ to\ be\ proved\ only\ for\ odd\ values\ of\ 'n'$

$because\ for\ even\ values\ of 'n' it\ will\ tend\ to\ an\ odd\ number.$

PROOF:-

$\underline{PART\ 1:-\ \ An\ analysis\ of\ numbers\ 1,5,21,85,\ etc}$

(4-1) $\{ 4^{r-1} + 4^{r-2} + 4^{r-3} + --- + 1 \} + 1 = 4^r$

$So,\ \ if\ \ n = \{4^{r-1} + 4^{r-2} + --- + 1\},\ \ f(n, 2r) = 1$

$Hence\ \ A(n) = B(n).$

$This\ constitutes\ the\ set\ S = \{ ar: ar = [(4^r - 1) \div 3], r \geq 1\}$

$And\ A\ (ar) = B(ar)\ .\ And\ S = \{ 1, 5, 21, 85, ---\ \}$

$\underline{PART\ 2:-\ \ An\ analysis\ of\ \ n = (4k+1)}$

Let us assume that $if,\ \ n \leq 4k+1\ then\ A(n) = B(n)$

Let, $n_1 = 4(k+r)+1;\ then,\ f(n_1, 3) = 3(k+r)+1$

The numbers of the set $'S'$ have value of $k = (0,1,5,21,---)$

$\underline{i)\ 'k'\ \ is\ odd\ \&\ \ 'r'\ \ is\ even}$

Then $(k+r) is\ odd\ and\ so\ f(k+r, 1) = 3(k+r)+1$

$A\ (n_1) = B\ (n_1),\ if,\ (\ k+r) \leq 4k+1\ or\ r \leq 3k+1$

So, $for\ \ k = 1,\ \ r = 2, 4\ \ \&\ \ k+r = 3, 5.$

$k = 5,\ \ r = 2, 4, 6, 8, ---, 16\ \&$

$k+r = 7, 9, 11, 13, ---, 21.$

$k = 21,\ \ r = 2,4, ----, 64$ and so on.

These generate the set $K_1 = \{ 1, 3, 5, ---\}$

$K_1 = \{ k: k = 2s-1, s \geq 1\}$

$\underline{ii)\ 'k'\ \ is\ \ odd\ \&\ \ 'r'\ \ is\ odd}$

$A(n_1) = B(n_1),\ \ if\ 3(k+r)+1 \le 4k+1\ ie\ 3r \le k$

$k = 3\,,5,7\ \ ;\ \ r = 1\ \ \&\ \ \ k+r = \ 4,6,8\ .$

So, $k = 9{,}11{,}13\ ;\ \ \ r = 1{,}3\ \ \&\ k+r = 10\,,12{,}14{,}16\ ;\ \ and\ so\ on.$

These generate the set $K_2 = \{\, k : k = 2s\ \&\ s \ge 0\,\}$

$(\ k = 0,2\ is\ included\ ,as\ it\ can\ be\ verified.)$

The set $K\ = K_1 U K_2 = \{\, k\colon k = s\ \&\ s \ge 0\,\}$

This set $'K'$ generates the set,

$N(2^2) = \{\,1,5,9,13,----\}$

$= \{\, n_2\ :\ n_2 = (4k+1),\ \ k \ge\ 0\}\ \ \ \&\ \ \ .$

$A(n_2) = B(n_2)$

$\underline{\ PART\ 3 : -\ \ An\ analysis\ of\ \ n = (4k+3\,)}$

Though it is assumed that n= 4k+1 is solved ,4k+1 and 4k+3 are Interconnected. It will be shown how (4k+3) becomes (4k+1) before Becoming '1'.

Let $n = (4k+3);'k'can\ be\ odd\ or\ even.$

Let, $k = 2k_3\ or\ 2k_3+1\ \&\ n_3 = 8k_3+3\ \&\ n_{31} = 8k_3+7\ .$

$\underline{(1)\ n_3 = (\ 8k_3+3\ )}$

$f(n_3,1) = f(2^3k_3+2^2-1{,}1) = 3.\,2^3k_3+2^3+2^2-3+1$

$f(n_3,2) = 3.\,2^2k_3+2^2+2-1 = 4(3k_3+1)+1$

$Therefore,\ \ A(n_3) = B(n_3).$

Thus, the following set is generated.

$N\ (2^3) = \{3, 11, 19, ---\} = \{n_3\colon n_3 = (2^3k + 3),\ k \geq 0\}$.

$In\ \ n_{31},\ \ k_3\ \ can\ be\ odd\ or\ even.$

Therefore let, $n_4 = 2^4k_4 + 7\ \ \&\ \ n_{41} = 2^4k_4 + 15$

$\underline{(2)\ n_4 = 2^4k_4 + 2^3 - 1}$

$f(n_4, 1) = 3.2^4k_4 + 2^4 + 2^3 - 3 + 1 = 2^4(3k_4 + 1) + 2^3 - 2$

$f(n_4, 2) = 2^3(3k_4 + 1) + 3\ :\ [\,(8k + 3)\,]$

$f(n_4, 4) = 2^2(\,3^2k_4 + 3 + 1) + 1$

$Therefore\ , A(n_4) = B(n_4)$

Thus, the following set is generated.

$N(2^4) = \{7, 23, 39, ---\} = \{n_4 : n_4 = (2^4k + 7),\ k \geq 0\}$

$In\ n_{41}, k_4\ \ can\ be\ odd\ or\ even.$

$Therefore\ let\ n_5 = 2^5k_5 + 15\ \&\ n_{51} = 2^5k_5 + 31$

$\underline{(3)\ n_5 = 2^5k_5 + 2^4 - 1}$

$f(n_5, 6) = 4(3^3k_5 + 3^2 + 3 + 1) + 1$

$Therefore,\ A(n_5) = B(n_5)$

Thus, the following set is generated.

$N(2^5) = \{15, 47, 79, --\} = \{n_5\colon n_5 = (2^5k + 15),\ k \geq 0\}$

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$let\ n_r = 2^r + 2^{r-1} - 1\ \ \&\ n_{r1} = 2^r + 2^r - 1$

(r-2):- $n_r 2^r k_r + 2^{r-1} - 1$

$f(n_r, 2r-4) = 4(3^{r-2}k_r + 3^{r-3} + 3^{r-4} + --- + 1) + 1$

$Therfore\ \ A(n_r) = B(n_r)$

Thus, the following set is generated.

$$N(2^r) = \{[(2^{r-1} - 1),\ ----,\ (2^{r-1} - 1 + k2^r),\ --]\}$$

$$= \{n_r : n_r = (2^r k + 2^{r-1} - 1),\ k \geq 0\}$$

PART 4 ; - The Conclusion

All these sets put together constitutes the master set N (2)

$$N(2) \equiv \{N(2^2)UN(2^3)U ----- U(2^r) ---;\ r \to \infty\}.$$

The Set $N(2) = \{1, 3, 5, --\} = \{n_2 : n_2 = (2k+1),\ k \geq 0\}$

$And\qquad A(n_2) = B(n_2)$

The above set represents all odd numbers. All even numbers tend to an odd number.

$$Therefore,\quad A(n) = B(n)\ \ \&\ n \in N$$

Thus, $COLLATZ'S\ \ CONJECTURE,$ Is proved for all Positive Integers.