Proof of the Riemann Hypothesis (Method using absolute values of image part of Xi function with some help from computers)

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Abstract

If s=a+bi, proving that the absolute value of the imaginary part of $\xi(s)=\zeta(s)\Gamma(s/2)\pi^{(-s/2)}$ is not 0 for $0<a<0.5$, $0.5<a<1$, $-\infty<b<\infty$ is equivalent to proving that the Riemann hypothesis is true. When searching for $0<b<x$ (x is near 0) for $0<a<0.5$, $0.5<a<1$ using a computer, it was confirmed that the absolute value of the integral term of $|Im(\xi(s))|$ is smaller than the absolute value of the additional term of $|Im(\xi(s))|$ and that the polarities of the two are the same. It was shown that even when $b\to\infty$, the absolute value of the integral term of $|Im(\xi(s))|$ remains smaller than the absolute value of the additional term of $|Im(\xi(s))|$ and converges. The polarities of the two are the same. As a result, the Riemann hypothesis was proven correct.

Contents

The analytically continued zeta function $\zeta(s)$ has poles at s=0 and s=1, trivial zeros at negative even points, and non-trivial zeros along the s=1/2 line (the Riemann hypothesis is that there are no non-trivial zeros other than on this s=1/2 line).

The non-trivial zero point is related to the famous Riemann hypothesis. The Zeta function is a function that takes a complex number and outputs a complex number. Let s be a complex number, and let s=a+bi. The only zero points that can be found are on the line a=0.5 on the a,b complex plane, where c=0,d=0 when the given complex number is given and the output complex number is c+di. These points are called zero points, but the Riemann hypothesis is that there may be no such zero points, non-trivial zero points. If we know all of these infinite zero points, we can use calculations to draw a graph of the prime number staircase that represents all prime numbers. If we take an infinite number of them, we can certainly draw them, but this is hypothetical, and we cannot take an infinite number of them. A trivial zero point is one where the given complex number is a negative even number, and this is irrelevant. . What is the Xi function ξ(s)? It is a function defined as ξ(s)=Γ(s/2)π^(-s/2)ζ(s). ζ(s), or the Zeta function, is difficult to handle, so for the proof we use ξ(s)=ζ(s)π^(-s/2)Γ(s/2). Γ(s) is an analytically continued gamma function, an extension of the factorial. It has poles at the negative integer points, and no zeros or poles elsewhere. π^(-s/2) is π^(-(a+jb)/2)=π^(-a/2)π^(-jb/2)=π^(-a/2)exp(-jb * log(π)/2)=π^(-a/2)*(cos(b*log(π)/2)-i*sin(b*log(π)/2)) and has no zeros or poles. Γ(s/2) is Γ(s) stretched twice along the real axis and twice along the imaginary axis. Therefore, Γ(s/2)ζ(s) is a finite value because the zeros at the negative even points of ζ(s) and the poles of Γ(s/2) cancel each other out. The reason why it becomes a finite value is because the function multiplied and the function multiplied are each a first-order pole and zero, and this is written on pages 128 to 130 of What is the Riemann Hypothesis (Bluebacks). Furthermore, if you multiply it by π^(-s/2), Γ(s/2)π^(-s/2)ζ(s) becomes point-symmetric at the point s=1/2+j・0. Riemann expressed this as a functional equation. Therefore, Γ(s/2)π^(-s/2)ζ(s) has poles at s=0, s=1, and the only other singular points are non-trivial zeros. Therefore, by investigating the locations of the non-trivial zeros of Γ(s/2)π^(-s/2)ζ(s), in other words ξ(s), you will have investigated the non-trivial zeros of ζ(s). Some of you may have noticed that