An Extension of the Schwarzschild Solution: Combined Gravitational Field of Two or More Sources

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Abstract

There is no direct mathematical tool that enables us to generalize Schwarzschild solution to cases where there is more than one source of gravitational field. In this paper, we explained that this generalization can be made if we develop an equivalent and parallel description to the geometric description based on the metric tensor, and we found that this method succeeds in generalizing the Schwarzschild solution for any number of gravitational sources that move relative to each other.

Introduction and Overview

The great work that Schwarzschild accomplished[1] in finding a simple and elegant relationship between space and matter has one flaw which is that the solution he arrived at is very rigid and can only be applied in one ideal case and cannot be dealt with using mathematical tools such as superposition, integration or statistics to derive solutions for general cases such as the presence of more than one source of gravitational field, that is because the metric tensor which represents the variable of gravitational field in general relativity cannot be combined by any mathematical tool, given multiple sources of gravity, each source corresponding to some metric , the "net metric" at a given point does not equal the sum of the individual metrics of each sources or any known function of these metrics. But the situation is not completely hopeless, as mathematics has many tactics other than these direct tools for deducing general relations from special and ideal cases. Here, we want to use one of this tactics which is to replace the variables of the relationship of the ideal case with other variables that can be dealt with by mathematical tools.

I want the reader not to rush to criticize the idea through the issue of whether the new description matches reality or not, because the intention is not to present an alternative to the geometric description based on the metric tensor of the gravitational field, this alternative description is merely a means to perform some mathematical operations and we quickly return to the original description.

Equivalence of Metric Tensor and Space Flow

Let's have a polar reference system with center O in a vacuum devoid of matter, in this case the metric tensor distributed around this point can be found from the formula:

$$ds^2 = C^2\, dt^2 - dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\Omega^2 \qquad (1)$$

So for a point **P(t , r , θ , Ω)** the metric is :

$$g_{tt} = C^2 \,, \quad g_{rr} = -1 \,, \; g_{\theta\theta} = -r^2 \,, \; g_{\Omega\Omega} = -r^2 \sin^2\theta \,,$$

and all other components equal zero.

Now suppose we put a mass M in point O. This mass will cause some changes in the space-time around the point.

Let's temporarily forget everything we know about Newton's law, general relativity and Schwarzschild's solution and look for a new way to address the problem guided only by the general principles of physics and special relativity and let us begin by making the following simple assumption: The presence of mass will cause the space around it to move inward in the direction to the center, so that the speed of any point of this space with respect to the center is given by:

$$\mathbf{v} = \sqrt{\frac{\mathbf{2GM}}{\mathbf{r}}} \quad \textbf{(2)}$$

This velocity will cause changes in the radial distance and time measurements as required by special relativity:

$$\mathrm{dr}' = \sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{C}^2}}\,\mathrm{dr} \;,$$

and

$$\mathrm{dt}' = \frac{\mathrm{dt}}{\sqrt{1 - \frac{\mathrm{v}^2}{\mathrm{c}^2}}} \,.$$

Substituting from equation (2) we get:

$$\mathrm{dr}' = \sqrt{1 - \frac{\boldsymbol{2GM}}{\boldsymbol{c^2 r}}}\,\mathrm{dr} \,,$$

and

$$\mathrm{dt}' = \frac{\mathrm{dt}}{\sqrt{1 - \frac{\boldsymbol{2GM}}{\boldsymbol{c^2 r}}}} \,.$$

Now by substituting into Equation (1) we get:

$$ds^2 = \sqrt{1 - \frac{2GM}{c^2 r}}\, C^2\, dt'^2 - \frac{dr'^2}{\sqrt{1-\frac{2GM}{c^2 r}}} - r'^2\, d\theta'^2 - r'^2 \sin^2\theta'\, d\Omega'^2 \quad (3)$$

This is the same result that Schwarzschild arrived at!

We also notice the equivalence between the two methods when we calculate the singularity points. According to the formula (2) , there is a singularity point when the speed of points of space reaches the speed of light. When calculating the radius with which this speed is achieved, we will find it to be the same as the Schwarzschild radius.

