

# Quantum wave entropy

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## Abstract

In quantum mechanics, particles have a new type of probabilistic property, which is quantum wave probability. Corresponding to this new probability, the particle has the property of quantum wave entropy, and it has the property of quantum wave temperature. Based on the quantum wave entropy, the Unruh formula, the black hole entropy formula, and the Verlinde entropy gravitational formula can be easily derived. It proves that these three formulas are not independent of each other, but are related to each other. These three formulas have the same physical origin, which is quantum wave entropy. The quantum wave temperature has similar properties to the Unruh temperature. The quantum wave temperature is not only directly proportional to acceleration, but also inversely proportional to velocity. The Unruh temperature is just a light speed case of quantum wave temperatures. Compared to the Unruh temperature, the quantum wave temperature is significantly larger and easier to test experimentally. All experiments to test the Unruh effect can be used to test the theory of quantum wave entropy. We can use experiments to test whether the theory is true. The quantum wave entropy can solve the contradiction between the time reversal symmetry of the dynamical equation and the law of entropy increase. The action corresponds to quantum wave entropy. The least action principle corresponds to the stationary quantum wave entropy principle. The quantum wave entropy creates a bridge between the dynamical equations and thermodynamics.

## Keywords

Particle wave, quantum wave probability, intrinsic degrees of freedom, quantum wave entropy, quantum wave temperature, Unruh effect, black hole entropy, Verlinde entropy gravity, Planck quantum gravity theory, time reversal symmetry, entropy increase, time arrow, least action principle, stationary quantum wave entropy principle.

## 1. Introduction

We all know that in quantum mechanics, there is a correlation between particle waves and the probability of distribution of particles. In quantum mechanics, particles do not exist fixedly at a certain point in space, particles exist in space in the form of probability waves. The amplitude of a probability wave represents the probability of a particle appearing in space. For a monochromatic plane wave, the probability of particles appearing at every point in space is same. This is the current understanding of particle waves [1]. But, does this understanding already represent the whole probabilistic property of particle waves? Is there an unknown probabilistic property of particle waves?

On the other hand, the relationship between gravity and entropy has been ambiguous since Beckenstein and Hawking proposed the concept of black hole entropy [2][3]. Next, Unruh proposed the Unruh effect [4]. Later, Verlinde proposed the entropy gravity theory [5]. What

is the relationship between black hole entropy, the Unruh effect, and the Verlinde hypothesis formula, we have not yet been able to give a clear answer. How can the problem of quantum gravity be solved? None of these questions have a definitive answer. In the field of quantum gravity, any new idea is worth thinking about and discussing.

In the author's previous paper on quantum gravity, the author proposed a new concept that the wavelength of particle waves represents a new kind of distribution probability [6]. Although the object of this concept is the wavelength of particles in the fourth spatial dimension, the author boldly proposes that the concept is a general concept. This new probabilistic property exists in particle waves in 3-dimensional space also. This paper is an in-depth explanation of this concept in 3-dimensional space. In this paper, the authors will elaborate entirely new concepts and perspectives that lead to completely unexpected results.

## 2. Quantum wave entropy and Unruh effect

The wavelength of the particle wave represents a new kind of distribution probability. In a fixed-length spatial range, for a free particle (monochromatic plane wave), the smaller the wavelength, the greater the probability of particle excitation; Conversely, the larger the wavelength, the smaller the probability of particle excitation. That is, for this new probability, the probability density is inversely proportional to the wavelength. This is a new type of probability. For this new type of probability, its probability density is expressed by the following formula (1.1).

$$dp = \alpha \frac{dr}{\lambda} \quad (1.1)$$

The  $\alpha$  is a proportionality constant. The  $\lambda$  is the wavelength of the particle. The  $dp$  actually represents the probability density within the length of  $dr$ . The value of the probability constant  $\alpha$  needs to be measured experimentally.

Since the distance  $r$  in full space can be seen as infinity, the probability of a particle in full space is infinite for this new type of probability. That is, the sum of the probabilities of the excitation of particles in full space is infinite. This is an obvious conclusion.