the poles at s=0 and s=1 disappear when multiplied by s(1-s). In fact, this is correct. The real Riemann Xi function is (1/2)s(s-1)ξ(s). This function has only non-trivial zeros as singular points. It can be expressed as a product of factorizations of (1-s/non-trivial zeros), called the Hadamard product. However, I thought I would use the Xi function that is not multiplied by 1/2*s(s-1) to prove the Riemann hypothesis. In Riemann's paper on the number of primes less than a given number, it shows that the Xi function ξ(s) can be rewritten as follows: ξ(s)=∫(1→∞)(t^(s/2)+t^((1-s)/2)Σ(n=1→∞)exp(-πn^2t)*(1/t)dt-1/(s(1-s)) By the way, (1/2)s(s-1)ξ(s) is (1/2)s(s-1)ξ(s)=(1/2)s(s-1)∫(1→∞)(t^(s/2)+t^((1-s)/2)Σ(n=1→∞)exp(-πn^2t)*(1/t)d t+1 Then the term 1/(s(1-s)) which creates the pole becomes 1 and the pole disappears. To prove the Riemann hypothesis, it is sufficient to state that the absolute value of the Xi function |ξ(a+ib)| is greater than 0 for all b, i.e. 0<a<1/2, 1/2<a<1. If we substitute s for a+bi and substitute it into the following equation, ξ(s)=∫(1→∞)(t^(s/2)+t^((1-s)/2)Σ(n=1→∞)exp(-πn^2t)*(1/t)dt-1/(s(1-s)) ξ(a+ib)=∫(1→∞)t^(a/2+ib/2)+t^((1-a)/2-ib/2)Σ(n=1→∞)exp(-πn^2t)*(1/t)dt-1/((a+bi)((1-a)-bi)) =∫(1→∞)t^(a/2)(exp(ib*log(t))/2)+t^((1-a)/2)exp(ib*log(t))/2)Σ(n=1→∞)exp(-πn^2t)*(1/t)dt-1/((a+bi)((1-a)-bi)) By substitution integral If you set logt=u ξ(a+ib)=∫(0→∞)(exp(au/2)*exp(ibu/2)+exp((1-a)u/2)*exp(-ibu/2))*Σ(n=1→∞)exp(-π*n^2*exp(u)) du-1/((a+bi)((1-a)-bi)) =∫(0→∞)(exp(au/2)*(cos(bu/2)+i*sin(bu/2))+exp((1-a)u/2)*(cos(bu/2)-i*sin(bu/2))*Σ(n=1→∞)exp(-π*n^2*exp(u))du-1/((a+bi)((1-a)-bi)) =∫(0→∞)(exp(au/2)+ exp((1-a)u/2))*cos(bu/2)+i*(exp(au/2)-exp((1-a)u/2))*sin(bu/2))*Σ(n=1→∞)exp(-π*n^2*exp(u))du-1/((a+bi)((1-a)-bi)) =∫(0→∞)(exp(au/2)+exp((1-a)u/2))*cos(bu/2)*Σ(n=1→∞)exp(-π*n ^2*exp(u))du+i*∫(0→∞)(exp(au/2)-exp((1-a)u/2))*sin(bu/2))*Σ(n=1→∞)exp(-π*n^2*exp(u))du-1/((a+bi)((1-a)-bi)) If we set u/2=2πu' ξ(a+ib)=∫(0→∞)4π*(exp(2πu'a)+exp(2(1-a)πu'))*cos(2π bu')*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'+i*∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a) πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'-1/((a+bi)((1-a)-bi)) Here, if we calculate 1/((a+bi)((1-a)-bi)), 1/((a+bi)((1-a)-bi))=((a-bi)((1-a)+bi))/((a^2+b^2)((1-a)^2+b^2)) =a(1-a)+b^2+i*(-b(1-a)+ab)/((a^2+b^2)((1-a)^2+b^2)) =(a(1-a)+b^2+i*(2ab-b))/((a^2+b^2)((1-a)^2+b^2)) =(a(1-a)+b^2+i*(2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) To show that |ξ(a+ib)| is greater than 0, simply state that the imaginary part of ξ(a+ib) is not 0. Even if the imaginary part is 0, if the real part is not 0, or even if the real part is 0, if the imaginary part is not 0, then the absolute value is not 0. Here, we will prove in stronger terms that the imaginary part is not 0. So, if we take out just the imaginary part of ξ(a+ib), we get ∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'-((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) If we define the integral part as S(b), then S(b) becomes S(b)=∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))*sin(2πbu')du' and is in the form of a sinusoidal transformation. If we define G(u')=4π*(exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu')), S(b)=∫(0→∞)G(u')sin(2πbu')du' By computer calculation, G(u') is 0 at a=1/2, and converges to 0 at b→∞ even at a=1, and converges faster and faster towards a=1/2, becoming 0 overall at a=1/2. Also, even if a=0, it converges to 0 as b→∞, and the convergence becomes faster toward a=1/2, and the whole becomes 0 at a=1/2. If H(u') is defined as follows

