On Complex Dynamics and the Schrödinger Equation

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Abstract

Complex Ginzburg-Landau equation (CGLE) is a paradigm of complex dynamics that holds for all spatially extended systems near the onset of oscillatory behavior. CGLE applies to a vast array of phenomena ranging from superconductivity and superfluidity, to Bose-Einstein condensation, astrophysics, nonlinear optics and spatiotemporal chaos. In particular, CGLE describes the formation of dissipative spacetime structures in Reaction-Diffusion (RD) processes. Here we bridge the gap between CGLE and the RD model of *evolving dimensional fluctuations*, the latter being conjectured to arise far above the electroweak scale. Our findings open an intriguing path connecting complex dynamics of dimensional fluctuations to Quantum Physics.

Key words: Complex Dynamics, Complex Ginzburg Landau equation, Reaction-Diffusion processes, dimensional fluctuations, Schrödinger Equation, Quantum Physics.

<u>Cautionary remarks</u>:

We caution from the outset that the sole intent of this paper is to lay the groundwork for further analysis and exploration. Independent work is needed to develop, validate, or reject the ideas presented here.

1. Introduction

Traditional textbooks on condensed matter and statistical physics typically cover the topic of large many-body systems in *thermodynamic equilibrium*. An implicit assumption of this classic paradigm is that small fluctuations away from equilibrium relax to zero in a finite lapse of time. The assumption extends to transient perturbations that do not impact the global stability of the system or produce unpredictable outcomes in the long run.

By contrast, the last decades have consistently shown that most many-body systems evolve in *out-of-equilibrium* conditions. These conditions apply to a vast array of collective phenomena, from early Universe cosmology and astrophysics to climate science, evolution of living organisms, social interactions, molecular dynamics, information networks, collective processes in fluids and plasma, and relativistic heavy ion collisions in high-energy physics. As of today, the consensus is that the science of *complex many-body systems* in non-equilibrium conditions has gained significant ground, with notable contributions across various disciplines.

Complex dynamics is customarily characterized by non-linear, dissipative and non-local interactions between components, by the emergence of instability and chaos, the tendency to self-organize and adapt, to generate unforeseen outcomes and patterns, to unfold on multiple scales, display memory effects and an endless game of cooperation and competition. Applying complex dynamics to theoretical physics requires unconventional tools such as the analysis of chaos and multifractal geometry, non-extensive statistics, self-organized criticality, and fractional dynamics.

In this context, *Complex Ginzburg-Landau equation* (CGLE) is a paradigm of complex behavior that holds for all spatially extended systems near the onset of generic oscillations [5, 11–13, 15]. CGLE has found applications to many phenomena including the formation of dissipative spacetime structures in Reaction-Diffusion (RD) processes [3–4, 6–13]. The goal of this work is to bridge the gap between CGLE and the RD model of *evolving dimensional fluctuations*, the latter being conjectured to develop far above the electroweak scale.

The paper is structured in the following way: next section is a review of [1], which contains an account of the RD model of evolving spacetime dimensions and dimensional fluctuations; section 3 elaborates upon the derivation of CGLE from dimensional fluctuations and its subsequent analogy with quantum theory. A brief discussion and the list of abbreviations are included in the last section.

2. RD model of evolving spacetime dimensions

For the sake of clarity and reader's convenience, we begin this section with a condensed review of [1].

RD processes are a subset of complex phenomena defined within the framework of Nonequilibrium Statistical Physics. These models are typically formulated in $d+1$ dimensions, where d is the dimension of the Euclidean manifold representing the physical space and t is the time coordinate. Ref. [1] develops a toy RD model acting on a two-dimensional lattice $(d=2)$, whose local variables are time-varying *dimensional fluctuations* $\delta\varepsilon(t)=\delta[2-d(t)]$. The model includes a *scattering* event at rate D, a *clustering* event at rate u and a *decay* (or *percolation*) event at rate $\kappa=\lambda-\lambda_c$, with λ being a control parameter nearing its critical value λ_c.

Up to a leading order approximation, the macroscopic properties of RD processes may be encoded in a *mean-field* (MF) equation, which quantifies the competition between losses and gains in a generic density parameter

$\rho(t)$. In particular, the decay/percolation process occurs with a rate proportional to $\kappa\rho(t)$ and leads to a gain in density. By contrast, the clustering process drops the density with a rate proportional to $u\rho^2(t)$. Ignoring diffusion, the resulting MF equation takes the form

$$\frac{\partial\rho(t)}{\partial t}=\kappa\rho(t)-u\rho^2(t) \tag{1}$$

In the context of [1] and this paper, the control parameter $\lambda(t)=\lambda[\delta\varepsilon(t)]$ represents the *density of dimensional fluctuations* $\delta\varepsilon(t)<<1$ while $\rho(t)$ denotes the *density of active (or unstable) lattice sites.*