This is a new type of probabilistic property of particles, which is completely different from the probabilistic amplitude properties of existing particle waves. Unless otherwise specified, the probabilities described later are all those of this new type. Readers need to pay attention to the distinction.

Because the probability density of this new type is determined by wavelength, the authors named this new type of probability by quantum wave probability.

With this new probabilistic object, we can discover very interesting and surprising results. Using this new probabilistic object, we can simply derive the Unruh formula, we can simply derive the Verlinde formula, and we can simply derive the black hole entropy formula.

In quantum mechanics, a particle has intrinsic properties, such as the spin of a particle. For example, the spin of a particle has two possibilities,  $+1/2$  and  $-1/2$ , that is, it has two degrees of freedom. This degree of freedom is the intrinsic degree of freedom of the particle. This is a special type of freedom in quantum mechanics. There must be other unknown intrinsic degrees of freedom in particles. We uniformly use  $n$  to identify the sum of all intrinsic degrees of freedom of a particle. The degrees of freedom and the number of microstates are correlated. The intrinsic degrees of freedom of a particle also represent a new number of

microscopic states of the particle. Each particle excited in space has intrinsic degrees of freedom  $n$ , and the probability of excitation of the particle in the range of the  $dr$  length is equation (1.1). So, the sum of the number of microstates of the particles in the length of  $L$  is equation (1.2).

$$N = n \int_0^L dp \quad (1.2)$$

Since having the number of microstates, entropy can be defined.

$$S = \kappa_B \ln N = \kappa_B \ln \left( n \int_0^L dp \right) = \int_0^L dp \times \kappa_B \times \ln(n) = D \kappa_B \int_0^L dp \quad (1.3)$$

where  $D = \ln(n)$ , which is related to the intrinsic degrees of freedom of the particle.  $\kappa_B$  is the Boltzmann constant. For the same type particles,  $D$  is a constant value. Different types particles may have different  $D$  values. For example, if we consider only the spin of the particle,  $n=2$ ,  $D=\ln 2$ .

This is a new type of entropy that corresponds to the new type probability. The new type probability is the quantum wave probability of the particle, so the author named it by quantum wave entropy.

The basis of quantum wave entropy is the intrinsic degree of freedom  $D$  of the particle. In classical physics, a particle does not have intrinsic degrees of freedom,  $n=1$ ,  $D=0$ . So in classical physics, particles does not have this entropy. Only in quantum mechanics, the particle have intrinsic degrees of freedom, and  $D$  is not equal to zero, so the particle has quantum wave entropy. So, this entropy only exists in quantum mechanics, which is a special property of quantum mechanics.

For monochromatic plane waves, the integral range is infinity, so the quantum wave entropy is infinity. Monochromatic plane waves correspond to free particles. Therefore, the quantum wave entropy of a free particle is infinity.

Physically, the entropy of infinity has no physical significance. However, the differentiation of entropy has a physical significance. By equation (1.3), we can get the differential form of entropy as equation (1.4).

$$dS = D \kappa_B dp = D \kappa_B \alpha \frac{dr}{\lambda} \quad (1.4)$$

To particles, where  $D$ 、 $\kappa_B$ 、 $\alpha$  are constants. In the previous paper [6], the authors derived  $\pi \alpha = 1$ . This is just a rough estimate. The specific value needs to be determined through experiments. To simplify the explanation, we defined a new constant  $\kappa_p$ . There is only one constant difference between this new constant and the Boltzmann constant  $\kappa_B$ . Readers need to pay attention to the distinction.

$$\kappa_p = D \alpha \kappa_B$$

So the equation (1.4) is simplified to equation (1.5).

$$dS = \kappa_p \frac{dr}{\lambda} \quad (1.5)$$

Note that the  $dr$  here is derived from equation (1.1) and is only derived from the quantum wave probability within the length of  $dr$ , which is only a differential form of the distance length  $r$ , and has no correlation with the increase or decrease of the length  $r$ . Regardless of whether the length  $r$  increases or decreases,  $dr$  is a positive value, and there is no negative  $dr$  value. So, the  $dS$  here is always positive. Please pay attention to the distinction.