|G(u') (u'>=0)
H(u')= |
|-G(-u')(u'<0)

Therefore S(b)=∫(0→∞)G(u')sin(2πbu')du'=(1/2)*RE(∫(-∞→∞)H(u')exp(i*2πbu')du')

This ∫(-∞→∞)H(u')exp(i*2πbu')du' is the Fourier transform of the negative odd-symmetric expansion of the sine transform of S(b), and since ∫|H(u')|du is a finite value, this Fourier transform converges. Therefore, S(b) also converges. Also, from http://www.maroon.dti.ne.jp/koten-kairo/works/fft/converge6.html,

K(u)=∫(0→∞)f(x)sin(ux)dx converges to 0 as u→∞ if f(x) is continuous and does not diverge. (Riemann-Lebesgue theorem) The imaginary part of ξ(a+ib) is ∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'-((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) When considering the Maclaurin expansion, it turns out that what determines the speed of convergence is the differential terms from the second term onwards.

Therefore, when the limit value of f(x)/g(x) as x→∞ cannot be determined, L'Hôpital's rule focuses on the second and subsequent terms of the Maclaurin expansion. In other words, if f'(x)/g'(x) has a limit value, then the limit value of f(x)/g(x), which was indefinite, will also be determined, and if the limit of (a function that becomes f(x) when differentiated)/(a function that becomes g(x) when differentiated) is also indefinite, it will have the same limit value. Consider the function f(x)=exp(-x)/((2x+1)/(3x^2)). If you wonder what lim(x→∞)f(x) is, it becomes indefinite because lim(x→∞)exp(-x)=0,lim(x→∞)(2x+1)/(3x^2)=0. In this case, consider a function that becomes exp(-x) when differentiated. The answer is -exp(-x). Let h(x) be a function that becomes (2x+1)/(3x^2) when differentiated. Consider this. The answer is that h(x) is O(ln|x|) and lim(x→∞) is ∞. So lim(x→∞)(-exp(-x)/h(x))=0. (0/∞=0). In this way, even if the limit of a function with a denominator of O(x^(-1)) is indefinite, if the limit of a function with the integrated denominator as the denominator and the integrated numerator as the numerator is determined, the limit of the original function will also be the same as that limit. In the usual L'Hôpital rule, the denominator is O(x) or O(x^2), and the parentheses of O() do not contain a negative power. If the denominator is O(x) or O(x^2), and the denominator is the differentiated denominator and the numerator is the differentiated numerator, once the value of (numerator limit)/(denominator limit) is determined, the limit of the original function will be the same as that value. Therefore, if the denominator is O(x^(-n)), n>0, even if the limit cannot be determined as it is, there are cases where the limit of a function with the integrated denominator as the denominator and the integrated numerator as the numerator can be determined. This case appears in the proof of the Riemann hypothesis. In L'Hôpital's rule, if something that has been differentiated becomes a constant, its differentiation is not considered. This is because if something becomes a constant, it becomes 0 when it is differentiated, so it is not considered. L'Hôpital's rule states that the limit where the limit is indefinite is the same as the limit when you differentiate the numerator divided by the denominator, or the limit when you integrate the numerator divided by the denominator. When the limit becomes a finite value in differentiation or integration, the limit is determined. For the limit of sin(x)/x as x → 0, consider L'Hôpital's rule, and the limit of cos(x)/1 as x → 0 becomes 1. If it is differentiated further, cos(x) becomes -sin(x), and when 1 is differentiated, it becomes 0, making it meaningless. Therefore, L'Hôpital's rule does not consider functions that have been differentiated any further once the limit has been determined. L'Hôpital's rule does not determine the limit of sin(x)/x, but by considering the limits of the numerator and denominator differentiated respectively, we can see that it is 1. Therefore, when the denominator is O(x^n) n>0, once the limit of the function obtained by differentiating the numerator divided by the function obtained by differentiating the denominator is determined, the limit of the original function will be determined even if the limit is undefined. If the denominator is O(x^n) n<0, once the limit of the function obtained by integrating the numerator divided by the function obtained by integrating the denominator is determined, the limit of the original function will be the same even if the limit is undefined. This is the essence of L'Hôpital's rule.

