Electron Size, Angular Momentum and g-Factor Re-Visited

by

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Although Quantum Mechanics (QM) defines the electron as a point particle, as evidenced by the multitude of diagrams representing electrons as spherical particles, the concept of a classical spherical electron would appear to be alive and well. The radial size of the spherical electron remains highly disputed, with various estimates in the range of 10^{-20} to 7 x 10^{-13}m. The CODATA radius of the electron, which represents "classical electron radius", is 2.82 $\times 10^{-15}$ m. Earlier estimates (M MacGregor 1992, 'The Enigmatic Electron', Klurer Academic) placed the radius of an electron in the range of 4 x 10^{-13} to 7 x 10^{-13} m. The 2015 Bowen and Mulkern estimate of 3.86 x 10^{-13} m is at the lower end of MacGregor's range and about 100 times larger than the classical CODATA estimate for an electron radius.

The QM definition of an electron as a point particle makes no logical sense, and results in the electron's momentum and electric charge being considered 'intrinsic' (i.e. of unknown cause), and suggests that the smaller an estimate of electron radius might be, the more acceptable it would be. The point-form definition is a necessary evil required to prevent unwanted singularities within the QM wave equations, and those wave equations model the electron's characteristics quite well mathematically, but do not represent a cohesive or realistic physical model for the electron.

But should an electron have spatial extent, as seems to be the case, does 'the smaller, the better' apply to estimates of electron size? For the classical spherical-electron model, the angular momentum S = v.m.R, which means that when the radius (R) gets too small, the rotation speed needed to generate the electron's known angular momentum would have to increase to ridiculous spin speeds. Using the classical CODATA electron radius estimate as an example, the tangential velocity at the electron's outer equatorial plane is:

v = S / (m_e*R) = 5.27 x 10^−35 / (9.1 × 10^−31 * 2.82 x 10−15) = 2 * 10^10 m/s

where R = 2.82 ×10^−15 m (CODATA classical spherical-electron radius),

S = 5.27 x 10-35 Js. is the QM estimate of intrinsic spin which is based upon the Bohr electron. It is half the reduced plank constant **(h-bar** or **ħ**) = ħ/2 = 0.5 x 1.054571817 × 10−34 = **5.27 x 10^-35** Js,

and m_e = the mass of an electron = **9.1 × 10^−31** kg.

Thus, for a spherical electron with the CODATA radius of 2.82 ×10^−15 m, the outer surface tangential velocity would be 2 * 10^10 m/s, which is about 100 times the speed of light (c = 3 x 10^8 m/sec). For a 10^−20 m radius (at the lower end of the electron radius range) that tangential speed would be more than a ten million (10^7) times the speed of light!! Because of this spin-related conundrum, conventional Science refrains from using a radial size estimate to calculate the electron's angular momentum (or spin), preferring to assert it to be 'intrinsic', and determining its value via the electron's precessional characteristics.

Experimentally, the **gyromagnetic ratio** for magnetic dipoles, inclusive of particles such as electrons, can be determined from their **precession**, called Larmor precession, which occurs when they are subjected to an externally applied magnetic field (**B** in teslas). When the particle's spin axis is oblique to the direction of the external field, the precession frequency (**f** in hertz) is proportional to the magnetic field strength, or specifically: f = ɣ/(2.π).B, which allows the gyromagnetic ratio ɣ to be accurately determined experimentally.

The gyromagnetic ratio (**ɣ**) is the ratio magnetic moment to angular momentum, calculated as **ɣ** = μ/S = q/(2m), where **μ** is the electron's magnetic moment and **S** its angular momentum. The CODATA estimate for μ is **9.285 x 10-24** J T^{-1} rounded, **S** = 5.27 x 10-35 Js (as discussed above), and q = charge of an electron **= 1.60218 x10^-19** C (coulomb):

Using μ and S, ɣ1 = μ/S = 9.285 x 10-24 / 5.27 x 10-35 = 17.62 x 10^10 (C/kg).

Using q and m_e, ɣ2 = q/(2.m_e) = 1.60218 x10^-19 / (2 x 9.1×10^−31) = 8.8 x 10^10 (C/kg).

Thus, ɣ as calculated from μ and S is approximately double that using q and m, leading to the introduction of the Landé **g**-factor (where **g** is a dimensionless constant) to correct the discrepancy as:

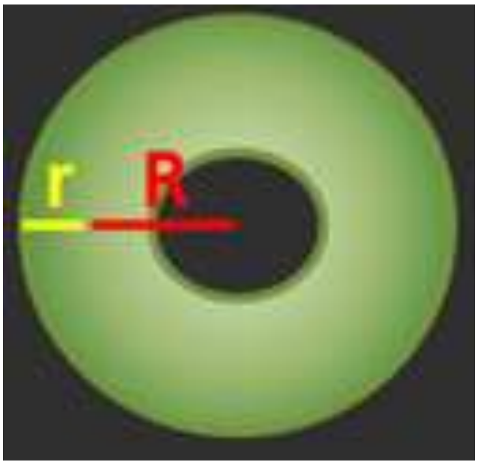
Gyromagnetic Ratio ɣ = μ / S = g.q/(2.m_e), or g = ɣ1/ɣ2 = 17.62 x 10^10 / 8.8 x 10^10 = 2.0023 (or = 2 rounded)