The  $dS$  here is actually an entropy density, which represents the magnitude of the quantum wave entropy that the particle has over the length of the  $dr$ .  $dS$  here does not actually mean a value of  $S$  increasing or decreasing. However, if the length range changes, bringing  $dr$  and  $dS$ , the actual  $dS$  can also be regarded as the value of the change in  $S$ . Since quantum wave entropy is represented by equation (1.5) and is related to  $dr$ ,  $dS$  actually has a double meaning, which is both the value of  $S$  change and the entropy density. This is the peculiarity of quantum wave entropy that differs from existing thermodynamic entropy. Readers need to pay special attention to this peculiarity.

In addition,  $dS$  only involves the value of wavelength  $\lambda$ , and does not involve the value of wavelength change  $d\lambda$ . This is because equation (1.5) is derived from equation (1.1). Equation (1.1) only deals with the wavelength  $\lambda$  and has no correlation with  $d\lambda$ . Therefore,  $dS$  is only related to the wavelength  $\lambda$  and not to  $d\lambda$ . It can be understood in this way. In quantum mechanics, if the wavelength of a particle changes, the particle is no longer the original particle, but a new particle, so the entropy of a particle must not be correlated with  $d\lambda$ .

We apply quantum wave entropy to the below equation of thermodynamics [7].

$$T = \frac{dE}{dS}$$

So we get formula (1.6).

$$T = \frac{dE}{dS} = \frac{\lambda}{\kappa_p} \frac{dE}{dr} \quad (1.6)$$

This is the temperature corresponding to the quantum wave entropy. This is a new type of temperature. We named this new type of temperature by quantum wave temperature.

The following formula exists.

$$\frac{dE}{dr} = F$$

In the case of an approximation at low speeds.

$$F = ma$$

$$\lambda = \frac{h}{mV}$$

Take above three formulas into (1.6), we can get formula (1.7).

$$T = \frac{h}{\kappa_p V} a \quad (1.7)$$

If we do the same for photons, assume that the above equation is also valid for photons. Considering where  $m$  is the equivalent mass of the photon, and the photon energy  $E = mC^2$ , so we get the equation (1.8).

$$T = \frac{h}{\kappa_p C} a \quad (1.8)$$

Equation (1.8) is the Unruh formula, with only constant differences. We found that based on the equation of quantum wave entropy (1.5), the Unruh formula can be easily derived. This is a very surprising result.

It can be found that the quantum wave entropy defined in equation (1.5) has important physical significance, and it is not a concept made up randomly.

The Unruh formula only works in the case of photons. However, we found that there is another, more generally-applicable formula (1.7). Equation (1.7) is valid for all particles. The

Unruh formula is just a special case of Equation (1.7).

Similar to the Unruh effect, for an accelerated observer, the observer will see the same background acceleration for all other particles. Therefore, all other particles have a temperature as defined by equation (1.7). So, this temperature can be seen as a background radiation temperature. But the background radiation temperature here is the quantum wave temperature of all other particles, not the vacuum radiation temperature. Compared with Unruh temperature, the two have different physical origins. The background radiation temperature caused by quantum wave entropy is not directly related to vacuum, and is not a property of vacuum. This is very different from the Unruh temperature. In addition, other particles do not move at the same speed, so the quantum wave temperature exhibited by different particles is not the same. However, looking at the statistical average of all particles, equation (1.7) can be thought of as an average background radiation temperature.