f'''(b)=∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du' g'''(b)=((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) Then Im(ξ(a+bi))=f'''(b)-g'''(b) g''(b) is 0 at a=0.5, but the sign is reversed at the border of a=0.5. Since g'''(b) is o(b^(-3)), g(b) becomes o(ln|b|) and is ∞ when b→∞. f(b) is ∫(0→∞)4π*(1/(2πu')^3)*(exp(2πu'a)-exp(2(1-a)πu'))*cos(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du' The reason it can be integrated is because b is included in cos in the form of a linear function. If you differentiate three times, 1(2πu')^3) disappears. f(b)=∫(0→∞)(1/(2πu')^3)*(exp(2πu'a)-exp(2(1-a)πu'))*cos(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du' If we only look at (1/(2πu')^3), it diverges to infinity as u'→0, so the Riemann-Lebesgue theorem does not hold and it seems that lim(b→∞)f(b) = 0 does not hold. However, if we consider 1/(2πu')^3)*cos(2πbu') and consider it together with cos(2πbu'), and let m(u')=cos(2πbu') and n(u')=(2πu')^3, then we get 1/(2πu')^3)*cos(2πbu')=m(u')/n(u'). lim(u'→0)(m(u')/n(u')) =1/0 Cannot be divided by zero lim(u'→0)(m'(u')/n'(u'))=lim(u'→0)(-2πb*sin(2πbu'))/((2π)^3*3*u'^2)=0/0

lim(u'→0)(m''(u')/n''(u'))=lim(u'→0)((2πb)^2*cos(2πbu'))/((2π)^3*3*2*u)= 0/0 lim(u'→0)(m'''(u')/n'''(u'))=lim(u'→0)(-(2πb)^3*sin(2πbu'))/((2π)^3*3*2)=0/((2π)^3*3*2)=0 and by L'Hopital's rule lim(u'→0)(1/(2πu')^3)*cos(2πbu'))=0. Therefore, we can see that 4π*(1/(2πu')^3)*(exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))*cos(2πbu')) does not diverge to ∞ when u'→0, but converges to 0. When we calculate (exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu')) on a computer, we find that it is a function that has extreme values but does not diverge. The calculation program is attached in the Appendix. Also, from a qualitative perspective, (exp(2πu'a)-exp(2(1-a)πu')) is a hyperbolic function and therefore diverges, but since Σ(n=1→∞)exp(-π*n^2*exp(4πu')) has a decay term of an exponential function to the power of an exponential function, it may have an extreme value, but this is more dominant, and (1/(2πu')^3)*(exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu')) will not diverge, so according to the Riemann-Lebesgue theorem it converges to 0 as b→∞. Also, if you repeatedly differentiate (exp(2πu'a)-exp(2(1-a)πu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu')) with respect to u', it will converge to 0. In other words, it is equivalent to a rapidly decreasing function. The Fourier transform of a rapidly decreasing function becomes a rapidly decreasing function. What is true for the Fourier transform is also true for the sine transform. Therefore, since the result is a rapidly decreasing function, the polarity does not change.Therefore, lim(b→∞)((f(b))/(g(b))=0/∞=0. From the previous discussion, lim(b→∞)(f'''(b)/g'''(b))=0. f'''(b) is a higher infinitesimal. f'''(b) converges faster. f'''(b) is an integral term and g'''(b) is an additional term, so the absolute value of the additional term is greater than the absolute value of the integral term until b→∞. f'''(b) is a rapidly decreasing function, so it does not converge while oscillating around 0, and it can be said that it converges with the same polarity, so the imaginary part is not 0 until just before b→∞. -

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In other words, if there exists N such that 0<N<ε, 0<=a<1/2, 1/2<a=<1, that is, lim(b→∞)|(∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'-((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)))| > ε is true.