3. From dimensional fluctuations to CGLE and Quantum Physics

A straightforward extension of (1) is given by the system of coupled partial differential equations [3-4]

$$\frac{\partial\rho_1(x,t)}{\partial t}=D_1\Delta\rho_1(x,t)+f(\rho_1,\rho_2,\kappa) \tag{2a}$$

$$\frac{\partial\rho_2(x,t)}{\partial t}=D_2\Delta\rho_2(x,t)+g(\rho_1,\rho_2,\kappa) \tag{2b}$$

The system (2) models a wide range of phenomena spanning from physics, chemistry, and biology, to ecology, social dynamics and data science. The behavior of its solutions depends on the diffusion coefficients D_1, D_2 and boundary conditions. According to [2–4], an arbitrary solution of (2) lying near the bifurcation point at $\kappa > \kappa_0$ can be expressed through a *complex-valued function* $W(r,\tau) = U(r,\tau) + i\,V(r,\tau)$ obeying the complex Ginzburg-Landau equation (CGLE) [2-4 , 9, 11–13]

$$\boxed{\frac{\partial W}{\partial \tau} = W + (1 + i c_1)\frac{\partial^2 W}{\partial r^2} - (1 + i c_2) W |W|^2} \tag{3}$$

Here, the set of new coordinates is given by [3–4]

$$r = \eta x \tag{4a}$$

$$\tau = \eta^2 t \tag{4b}$$

where,

$$\eta = (\kappa - \kappa_0)^{1/2} \propto (\lambda - \lambda_c)^{1/2} << 1 \tag{5}$$

Upon taking $c_1, c_2 \to \infty$ and performing few rescaling operations, (3) turns into a dissipative extrapolation of the standard Schrödinger equation, called *nonlinear Schrödinger equation* (NSE) [13]

$$i\frac{\partial W}{\partial \tau} = \frac{\partial^2 W}{\partial r^2} \pm W|W|^2 \qquad (6)$$

It is important to note here that NSE describes *deterministic nonlinear wave phenomena* in classical fluid mechanics and optics. As such, it stands in direct contrast with the conservative Schrödinger equation, which describes the unitary evolution of quantum wavefunctions.

There is a couple of ways one can convert (6) into the standard Schrödinger equation, namely,

a) Using the linear approximation of the cubic term of (6) either through the tangent line approximation or cubic Hermite splines [16]

b) Imposing the *unitary norm constraint* on the function W,

$$\boxed{|W|^2 = WW^* = U^2 + V^2 = 1} \tag{7}$$

Choosing option b), one is led to a Schrödinger-type equation with a constant probability density and a uniform potential term,

$$\boxed{i\frac{\partial W}{\partial \tau} = \frac{\partial^2 W}{\partial r^2} \pm W} \tag{8}$$

It is now apparent that (8) points to the formal analogy

$$\boxed{W(r,\tau) \Leftrightarrow \psi(r,\tau)} \tag{9}$$

One recalls that the wavefunction $\psi(r,\tau)$, aside from being defined and continuous everywhere, must be square-integrable and normalized to unity according to

$$\int |\psi(r,\tau)|^2 d^3r = 1 \tag{10}$$

It follows from (7) that condition (10) amounts to a normalization of the integration space, that is,

$$\int d^3 r = 1 \tag{11}$$

In summary, by (8)–(11) and per the diagram below, our findings indicate that Quantum Physics may be interpreted as *limit case* of complex dynamics under suitably defined conditions.

Dimensional Fluctuations → **CGLE** → **Quantum Physics**

3. Discussion

Several key remarks and caveats are in order, as detailed below:

1) While both CGLE and NSE (3) and (6) reflect an entirely *deterministic* framework, the standard Schrödinger equation is based upon a *probabilistic* framework. One needs to keep in mind, however, that the derivation of (3) and (8) starts from (1), which is a model of *dimensional fluctuations*, inherently statistical in nature.
2) To the extent that the CGLE (3) can be tied to the evolution of a many-body Hamiltonian systems, one can appeal to the so-called mechanism

of *Arnold diffusion* (AD) to motivate the quantization of action and the existence of Planck's constant [17]. As a result, the appearance of Planck's constant in the standard form of the Schrödinger equation is a natural outcome of AD.

3) A particular embodiment of CGLE (3), the so-called Stuart–Landau equation, may be shown to provide the basis of the *spin-statistics theorem* and the formation of Dark Matter as a topological condensate with undefined spin [18]
4) The bifurcation structure of the Standard Model may be sequentially derived from the real Ginzburg-Landau potential [10].
5) At least in principle, using the Bohm potential and Madelung transformation, one can extrapolate from the NSE to Navier-Stokes and the General Relativity equations [14].
6) There are well-motivated hints that bifurcations, pattern formation and complex behavior underlie the composition of the Standard Model for particle physics [5-6, 10-11].

7) RD theory can be cast in the language of *creation-annihilation* parameters, [7–8] This conclusion reinforces the bridge between complex dynamics and Quantum Physics.

8) Finally, RD processes and CGLE are tied to *self-organization*, which is a hallmark feature of complex dynamics [2, 19].

Abbreviations

RD = reaction-diffusion

CGLE = Complex Ginzburg-Landau Equation

NSE = Nonlinear Schrödinger Equation

AD = Arnold diffusion

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