Because the velocity in the Unruh formula is the speed of light  $C$ , the temperature is very small, so it is very difficult to measure experimentally. Conversely, the velocity in equation (1.7) may be small, the temperature will be large, and it will be easier to measure experimentally. For example, if the velocity of the experimental instrument is in the range of 1-10 m/s and the acceleration is about 10 m/ss, the quantum wave temperature measured by the instrument will be about  $10^{-10}$  degrees according to equation (1.7). Under the same conditions, according to equation (1.8), the Unruh temperature is about  $10^{-18}$  degrees. The quantum fluctuation temperature is about  $10^8$  times to the Unruh temperature. Obviously, quantum wave temperatures are easier to test experimentally. Therefore, the experiment used to test the Unruh temperature can also be used to test the quantum wave temperature, and the effect of the quantum wave temperature is more significant.

Therefore, we need to take a closer look at the experiments that detect the Unruh effect. Experimental data may be more consistent with equation (1.7). If the accelerating instrument only detects the phenomenon of background radiation temperature, it does not prove the Unruh effect. The measured temperature values must also conform to the Unruh formula in order to prove the Unruh effect. If the accelerating instrument only detects the temperature phenomenon of background radiation, but the measured temperature value is significantly larger than the value calculated by the Unruh effect, and the temperature value is more consistent with the result of equation (1.7), then the experiment is not proving the Unruh effect. Instead, the experiment is proving Equation (1.7). Therefore, the concept of quantum wave entropy brings new thinking and new physical meaning to the experiment of the Unruh effect.

If the instrument is accelerating from rest with the same acceleration, in this case,  $V=at$ , take it into formula (1.7), get the following result.

$$T = \frac{h}{\kappa_p t} \quad (1.9)$$

The quantum fluctuation temperature decreases with time. There is no correlation between this formula and acceleration, and all cases of acceleration follow this formula. Experiments can also be designed according to equation (1.9) to test whether the formula is consistent with the actual situation and whether the quantum wave entropy is valid. However, it is important to note here that the temperature is not infinity when  $t=0$ . The formula  $V=at$  holds only in the case of macroscopic classical physics. If the time interval  $t$  is very small, it is already

a microscopic state, and according to quantum mechanics, the formula  $V=at$  is no longer true. It's just that in the case of approximation of macroscopic classical physics, the above formula is valid.

According to equation (1.7), we can get a result. If the acceleration remains constant, the speed of the experimental instrument will continue to increase as the acceleration continues, and the value of the background radiation temperature measured by the instrument will continue to decrease. Conversely, if the acceleration remains constant, the temperature predicted by the Unruh effect is a constant value. Based on this difference, we can also use experiments to check whether the results are consistent with equation (1.7) or the Unruh effect.

Based on equation (1.5), we can also find that the Verlinde entropy gravitational formula can be easily derived. From formula (1.5), we can get formula (1.10).

$$\frac{dS}{dr} = \frac{\kappa_p}{\lambda} = \frac{\kappa_p}{\frac{h}{mV}} = \kappa_p \frac{mV}{h} \quad (1.10)$$

Similarly, if we generalize the  $V$  in equation (1.10) to the speed of light  $C$ , we get equation (1.11).

$$\frac{dS}{dr} = \frac{\kappa_p m C}{h} \quad (1.11)$$

Equation (1.11) differs only from Verlinde's entropy gravitational equation by constants. So, we can see that the Verlinde entropy gravitational formula is also only a special case of equation (1.10). More generally, it is the formula (1.10). The Verlinde entropy gravitational formula is also only a derivation of quantum wave entropy. The entropy in Verlinde's entropy gravitational formula is not a conventional thermodynamic entropy, but a quantum wave entropy.

From the above derivation process, we can also see that there is a correlation between the Verlinde entropy gravitational formula and the Unruh formula. The two formulas do not stand on their own. Both formulas are actually the results of quantum wave entropy. This is a completely new realization. We need to be cautious about using both of these formulas. If we use both formulas, we may be using the same concept repeatedly, thus forming a logical loop that makes the derivation deceptive.

The above derivation process does not involve any specific type of force, and is a general derivation result that is valid for all forces.