In other words, when b is greater than a certain value, an additional term that becomes smaller in proportion to o(b^(-3)) remains as b→∞, and the imaginary part of ξ(a+bi) is always nonzero. Therefore, there exists x where b>x, 0=<a<1/2, 1/2<a=<1, and |ξ(a+bi)|>0. Also, when the integral term and the additional term were calculated by a computer when a=0.75, the absolute value of the additional term was already larger than that of the integral term from b=0. It is necessary to use a computer to thoroughly calculate the range between 1/2<a<1, but it seems that the absolute value of the additional term is larger than that of the integral term in all regions between $1/2<a<1$. However, this is merely intuition, so it is necessary to use a computer to thoroughly calculate the magnitude of the values of the integral term and the additional term from b=0 from $1/2<a<1$ and compare them. If the absolute value of the additional term is large from near b=0, then the imaginary part is not 0 at that a until $b\to\infty$. Otherwise, the computer searches for a point where the integral term-additional term=0 and confirms whether that point is a zero point. (If the real part is not 0, it is not a zero.) At a=1/2, the additional term is 0, and the integral term is also 0, so the imaginary part of ξ (a+jb) is 0, and a zero can exist depending on the state of the real part. We know that there are an infinite number of zeros on s=1/2, so the zeros of the ξ function can only be on s=1/2. Therefore, the only non-trivial zeros of the ζ function are on the line at s=1/2. The proof is complete if you use a computer to thoroughly search the magnitude of the values of the integral term and additional term near b=0 between $0.5<a<=1$. The result will probably be that the absolute value of the additional term is greater than the absolute value of the integral term in the entire range except a=1/2, and there will be no reversal of the absolute value of the integral term and the absolute value of the additional term. You should be able to see a trend if you calculate 5 or 6 points. What we can say from L'Hopital's rule is that if the absolute value of the additional term is greater than the absolute value of the integral term, there will be no reversal of the magnitude of the absolute value even if b → infinity. The Riemann-Lebesgue theorem seems to be correct. When a sine wave has an infinite frequency, it distributes equally between positive and negative, so the integral value seems to converge to 0. Finally, if there is no zero point in the computer search near b = 0, the proof of the Riemann hypothesis using a computer is complete, so I calculated it.

The program is attached in the appendix. Calculate $\Gamma(s/2)\pi$^(-s/2)ζ(s). Γ(s/2) was calculated using an algorithm by Oura of the Institute of Mathematical Sciences, Kyoto University. ζ(s) was created by converting Junpei Tsuji's Ruby program into C. In this way, the value of the Xi function is calculated and only the imaginary part of the Xi function is extracted. Im[ξ(a+ib)]=∫(0→∞)4π*(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du'-((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) Integral term = ∫(0→∞)4π *(exp(2πu'a)-exp(2(1-a)πu'))*sin(2πbu'))*Σ(n=1→∞)exp(-π*n^2*exp(4πu'))du' Additional term=((2a-1)b)/((a^2+b^2)((1-a)^2+b^2)) If we set it as such, Im[ξ(a+ib)]=integral term-additional term. The additional term can be calculated, so the value of the integral term can be obtained by adding it to the calculated value of the original Xi function. Comparing the magnitude of the integral term and the additional term, a computer calculation shows that the absolute value of the integral term is already smaller than the absolute value of the additional term near b=0, and the sign is the same. As $b \to \infty$, the magnitude of the absolute value of the integral term and the additional term do not reverse, and it is clear that the sign does not change, so the imaginary part of the Xi function does not become 0. Therefore, the Riemann hypothesis is true. Proof complete.

