Infinite tree branch hypothesis

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12/06/2024

Abstract
The Infinite Tree Branch Hypothesis explores the transformation of infinitely branching trees across dimensions. In 2D, infinite branches converge into a circle, while in 3D and 4D, they form a sphere and hypersphere, respectively, with growth dictated by dimensional scaling. Using a what I think is novel Tree(n-branch)funtion, although agreebly this could have been invented before in fractal geometry or graph theory (but I could not find such a function). This hypothesis mathematically formalizes the convergence of such structures, leveraging properties of spherical coordinates and Gaussian integrals to define their limits. Novel implications include a reinterpretation of infinite branching within quantum mechanics, suggesting that seemingly independent "branches" in the Many-Worlds Interpretation may converge into a unified hyper continuum in higher dimensions. This hypothesis introduces a unique perspective on infinite systems, connecting fractal geometry, graph theory, and quantum cosmology.

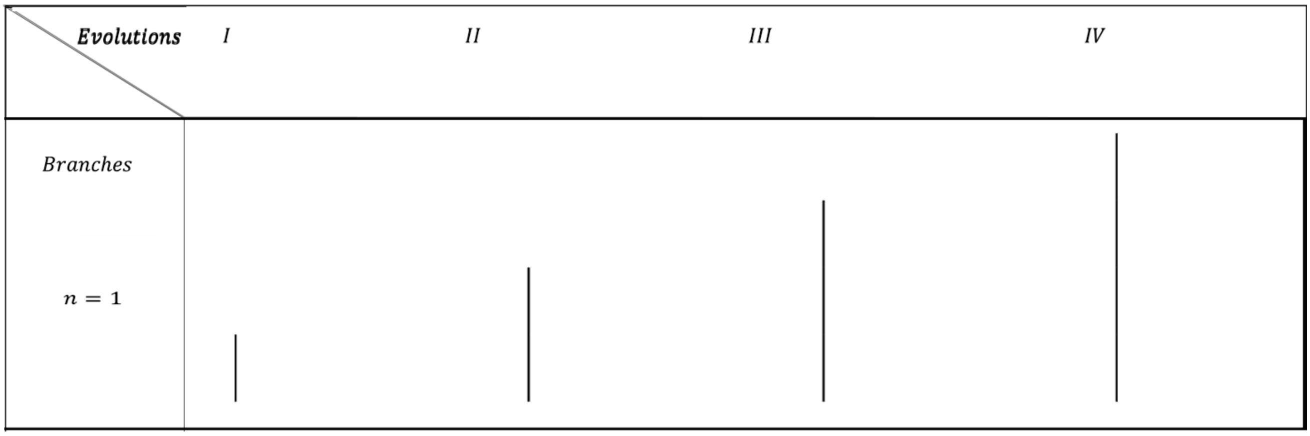
Hypothesis

If we would solely to increase the number of branches a tree in the plane (2D) had and if for each evolution we would carry on that same amount of branches something remarkable would happen. The tree with ifinetely many branches would become perfectely circular, become a circle with area, because the respective angles θ between each branch to its adjacent would go aigainst 0 as n (number branches) goes against infinity. With each passing evolution the planar tree with infinetely many branches that has become circle by now would grow in radius and thus in area.