### 3. Derive black hole entropy

Based on the quantum fluctuation entropy, we can also very easily derive the black hole entropy formula. We apply equation (1.7) to the case of gravitational force in a 3-dimensional space (4-dimensional space-time). Gravitational acceleration is

$$a = \frac{dE}{dr} = \frac{GM}{r^2}$$

Take the result into formula (1.7), get formula (2.1).

$$T = \frac{h}{\kappa_p V} a = \frac{h}{\kappa_p V} \frac{GM}{r^2} \quad (2.1)$$

This is the quantum wave temperature of the particle in the gravitational field generated by the gravitational source  $M$ . Equation (2.1) is a common situation.

We generalize the equation (2.1) to the speed of light  $C$ ,  $V=C$ , and we get the following

formula.

$$T = \frac{h}{\kappa_p C} \frac{GM}{r^2}$$

Then set  $r$  to the radius of the black hole  $R$  of the gravitational source  $M$ . In Planck gravity theory, the radius of a black hole is expressed by the following formula [8]. In general relativity, the radius of a black hole is twice that of this formula [9], with only a constant difference. We use the radius of a black hole in Planck gravity theory to calculate.

$$r = R = \frac{GM}{C^2}$$

So can get.

$$T = \frac{h}{\kappa_p C} \frac{GM}{r^2} = \frac{hC}{\kappa_p R} \quad (2.2)$$

Equation (2.2) differs only from the black hole temperature formula by constants. This is yet another surprising result. By applying the quantum wave entropy to a particle located at the radius of a black hole, we can deduce the quantum wave temperature of the particle, and we get the result of the temperature of the black hole.

Directly from equation (1.8), assuming that the gravitational acceleration at the radius of the black hole still satisfies  $a = \frac{GM}{r^2}$ , the same temperature formula can be obtained.

$$T = \frac{h}{\kappa_p C} a = \frac{h}{\kappa_p C} \frac{GM}{r^2} = \frac{hC}{\kappa_p R}$$

If we assume that the temperature of the particles at the radius of the black hole is the same as the temperature of the black hole, it is a temperature equality condition. Then the temperature of the black hole should also be equal to the result of equation (2.2). If we then assume that the energy of the black hole satisfies.

$$E = MC^2$$

We can get the black hole entropy formula (2.3). This formula differs from the Hawking-Beckenstein black hole entropy formula only by a constant.

$$S = \frac{E}{T} = \frac{MC^2}{\frac{hC}{\kappa_p R}} = \frac{\kappa_p R^2}{L_p^2} \quad (2.3)$$

This is a very obvious result. Now that we have the black hole temperature formula, we must be able to get the black hole entropy formula.

Let's look at equation (2.1) again. If we assume that the velocity of a particle is obtained entirely by gravity, it is the particle that is initially at rest that begins to accelerate under the gravitational force. In the case of a low-speed approximation, the velocity  $V$  satisfies the following equation.

$$V = \sqrt{\frac{2GM}{r}}$$

Take the formula into formula (2.1), then can get formula (2.4).

$$T = \frac{hV}{2\kappa_p r} \quad (2.4)$$

Equation (2.4) is the quantum wave temperature formula for the motion of a particle in a gravitational field at a low velocity approximation. It has a similar form to the black hole temperature formula. Comparing equations (2.4) and (2.2), it can be seen that the black hole

temperature is the highest temperature.

Why is it possible to derive the same form of black hole temperature formula and black hole entropy formula based on quantum wave entropy? What is the real physical meaning of black hole entropy? We need to re-examine the physical meaning of black hole entropy and black hole temperature. What is the physical meaning of black hole radiation? Quantum wave entropy is the result of a new type of quantum probability. This new type of probability is the probability represented by the wavelength of the quantum wave. Any particle with quantum wave properties has this quantum wave entropy, which is not directly related to a black hole. Black hole entropy, on the other hand, is just a special case of this quantum wave entropy. Can we understand the entropy of black holes in this way? Is the new understanding correct? We need to think new about the physical meaning of black hole radiation. This is a subject that needs to be studied in depth.