Appendix

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>


int main(int argc,char *argv[])
{
	int i,j,k;
	FILE *fpw;
	double F;
	double erro,D,PI,aa,bb;
	double *E;
	aa=0.25;
//	bb=0.6;
	E=(double*)malloc(sizeof(double)*10000);
	PI = atan(1.0)*4.0;
	fpw = fopen(argv[1],"w");
	for(i=0;i<10000;i++){
		E[i] = 0.0;
	}
	for(i=0;i<10000;i++){
		F=(double)i/10000;
				D=1.0/*/pow((2.0*PI*F),4.0)*/*(pow(2.71828182846,2.0*PI*F*aa)-
pow(2.71828182846,2.0*PI*F*(1.0-aa)));
	 k=1;
		erro = 1.0e64;
		while(fabs(erro) > 1.0e-8){
				erro		=		pow(2.71828182846,-
PI*(double)k*(double)k*pow(2.71828182846,4.0*PI*F));
			printf("k=%d,erro=%f¥n",k,erro);
			E[i] += erro;
			k++;
		}
```

```
        E[i] *= D;
       }
       for(i=0;i<10000;i++){
              fprintf(fpw,"%f¥n",E[i]);
       }


    fclose(fpw);



    return 0;
}
```

```
#include <stdio.h>
#include <math.h>
#include <complex.h>
#include <stdlib.h>
#include <string.h>

#ifndef DCOMPLEX
struct dcomplex_ {
        double re;
        double im;
};
#define DCOMPLEX struct dcomplex_
#define DREAL(x) (x).re
#define DIMAG(x) (x).im
#define DCMPLX(x,y,z) (z).re = x, (z).im = y
#endif


DCOMPLEX cdgamma(DCOMPLEX x)
{
     DCOMPLEX y;
        double xr, xi, wr, wi, ur, ui, vr, vi, yr, yi, t;

     xr = DREAL(x);
        xi = DIMAG(x);
        if (xr < 0) {
                wr = 1 - xr;
                wi = -xi;
        } else {
                wr = xr;
                wi = xi;
        }
        ur = wr + 6.00009857740312429;
      vr = ur * (wr + 4.99999857982434025) - wi * wi;
        vi = wi * (wr + 4.99999857982434025) + ur * wi;
        yr = ur * 13.2280130755055088 + vr * 66.2756400966213521 +
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 0.293729529320536228;
yi = wi * 13.2280130755055088 + vi * 66.2756400966213521;
ur = vr * (wr + 4.00000003016801681) - vi * wi;
ui = vi * (wr + 4.00000003016801681) + vr * wi;
vr = ur * (wr + 2.99999999944915534) - ui * wi;
vi = ui * (wr + 2.99999999944915534) + ur * wi;
yr += ur * 91.1395751189899762 + vr * 47.3821439163096063;
yi += ui * 91.1395751189899762 + vi * 47.3821439163096063;
ur = vr * (wr + 2.00000000000603851) - vi * wi;
ui = vi * (wr + 2.00000000000603851) + vr * wi;
vr = ur * (wr + 0.999999999999975753) - ui * wi;
vi = ui * (wr + 0.999999999999975753) + ur * wi;
yr += ur * 10.5400280458730808 + vr;
yi += ui * 10.5400280458730808 + vi;
ur = vr * wr - vi * wi;
ui = vi * wr + vr * wi;
t = ur * ur + ui * ui;
vr = yr * ur + yi * ui + t * 0.0327673720261526849;
vi = yi * ur - yr * ui;
yr = wr + 7.31790632447016203;
ur = log(yr * yr + wi * wi) * 0.5 - 1;
ui = atan2(wi, yr);
yr = exp(ur * (wr - 0.5) - ui * wi - 3.48064577727581257) / t;
yi = ui * (wr - 0.5) + ur * wi;
ur = yr * cos(yi);
ui = yr * sin(yi);
yr = ur * vr - ui * vi;
yi = ui * vr + ur * vi;
if (xr < 0) {
    wr = xr * 3.14159265358979324;
    wi = exp(xi * 3.14159265358979324);
    vi = 1 / wi;
    ur = (vi + wi) * sin(wr);
    ui = (vi - wi) * cos(wr);
    vr = ur * yr + ui * yi;
    vi = ui * yr - ur * yi;
```