The realisation that the spin-related gyromagnetic ratio of an electron is 2, rather than being 1 as expected for the classical spherical-electron using Newtonian Physics, has been problematic, and long considered a quantum-related oddity. Richard Feynman, using **L** for an electron's angular momentum rather than **S**, stated that μ = q.**L**/(2.m_e) '*is true for orbital motion, but that's not the only magnetism that exists. The electron also has a spin rotation about its own axis (something* *like the earth rotating on its axis), and as a result of that spin it has both an* ***angular momentum*** *and a* ***magnetic moment****. But for reasons that are* ***purely quantum-mechanical****—there is no classical explanation—the ratio of μ to* ***L*** *for the electron spin is twice as large as it is for orbital motion of the spinning electron*' The Feynman Lectures on Physics, vol. 3, chapter 34, p34-6.

Considering the classical spherical electron model, should the maximum speed at its equatorial perimeter be the speed of light (c = 3 x 10^8 m/sec), then its radius R = S/(v. m_e) = 5.27 x 10-35 / (3 x 10^8 x 9.1 × 10^−31) = 1.93 x 10^-13m. Although such an electron model would generate the required angular momentum, it would also generate unwanted singularities in the QM wave equations because it cannot validly be represented as a point-particle, and would thus have to be discarded. However, should a torus-based (or toroidal) model of the electron be used, the story is very different.

A torus-based electron model has nothing that contributes to its mass or composition at its centre of mass, and can thus validly be represented as a point-form particle to satisfy the mathematical requirements of the QM wave equations without generating those unwanted singularities. Several toroidal models have been documented: the solenoidal toroidal model such as described by O Consa; the rotating charge loop model of Bowen and Mulkern, and the STEM electron model. We will concentrate on the latter.

The STEM electron model consists of a toroidal energy core that spins or flows, and contains the bulk of the electron's mass, plus an outer torus of electromagnetic field-energy that is chiral, and which determines whether it is an electron or a positron. It is a physical model that satisfies the QM wave equations and provides a geometry and size estimates for the electron, which allow the electron's angular momentum to be determined using Newtonian (or classical) Physics.



The torus-shaped energy core contains the bulk of the mass of the electron,

and the torus's large radius **R** = 0.24 pm = 0.24 x 10^-12 m = 2.4 x 10^-13 m;

and its small radius **r** = 1.6 x 10^-13 m, as represented in the figure right.

The outer equatorial radius = R + r = 4 x 10^{-13} m.

Assuming q = charge of an electron = 1.60218 x 10^-19 coulomb,

m_e = the mass of an electron = 9.1 × 10^−31 kg,

and v = central spin/flow speed of energy core at R = 1.8 x 10^8 m/sec,

Then Angular Momentum S = I.ω where I = moment of inertia = m_e . (3/4 . r^2 + R^2) for a torus,

and ω = angular velocity = v/R radians/sec

= v . m_e . (3/4 . r^2 + R^2) / R

= 1.8 x 10^8 x 9.1 × 10^−31 x (0.75 x (1.6 x 10^-13)^2 + (2.4 x 10^-13)^2) / 2.4 x 10^-13

= 5.24 x 10^-35 Js, which is close to the QM estimate 5.27 x 10-35 Js based upon ħ/2.

Using the CODATA estimate for μ = 9.285 x 10-24 J T^{-1} and STEM S = 5.24 x 10^-35 Js as calculated above, then:

ɣ1 = μ/S = = 9.285 x 10-2 / 5.24 x 10^-35 = 1.771 x 10^11, and

ɣ2 = q/(2.m_e) = 8.8 x 10^10 (C/kg) as calculated earlier.

Thus g = ɣ1/ɣ2 = 1.771 x 10^11 / 8.8 x 10^10 = 2.012

The angular momentum determined from the geometry and size statistics proposed for the STEM electron model is very close to QM's estimate of 'intrinsic' spin, as is the associated electron g-factor, which is 2.012 compared to QM's estimate of 2.00232 (note that should the speed v at radius R be increased by a mere 0.5% from 1.8 x 10^8 to 1.81 x 10^8, then the estimates for S and g would be identical to the QM estimates). A toroidal electron model, such as that promoted by STEM, satisfies the QM wave equations and produces a classical (or Newtonian Physics) estimate of angular momentum (and g-factor) that corresponds to QM's 'intrinsic' spin estimate.

Richard Feynman can now rest in peace assured that, when an appropriate physical model of the electron is used, there is a '*classical explanation*' for why the electron g-factor is 2. Also, having an appropriate physical electron model opens many new doors to modelling and explaining many other aspects of nuclear Physics and Chemistry.