Another method can be used to derive the entropy of a black hole. According to equation (1.5), we assume that the wavelength of a particle is equal to its Compton wavelength, and then assume that the value of  $dr$  is the diameter of the particle's black hole.

$$\lambda = \frac{h}{MC}$$

$$dr = 2R = \frac{2GM}{c^2}$$

Take the two formulas into (1.5), then can get.

$$dS = \kappa_p \frac{dr}{\lambda} = \kappa_p \frac{\frac{2GM}{c^2}}{\frac{h}{MC}} = 2\kappa_p \frac{R^2}{L_p^2} \quad (2.5)$$

There is only a constant difference between this equation and (2.3). In this derivation, the particle wavelength is easily understood. If a black hole is treated as a single particle, then the particle wavelength of the black hole is equal to the Compton wavelength of the black hole. For a black hole, the quantum wave entropy within the length of its diameter is just the black hole entropy. How to understand this result needs to be further studied.

As can be seen from equation (2.5), the black hole entropy formula is just a differential form of the quantum wave entropy of a black hole. The black hole entropy formula (2.5) does not represent the full entropy of a black hole. At the full-space scale, the total quantum wave entropy of the black hole is the integral of  $dS$  in the full-space, so it is infinity. Therefore, the derivation process of equation (2.5) has prompted us to rethink the physical meaning of black hole entropy. This is a topic that needs to be studied in depth.

From the above derivation process, we can find a result. Using the concept of quantum wave entropy to derive does not require the assumption that a black hole has an internal structure, nor does it need to assume that the black hole event horizon emits radiation. Treating a black hole as a particle, we can derive the black hole entropy and the black hole temperature. So, we need to revisit the concepts of black hole entropy and black hole temperature. Does black hole entropy really mean that black holes have an internal structure?

From the above derivation process, we can see that the black hole entropy formula, the Unruh formula, and the Verlinde entropy gravitational formula are actually related to each other, and they are not independent of each other, and all three can be derived based on quantum wave entropy. Therefore, the Verlinde entropy gravitational hypothesis is not correct. In fact, in the process of deriving the black hole entropy formula, the radius or diameter of



the black hole must be introduced in order to derive the result of the black hole entropy formula. And the radius of the black hole is actually already the result of gravity. Without gravity, there is no black hole radius. Therefore, the derivation of gravity from the entropy of a black hole is a logical cycle, and such a derivation process cannot be established.

Let's look at the problem of black hole information loss. A particle falls into a black hole B, forming a larger black hole C, and there seems to be a loss of information. However, from the perspective of quantum wave entropy, there is no problem of information loss.

Based on equation (1.5), for particle A falling into a black hole, its velocity is approximately equal to the speed of light. We identify its equivalent mass by  $M_1$ .

$$E = M_1 C^2$$

$$\lambda = \frac{h}{M_1 C}$$

Take into formula (1.5), so get.

$$dS_1 = \kappa_p \frac{dr}{\lambda} = \frac{\kappa_p C dr M_1}{h}$$

To the black hole, there exist formula.

$$\lambda = \frac{h}{M_2 C}$$

So get.

$$dS_2 = \kappa_p \frac{dr}{\lambda} = \frac{\kappa_p C dr M_2}{h}$$

So get.

$$dS_1 + dS_2 = \frac{\kappa_p C dr (M_1 + M_2)}{h} = \frac{\kappa_p C dr M}{h} = dS$$

Therefore, if the particle falls into the black hole, the total quantum wave entropy does not decrease. So there is no problem of information loss.

Of course, the above is only a rough derivation process, and this is only a rough proof. For rigorous proof, further in-depth research is needed.

We can also rethink the argument about whether entropy increases or decreases under gravitational attraction. The particles automatically move closer to the center of gravity in the gravitational field, and there seems to be a decrease in entropy. However, if the quantum wave entropy of the particle is considered, the speed of the particle's motion increases, its quantum wavelength decreases, and according to equation (1.5), the quantum wave entropy  $ds$  of the particle will increase. So, the entropy of the particle does not decrease. There is no problem of entropy reduction under gravitational attraction. Therefore, we find that by introducing the concept of quantum wave entropy, many of the original fuzzy and confused problems become clear, and the law of entropy increase in thermodynamics can be clearly proved.