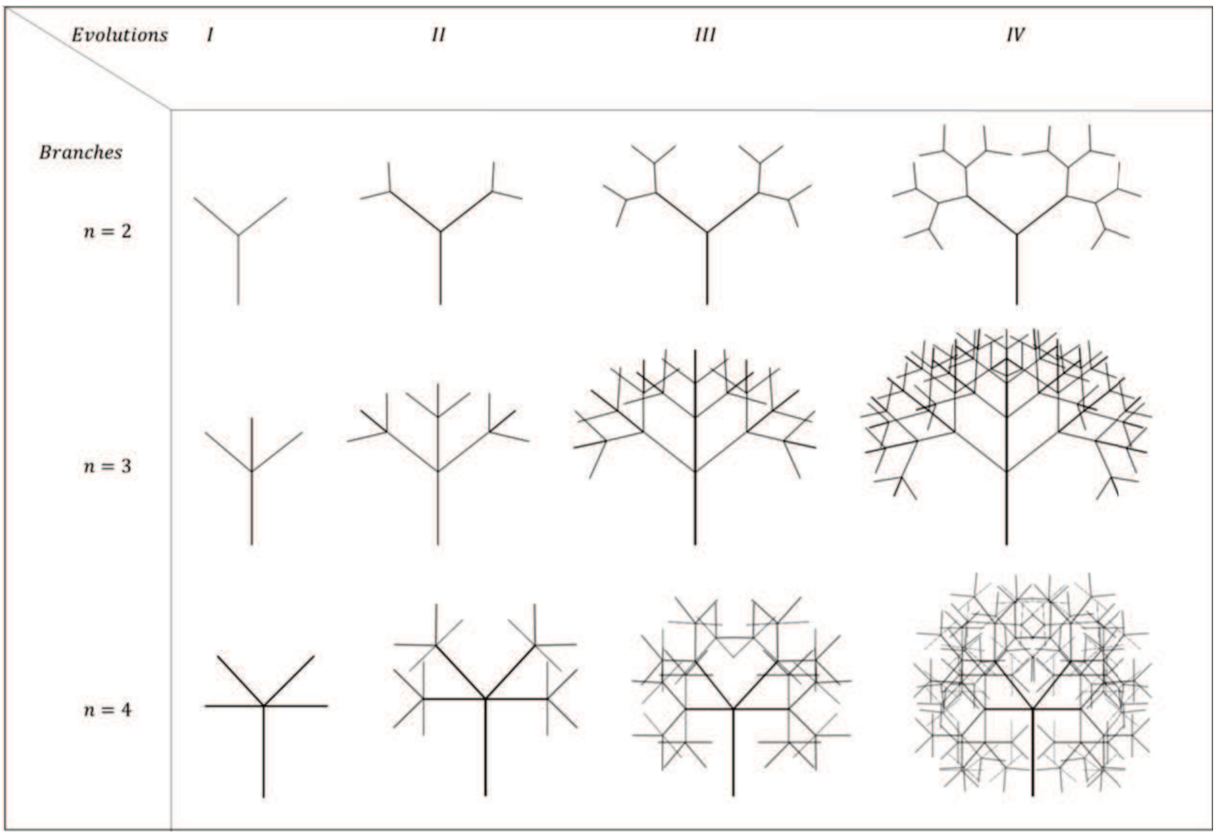
A 3D tree has three dimensional branching and would become a sphere with volume.if for each evolution we would carry on that same amount of 3 dimensional branches the angles would as well as in the planar tree, go against zero, thus the tree with ifinetely many 3D branches in the successing evolution would also be spheres with volume. A sphere with infinetely many sheres on its surface just goes against a bigger sphere, one with a bigger radius, and thus bigger volume.

Graphical example of 4 evolutions up to n=4

Special case of n=1 where only the line extends in length about its own length, or multiples of 1. This is not considerable a tree, but important not to oversee as well.



Graphical example of cases of n $\in \mathbb{N}$ there n$\neq$ $0, n \neq 1$.



Proof:

Mathematically this can be shown by introducing a tree function that scales as its argument scales. We define it as $Tree_{2D}(n \cdot branches)$ in 2D and it as $Tree_{3D}(n \cdot branches)$. Also it as $Tree_{nD}(n \cdot branches)$ for n-D.

For 1D we obtain:

$$\lim_{n\to\infty} Tree_{1D}(n \cdot branches) = 0 \vee 1$$

For 2D we obtain:

$$\lim_{n\to\infty} Tree_{2D}(n \cdot branches) = \pi r^2$$

For 3D we obtain:

$$\lim_{n\to\infty} Tree_{3D}(n \cdot branches) = \frac{4}{3}\pi r^3$$

For 4D we obtain:

$$\lim_{n\to\infty} Tree_{4D}(n \cdot branches) = \frac{\pi^2}{2} r^4$$

For n-D we obtain:

$$\lim_{n\to\infty} Tree_{4D}(n \cdot branches) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} r^n$$

The radius of the $Tree_{nD}(n \cdot branches)$ is dependent on the length of each brach as well as their displacement from beeing orthogonal to the origin. Thereafter it depends on the number of evolutions. The esiest case is one evolution and as $n \to \infty$ the angle $\theta \to 0$, so we assume they become othogonal to the origin (wich is a single point untop of the line of the «Tree-stem»), and if they have the same length as the «Tree-stem» wich we assume to be r=L=1 for simplicity, they form a circle of the radius one, a sphere of radius 1, a hyperspher of radius 1 and so forth. For each evolution, assumed the length of the branches does not change, groes bigger by one.

That means after the second evolution r=2, after the third evolution r=3, after the forth r=4 and so forth.