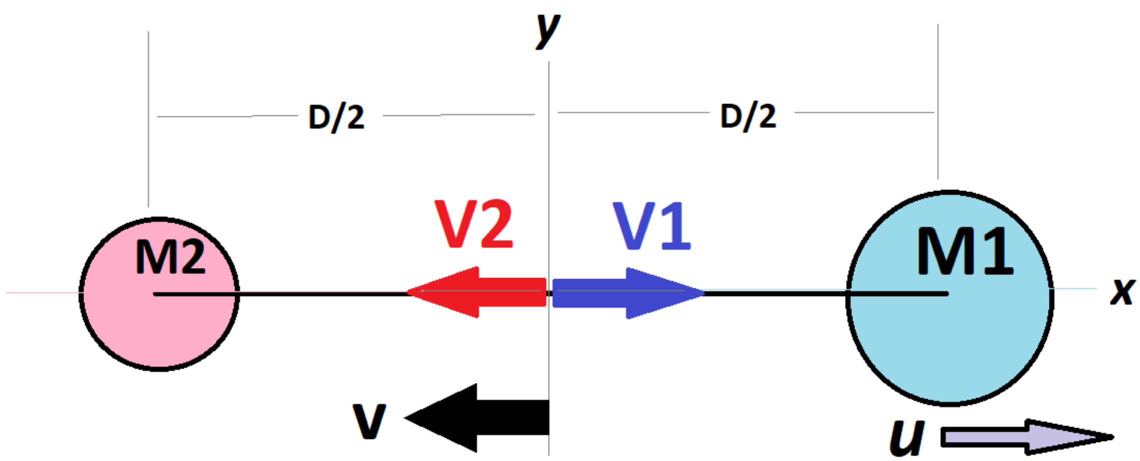
[This formula is attributed to Schwarzschild, although the first person to arrive at this exact formulation was Droste, [2] but it was Schwarzschild who developed the general method that leads to this formula and the formula close to it that he himself arrived at, which does not differ from Droste's formula except in the assumption that determines the point of singularity.]

It is truly amazing that such a simple assumption can lead to the same formula deduced by Schwarzschild from the principles of general relativity, and it is worth noting that if we consider equation (2) to be a modified version of Newton's law of universal gravitation, it will be an equivalent formula to general relativity, at least in applications to the gravitational field outside the gravitational source. But in reality, although this formula bears apparent resemblance to newton's law of gravitation, we find this formula radically different from it because it does not represent a force field, but rather represents the effect of matter on the space around it in the form of movement off all points of space (space flow) at certain speeds to be reflected on the metric tensor and meet general relativity.

More importantly, the velocity, which we have used as a means by which we can calculate the effect of a gravitational source on the metric tensor, can be used to solve a major dilemma in general relativity, namely the summation of gravitational fields, since the metric tensor produced by two gravitational sources is not equal to the sum of the two metric tensors produced by each source separately and we do not know any way to find the resultant. We do not know any way to find the resultant metric in the case of two sources, but we do know how to add the velocities, so in the case of more than one source of gravitation, we can add the velocities resulting from each source at a certain point and then convert this velocity into metric tensor and so we have a complete map of the distribution of the metric tensor around the sources of gravitation. Not only that, but we can also calculate this for all cases of gravitational source motion because we know the effect of the velocity of the gravitational source on the velocity of the points where we want to study the gravitational field.

Let's illustrate this with a simple example where we apply these ideas:

Suppose we have two objects of mases M1 and M2 and let them be separated by a distance D, and that M2 is at rest and M1 is moving with speed u away from M2, and we want to calculate the effect of these two object on the metric tensor at the midpoint between them:



The Resultant Velocity

We have:

$$\mathbf{v2} = \sqrt{\frac{\mathbf{4G(M2)}}{\mathbf{D}}} \quad \text{and}$$

$$\mathbf{v1} = \frac{\sqrt{\frac{\mathbf{4G(M1)}}{\mathbf{D}}} + \mathbf{u}}{\mathbf{1} + \sqrt{\frac{\mathbf{4G(M1)}}{\mathbf{D}}}\ \mathbf{u/C^2}}$$

The resultant velocity $\mathrm{V} = \mathrm{V2} - V2$ in X direction, the other direction will not be affected.

Then if we use Cartesian coordinates shown in the figure we get:

$$g_{xx} = \frac{-1}{\sqrt{1-\frac{v^2}{c^2}}}, \quad g_{yy} = -1, \ g_{zz} = -1, \ g_{tt} = C^2\sqrt{1-\frac{v^2}{C^2}},$$

and all other components equal zero.

Conclusions and Outlook

It is clear that the idea of explaining gravitation on the basis of geometry is a beautiful and very attractive to the point of discouraging physicists from thinking about alternatives to it, but this does not prevent us from trying to address the shortcomings and fill the gaps in this explanation by using other concepts that share the same results.

References

[1] On the Gravitational Field of a Point-Mass According to Einstein's Theory. Karl Schwarzschild, Jan1916.

[2] The Gravitational field of one or more bodies according to Einstein's Theory. Johannes Droste, Des.1916.