#### 4. Time reversal symmetry and time arrow

The basic dynamical equations have the time reversal symmetry. For example, Newton equation of mechanics and Schrödinger equation both have time reversal symmetry. However, in the natural state where there is no external work, a physical system always evolves in the direction of entropy increase, which is called the arrow of time. There seems to be a

contradiction between the two. This has always been a big problem in physics and has not been answered definitively.

From the perspective of quantum wave entropy, there is a clear answer. For the time inversion operation, the momentum  $P$  is required to be converted to  $-P$ . Only when this condition is met can the time reversal be established.

$$P(-t) = -P \quad (3.1)$$

For example, if the momentum is increasing in real physics, the momentum is decreasing during time reversal.

But there exists formula (1.5) in quantum mechanics. And the wavelength satisfies the formula (3.2).

$$\lambda = \frac{h}{P} \quad (3.2)$$

From (1.5) and (3.2), so we can get formula (3.3).

$$P = \frac{h}{\kappa_p} \frac{dS}{dr} \quad (3.3)$$

We assume that the momentum changes from  $P_1$  to  $P_2$  during the time inversion, and the momentum is decreasing during time reversal, so  $P_1 > P_2$ . The quantum wave entropy corresponding to momentum are  $S_1$  and  $S_2$ . Obviously, during this time reversal, the quantum wave entropy is decreasing, so  $S_1 > S_2$ . That is  $\Delta S < 0$  during the time inversion. But this necessarily violates the law of entropy increase. Therefore, the probability of this time reversal process occurring is less than that of the real physical process. Including quantum wave entropy, we find that the probability of the forward evolution process is not equal to the probability of the inversion process, and the dynamical equation does not have true time-inversion symmetry.

Therefore, the quantum wave entropy is the key to solving this problem. Why there is a contradiction between the time reversal symmetry and the law of increasing entropy. That's because there is a missing logical link in the whole reasoning process. The quantum wave entropy is just the missing logical link. When we consider this new concept of the quantum wave entropy, this contradiction disappears. The quantum wave entropy adds an additional limit to the evolution of the dynamical equations. Although the dynamical equation has the time reversal symmetry, but the probability of the direction of the quantum wave entropy increase is greater, so a time arrow is brought to the dynamical equation.

Here we can discover an important value of this new concept of quantum wave entropy. In formula (3.3), the momentum is associated with dynamical equations, and the entropy is associated with thermodynamics. The quantum wave entropy creates a bridge between the dynamical equations and thermodynamics. This aspect deserves in-depth study.

## 5. Action and quantum wave entropy

In classical mechanics, the formula for the action is (4.1).

$$I = \int L dt = \int (T - \phi) dt \quad (4.1)$$

$$T = \frac{p^2}{2m}$$

$$I = \int \left( \frac{p^2}{2m} - \phi \right) dt = \int \frac{p^2}{2m} dt - \int \phi dt \quad (4.2),$$

To the first item, there is a derivation process below.

$$\int \frac{P^2}{2m} dt = \int \frac{P}{2m} P dt = \int \frac{P}{2m} m v dt = \int \frac{P}{2} dr$$

Take formula (3.3) into the formula above, so we get.

$$\int \frac{P^2}{2m} dt = \int \frac{1}{2} \frac{h}{\kappa_p} \frac{dS}{dr} dr = \frac{h}{2\kappa_p} \int dS \quad (4.3)$$

So we can found an amazing result. The first item is just an integral about quantum wave entropy.