I here rely on the proof by using Gaussian integrals, consider the function:

$$f(x_1, x_2, x_3, \dots, x_n) = \exp\left(-\frac{1}{2}\sum_{i=1}^{n} x_i^2\right)$$

This function is both rotationally invariant and a product of functions of one variable each. Using the fact that it is a product and the formula for the Gaussian integral «gives:

$\int_{R^n} f dV = \prod_{i=1}^{n} \left(\int_{-\infty}^{\infty} \exp\left(-\frac{1}{2} x_i^2\, dx_i\right) \right) = (2\pi)^{n/2}$,

where dV is the n-dimensional volume element. Using rotational invariance, the same integral can be computed in spherical coordinates:

$\int_{R^n} f dV = \int_0^{\infty} \int_{S^{n-1}(r)} \exp\left(-\frac{1}{2} r^2\right) dA dr$,

where $S^{n-1}(r)$ is an $(n - 1)$ sphere of radius r (being the surface of an n-ball of radius r) and dA is the area element (equivalently, the $(n - 1)$ dimensional volume element). The surface area of the sphere satisfies a proportionality equation similar to the one for the volume of a ball: If $A_{n-1}(r)$ is the surface area of an $(n - 1)$ sphere of radius r, then: $A_{n-1}(r) = r^{n-1} A_{n-1}(1)$.

Applying this to the above integral gives the expression:

$(2\pi)^{n/2} = \int_0^{\infty} \int_{S^{n-1}(r)} \exp\left(-\frac{1}{2} r^2\right) dA dr = A_{n-1}(1) \int_0^{\infty} \exp\left(-\frac{1}{2} r^2\right) r^{n-1} dr$
Substituting for $t = \frac{n}{2}$:

$$\int_0^{\infty} \exp\left(-\frac{1}{2} r^2\right) r^{n-1} dr = 2^{(n-2)/2} \int_0^{\infty} e^{-t} t^{(n-2)/2}\, dt$$

The integral on the right is the gamma function evaluated at $\frac{n}{2}$. Combining the two results shows that:

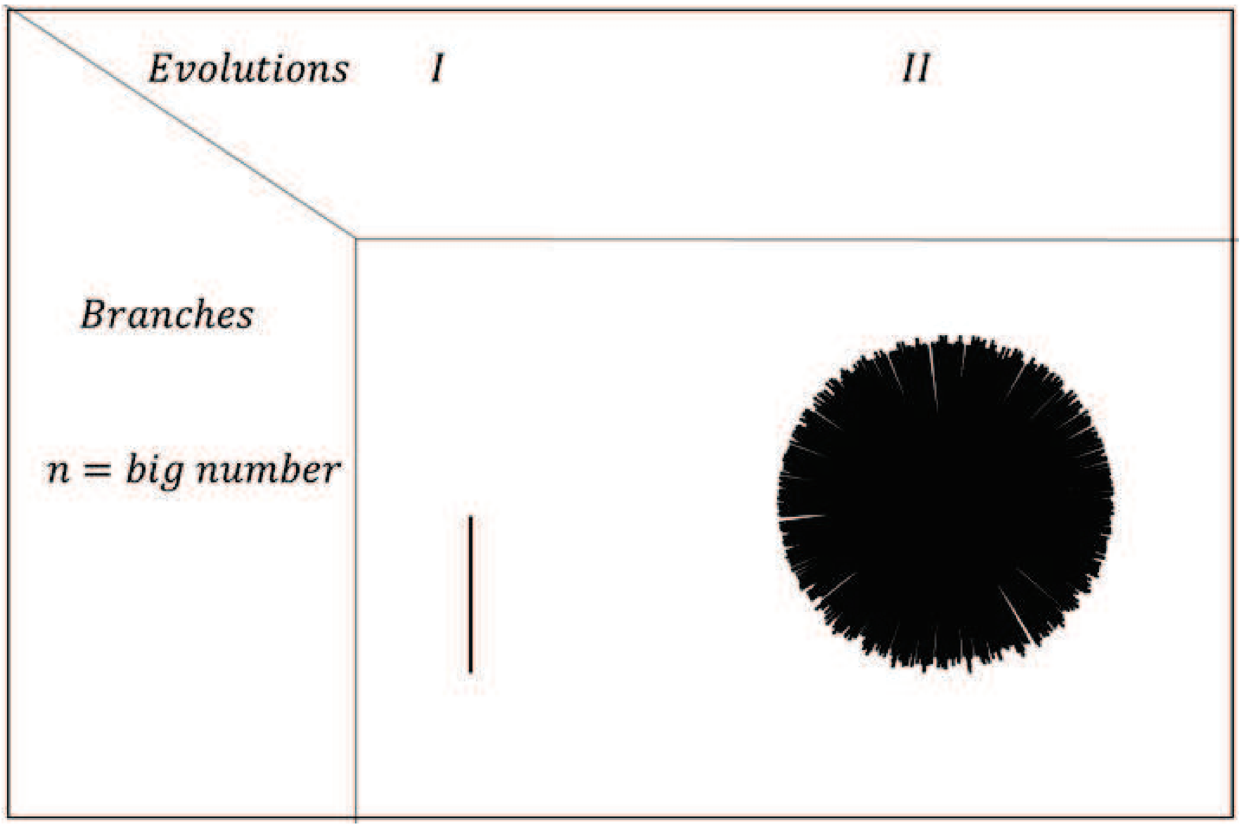
$A_{n-1}(1) = \frac{2\pi^{n/2}}{\Gamma(\frac{n}{2}+1)}$