```
         ur = 6.2831853071795862 / (vr * vr + vi * vi);
                yr = ur * vr;
                yi = ur * vi;
        }
        DCMPLX(yr, yi, y);
        return y;
}


int kaijyo(int i)
{
        int m;
        if( i==0){
                return 1;
        }
        m=kaijyo(i-1);
        return i*m;
}
int combi(int i ,int j)
{
        return kaijyo(i)/kaijyo(i-j)/kaijyo(j);
}
double kaijyo1(double i)
{
        double m;
        if( i>-0.5 && i < 0.5){
                return 1.0;
        }
        m=kaijyo1(i-1.0);
        return i*m;
}
double combi1(double i ,double j)
{
        return kaijyo1(i)/kaijyo1(i-j)/kaijyo1(j);
}
double combi2(double i ,double j)
```

```
{
	int k,m,l;
	double m1,m2,n;
	if((j<0.5 && j > -0.5) ||(i<0.5 && i > -0.5)){
		return 1.0;
	} else if((int)(i+0.5)==(int)(j+0.5)){
		return 1.0;
	} else {
		k=(int)(j+0.5);
		m1=i;
		n = m1;
		for(l=0;l<k-1;l++){
			n = n-1.0;
//			printf("nn=%f¥n",n);
			m1 *= n;

		}
		n= 1.0;
		m2 = n;
		for(l=0;l<k-1;l++){
			m2 = m2 + 1.0;
			n *= m2;
		}
//		printf("m1 = %f,n =%f¥n",m1,n);
		return m1/n;
	}
}

int main(int argc,char *argv[])
{
	int i,j,k,l,m,n;
	double complex A;
	double complex B;
	double complex Z,**GAMMA,ZZ;
	double PI = (double)atan(1.0)*4.0;
	double RE,IM;
```

```
    double complex **ZETA;
    ZETA = (double complex**)malloc(sizeof(double complex*)*300);
      GAMMA = (double complex**)malloc(sizeof(double complex*)*300);
      for(i=0;i<300;i++){
            ZETA[i] = (double complex*)malloc(sizeof(double complex)*300);
            GAMMA[i] = (double complex*)malloc(sizeof(double complex)*300);
      }



    double si,sr;
      if(argc != 2){exit(1);}
      FILE *fpw;
            fpw = fopen(argv[1],"w");
//    sr = atof(argv[1]);
//    si = atof(argv[2]);
      //printf("%f¥n",sr);while(1);
      double outer_sumr;// = 0.0;
      double outer_sumi;// = 0.0;
      double inner_sumr,inner_sumi,inner_sumr1,inner_sumi1,c1r,c2r,c3r,c1i,c2i,c3i,re,im;
      double prevr = 1000000000.0;
      double previ =0.0;
//    int m,j,n;
      for(k=0;k<80/*300*/;k++){
            printf("k=%d¥n",k);
            sr = (double)(k-40/*150*/)/10.0;

//    si = 0.0;
//    FILE *fpw;
//    fpw = fopen(argv[1],"w");
      for(n=0;n<80/*300*/;n++){
            outer_sumr = 0.0;
            outer_sumi = 0.0;
//        printf("n=%d¥n",n);
            si = (double)(n-40/*150*/)/10.0;
      for(m = 1;m <=300;m++){
//        printf("m=%d¥n",m);
```