To the second item, there exists formula (4.4).

$$\frac{dP}{dt} = -\frac{d\phi}{dr} \quad (4.4)$$

$$\frac{dr dP}{dt} = v dP = \frac{P}{m} dP = \frac{d(P^2)}{2m} = -d\phi$$

$$\frac{P^2}{2m} = -\phi \quad (4.5)$$

It is just the first item, so the second item is formula (4.6).

$$-\int \phi dt = \int \frac{P^2}{2m} dt = \frac{h}{2\kappa_p} \int dS \quad (4.6)$$

So we get

$$I = \frac{h}{2\kappa_p} \int dS + \frac{h}{2\kappa_p} \int dS = \frac{h}{\kappa_p} \int dS \quad (4.7)$$

So we can found an amazing result. The action corresponds to quantum wave entropy. The action of each motion curve is actually the quantum wave entropy of that curve. There is an unknown connection between quantum wave entropy and dynamical equations. This is a very surprising result and well worth further study.

The least action principle is

$$\delta I = 0 \quad (4.8)$$

So correspondingly exist stationary quantum wave entropy principle.

$$\delta S = 0 \quad (4.9)$$

So the actual physical process is the fastest way to increase or decrease quantum wave entropy. If the momentum increases, it is the maximum entropy principle. If the momentum decreases, it is the minimum entropy principle.

Conversely, we can derive the least action principle from the entropy increase principle. Quantum wave entropy comes from the probability of a particle wave. The path of the actual physical process is the path of probability maximization. The path that maximizes probability is the path where quantum wave entropy grows the fastest. The path with the fastest growth in quantum wave entropy is the path with the least action. If the quantum wave entropy decreases, it is the opposite path. The path of the actual physical process is the path where the quantum wave entropy decreases the fastest. Again, we find the probabilistic origin of the least action principle in quantum mechanics. The least action principle is the result of the evolution of probability.

Again, we find that although the dynamical equation of the action has time-inversion symmetry, the actual physical process does not have time-inversion symmetry because the action corresponds to quantum wave entropy. Quantum wave entropy is also a type of

entropy, which is still limited by the entropy increase principle. If there is no external work done, quantum wave entropy also follows  $\Delta S \geq 0$ . Such it brings the time arrow. The dynamical equation is not only the dynamical equation, but also contains the property of quantum wave entropy. By including the properties of quantum wave entropy, the dynamical equations no longer have time-inverse symmetry. If the time reversal process is to be realized, work must be done on it. This aspect deserves in-depth study.

## 6. Conclusion

In quantum mechanics, there is a new type of probability. It is the quantum wave probability of the particle, which is expressed by equation (1.1). This new probability corresponds to a new type of entropy, the quantum wave entropy of the particle, which is represented by equation (1.5). Based on the concept of quantum wave entropy, it is possible to deduce the result that particles that are moving at an accelerated rate have a new temperature property. It is the quantum wave temperature, which is represented by the equation (1.7). The quantum wave temperature is inversely proportional to the velocity of the particle and directly proportional to the acceleration of the particle. The quantum wave temperature at the speed of light is just the Unruh temperature. The Unruh effect can be derived from the concept of quantum wave entropy. At very small particle velocities, the quantum wave temperature is much larger than the Unruh temperature. Therefore, the quantum wave temperature can be more easily tested experimentally. Therefore, it is easier to judge the correctness of the concept of quantum wave entropy through experiments. Based on the quantum wave entropy, the Verlinde entropy gravitational formula and the black hole entropy formula can be simply derived. There is actually a correlation between the black hole entropy formula, the Unruh formula, and the Verlinde entropy gravitational formula. The three are not independent of each other, but all three actually originate from quantum wave entropy. Therefore, the theory of entropy gravitational force does not hold. The quantum wave entropy can solve the contradiction between the time reversal symmetry of the dynamical equation and the law of entropy increase. The quantum wave entropy creates a bridge between the dynamical equations and thermodynamics. The action corresponds to quantum wave entropy. The least action principle corresponds to the stationary quantum wave entropy principle. The actual physical process is the fastest way to increase or decrease quantum wave entropy. Quantum wave entropy is a new concept that needs to be studied in depth.

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