To derive the volume of an n-ball of radius r from this formula, integrate the surface area of a sphere of radius r for $0 \le r \le R$ and apply the functional equation $z\Gamma(z) = \Gamma(z+1)$:

$$V_n(R) = \int_0^R \frac{2\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} r^{n-1} dr = \frac{2\pi^{n/2}}{n\Gamma(\frac{n}{2})} R^n = \frac{2\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} R^n$$

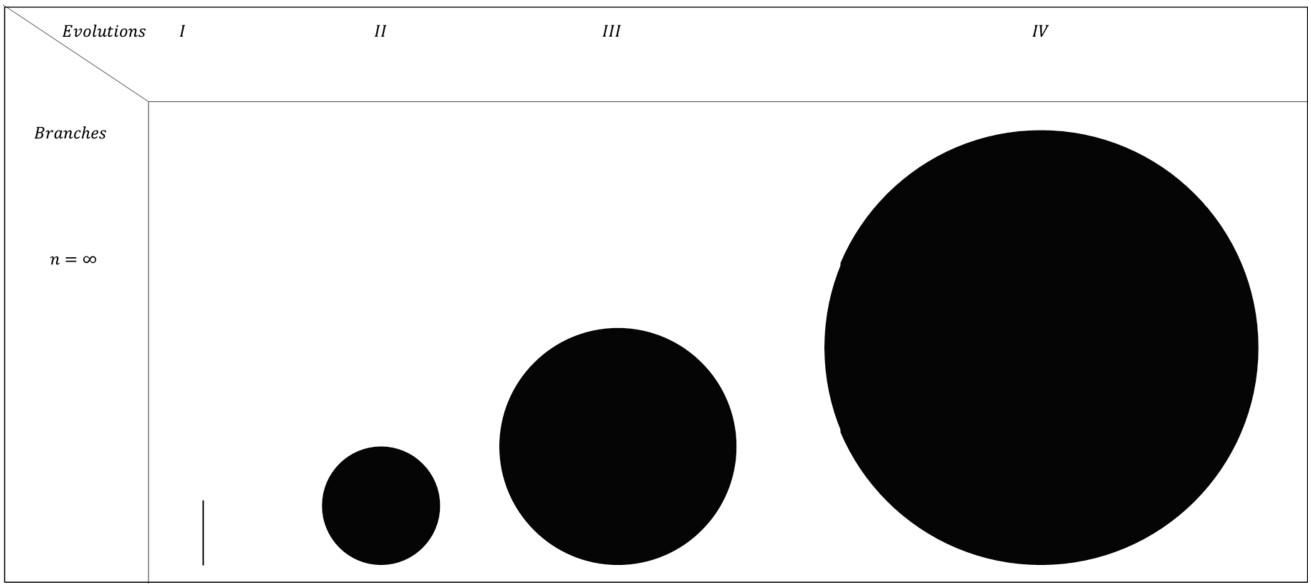
» Proof by Hayes, Brian. 2011. *An adventure in the N-th Dimension*, American Scientist, Volume 99, Number 6, Page 422, doi:10.1511/2011.93.442

Graphical examples of $ree(n - branches)$ $as\ n =$some arbitrarily big number



This example is of purely pedagogical character to show that w eget closer and closer to the shape of the cirlce, and thus the area of it as n goes against some arbitrarily big number before it goes against infinity.

Graphical examples of $Tree(n - branches)\ as\ n \to \infty$ from the I to the II evolution and to the III and IV



Here it is shown what happens if a tree with infinetely branches has sveral evolutions where on top of every single one of those branches are added infinetely many more branches. The area increases about one radius length, that the original «Tree-stem» had. The «Tree-stem» is the original line we start iterating on in the first evolution. This is easiest to show in 2D since the Author is limited in increasing the dimensions by his tools, but still is true for up to n-dimensions or evolutions as we call it here.

New Insights

Trees have been explored in both Fractal Geometry and in Graph Theory. I could not find in any forum what would happen if the number of branches would grow and even og against infinity. A new insight is that the trees become circular, spherical, hyperspherical and so forth, and that they approach a limit.

Discussion

An speculative but interesting thought is applying the rule of increasing amount of branches to causality. Applying this to causality and quantum mechanical interpretations such as ‘the many worlds theory’ we would gain the astounding insight that even if there are multiple universes created where each contains a single outcome of an event, they finally would go against the same boundary. No matter how many ‘worlds’ there would be they always would be bounded by the same upper bound.