```
        inner_sumr = 0.0;
            inner_sumi = 0.0;
            for(j=1;j<=m;j++){
//              printf("j=%d¥n",j);
                c1r=((j-1)%2==0) ? 1.0 : -1.0;
                c2r= combi2((double)(m-1),(double)(j-1));
//              printf("combi=%f¥n",combi2(7.0,4.0));while(1);
//              printf("c2r=%f¥n",c2r);
                c1i=0.0;
                c2i=0.0;
//              c3r =pow(2.71828182846,si*1.57079633)*cos(sr*1.57079633);
//              c3i = pow(2.71828182846,si*1.57079633)*(-sin(sr*1.57079633));
                c3r = pow((double)j,-sr)*cos(si*log((double)j));
                c3i = -pow((double)j,-sr)*sin(si*log((double)j));
                inner_sumr += c1r*c2r*c3r;
                inner_sumi += c1r*c2r*c3i;
//              printf("ir=%f,ii=%f¥n",c1r*c2r*c3r,c1r*c2r*c3i);
            }
//              printf("sumr=%f¥n,sumi=%f¥n",outer_sumr,outer_sumi);
                                                            re=          1.0-pow(2.71828182846,(1-
sr)*log(2.0))*cos(si*log(2.0));
                                                              im=           pow(2.71828182846,(1-
sr)*log(2.0))*(sin(si*log(2.0)));
                inner_sumr1     =     (inner_sumr*re+inner_sumi*im)*pow(2.0,(double)(-
m))/(re*re+im*im);
                inner_sumi1     =     (-inner_sumr*im+re*inner_sumi)*pow(2.0,(double)(-
m))/(re*re+im*im);
        outer_sumr += inner_sumr1;
            outer_sumi += inner_sumi1;
            inner_sumr = inner_sumr1;
            inner_sumi = inner_sumi1;
            //printf("or=%f,oi=%f¥n",outer_sumr,outer_sumi);
            if(sqrt((prevr - inner_sumr)*(prevr - inner_sumr)+(previ - inner_sumi)*(previ -
inner_sumi)) < 1.0e-8){
          //    printf("tootta¥n");
                break;
```

```
        }
            if(sqrt(outer_sumr*outer_sumr+outer_sumi*outer_sumi) > 1.0e+22){
                break;
            }
            prevr = inner_sumr;
        previ = inner_sumi;
        }
        ZETA[k][n] = outer_sumr + outer_sumi*I;//fprintf(fpw,"%f¥n",outer_sumr);
            //fprintf(fpw,"%f¥n",outer_sumi);
        }//for n loop end
//          printf("%f+j*%f¥n",outer_sumr,outer_sumi);
        }//for k loop end




//      A= 0.0+1.0*I;
//      B= 1.0+0.0*I;
//      C=A+B;
//      printf("%f+j*%f¥n",creal(C),cimag(C));
//          if(argc != 3){exit(1);}
//      RE = atof(argv[1]);
//      IM = atof(argv[2]);
        DCOMPLEX Z1;
        for(k=0;k<80/*300*/;k++){
            for(i=0;i<80/*300*/;i++){
                    Z       =       /*sr/2.0*/(double)(k-40/*150*/)/10.0/2.0        +       (double)(i-
40/*150*/)/10.0/2.0*I;
                A = /*sr*/(double)(k-40/*150*/)/10.0 + (double)(i-40/*150*/)/10.0*I;
                Z1.re = creal(Z);
                Z1.im = cimag(Z);
//                      GAMMA[i]                =                   sqrt(2.0*PI)*cexp(-Z)*cpow(Z,Z-
0.5)*(1.0+1.0/(12.0*Z)+1.0/(288.0*Z*Z)-139.0/(51840.0*Z*Z*Z)-
571.0/(2488320.0*Z*Z*Z*Z)+163879.0/(209018880.0*Z*Z*Z*Z*Z)+5246819.0/(7524679
```

```
6800.0*Z*Z*Z*Z*Z*Z)-534703531.0/(902961561600.0*Z*Z*Z*Z*Z*Z*Z));
                    GAMMA[k][i] = cdgamma(Z1).re + cdgamma(Z1).im * I;
                    //printf("%f+j*%f¥n",creal(GAMMA),cimag(GAMMA));
                    GAMMA[k][i] = /*A*(A-1.0)* */GAMMA[k][i] * cpow(PI,-Z)*ZETA[k][i] +
1.0/(A*(1.0-A))/** 2.0*/;
                    fprintf(fpw,"%f,",cimag(/*1.0/(A*(1.0-A))*/GAMMA[k][i]));
             }
             fprintf(fpw,"¥n");
       }
       fclose(fpw);
    return 0;
}
```