Inertial and Fictitious Forces in Physics

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Abstract: In this paper we investigate "inertial forces" and "fictitious forces" and the relationship between them as understood from the literature of physics where the majority seem to consider them as two labels for the same thing while some seem to suggest they are not identical. Our view and preference is to distinguish between them where fictitious forces should be considered a subset of inertial forces. We also investigate the issue of the reality of fictitious forces and whether they have authentic existence in the physical world or they are a kind of "delusion". Our view is that what is called "fictitious forces" can be either. In this regard, we identify and propose several physical characterizations and distinctions between the two types (i.e. the "apparently fictitious forces" which are physically real and the "really fictitious forces" which are illusory because they are purely frame-dependent as viewed by a non-inertial observer). This is consistent with our preference that fictitious forces should be considered a subset of inertial forces, i.e. "inertial forces" which are real should not be classified as "fictitious" in the real sense while "inertial forces" which are not real should be classified as "fictitious" in the real sense.

Keywords: Inertial forces, fictitious forces, reality of fictitious forces, epistemology of science, philosophy of science, contemporary physics, fundamental physics, modern physics, classical mechanics, Newtonian mechanics.

Contents

1 Introduction

"Inertial forces" is a central subject in classical mechanics and in its Newtonian formulation in particular. These forces are generally understood (and generically justified and defined) to be caused by "inertia" which is the essence of Newton's first law. In fact, "force" (which, in its basic meaning, is a push or pull) and "inertia" (which is supposedly an intrinsic property of material objects that represents their inertness and resistance to change of their state of rest and motion) are the most important themes and concepts in the dynamic conceptualization and formulation of the Newtonian formulation of classical mechanics. Hence, "inertial forces" (which is a combination of "inertia" and "force") should be a central issue in classical mechanics. Moreover, because of the strong influence of classical mechanics (due mostly to historical reasons) on the main branches of modern physics (such as Lorenz mechanics, gravitation and quantum mechanics), it is natural to expect the presence of "inertial forces" in branches of contemporary physics other than classical mechanics.

"Inertial forces" are frequently labeled in the literature of physics as "fictitious forces" among other similar labels and names (such as "pseudo forces" or "false forces" or "apparent forces" or "d'Alembert forces") which refer to their "fictitious" or hypothetical nature. However, on inspecting their conceptions and instances it seems more appropriate to make some distinction between them (i.e. between "inertial forces" and "fictitious forces") as some scholars seem to suggest. In fact there are many definitions, conceptualizations and characterizations of the above terms (i.e. "inertial forces" and "fictitious forces" and their alike), and there are even more and deeper differences and disputes among physicists about their physical nature and essence as well as their causes and origins (see for instance [1–7] and the references therein). These issues are related to and rooted into the theoretical framework and foundations of classical mechanics which naturally extend sometimes even to non-classical mechanics and issues (as demonstrated for instance in general relativity where gravity is classified as fictitious or inertial force; see for instance [8]).

Most of the "chaos" or looseness in the literature and the disputes among the scholars about the subject of inertial forces and fictitious forces originate from the fact that "inertial forces" (and even "fictitious forces" to some extent) is a loose and not properly and strictly defined term, and part of this originates from the looseness or lack of clarity about "inertia" and the conflicting views about its nature, types, causes and origin. The looseness of "inertial forces" may be demonstrated by inspecting its instances and types (as well as its conceptions and characterizations) in the literature of physics and the wide range that

these instances and types represent conceptually and realistically. The following list gives some common types or instances of the so-called "inertial forces" (which are also labeled very often as "fictitious forces") that can be found in the literature (noting that we will investigate the various definitions and characterizations of these forces later on; see § 2 and § 4):

• The force of inertia (or resistance of inertia) which appears in all frames and may be seen as the essence of "inertia".

• The reaction force in linearly (or translationally) accelerating frame. This may be seen as the simplest type of inertial forces.

• The centrifugal force which appears in rotating frames.

• The Coriolis force which appears in rotating frames.

• The Euler force which appears in non-uniformly rotating frames.

• The gravity in the theory of general relativity noting that gravity is commonly considered as fictitious force (and even as inertial force) in the literature of this theory (where these labels are justified by the presumption about the equivalence or resemblance between the effects of gravity and the effects of acceleration; also see § 3). In fact, gravity in general (i.e. including Newtonian gravity) is often considered and classified as a type of inertial forces.

An important issue that is related to inertial forces and their relationship to fictitious forces is the physical reality of fictitious forces (and hence the physical reality of inertial forces or at least some types of inertial forces). There are conflicting views about this issue where some scholars consider fictitious/inertial forces as real forces (i.e. they have physical origin and cause) while others do not, i.e. they have no physical origin and cause and hence they are illusory and an artifact of (non-inertial) frames of reference. This issue could be one reason (among other reasons) for the necessity of distinguishing between "inertial forces" and "fictitious forces" where in our view the latter should refer more appropriately to a certain type (or class or subset) of the "inertial forces".

In this paper we investigate inertial and fictitious forces and the relationship between them, i.e. whether they should be considered and treated as two labels for the same thing or they should be considered and treated differently and hence it may be more appropriate to consider fictitious forces as instances representing a subset of inertial forces (and therefore they should be defined and characterized accordingly). We will also investigate the issue of the reality of fictitious forces (and hence potentially the physical authenticity of some types of the so-called "fictitious forces" or/and "inertial forces" at least) which we indicated already. We will also touch on the issue of potential ambiguities and confusions

about all theses forces conceptually or/and realistically (mainly from an epistemological perspective).

This study essentially belongs to classical mechanics (and is actually part of "The Epistemology of Contemporary Physics" series). However, we will go in this paper beyond classical mechanics (which is the historical and natural venue for the investigations of inertial and fictitious forces and related issues) in a few places and topics but this is dictated by the necessity of completeness and thoroughness (noting that the same issues will be investigated further in forthcoming papers of this series within fields and contexts other than classical mechanics). We also highlight in this investigation some potential sources and causes of confusion and ambiguity as well as potential misconceptions (particularly from epistemological perspectives) in the treatment of these subjects in the literature of contemporary physics.

2 Definitions of Inertial and Fictitious Forces

"Inertial forces" are defined and conceptualized in the literature of classical mechanics (and the literature of contemporary physics in general such as the literature of gravitation, cosmology and relativity theories) in various ways. These definitions and conceptualizations reflect various views and opinions (as well as various tastes, conventions, preferences and so on) although these views in many cases are not necessarily different in essence or contradicting each other (noting that they are so in many other cases). However, these differences in general can be a source of confusion and ambiguity especially to the casual readers of this literature.

In fact, the mess and confusion (and even contradiction) about inertial forces seem to be deeper than just by having differences in definitions, characterizations and conceptualizations. We can actually find other sources of mess and confusion. One example of these sources is the difference in convention between different branches and disciplines of mechanics (see for instance [9]). Another example of these sources is the aforementioned ambiguity about the relationship between "inertial forces" and "fictitious forces" which also contributes to the latter confusion caused by that ambiguity. A third example is the ambiguity about "inertial forces" and "inertia forces" and if they refer to the same thing or not and if there are different conventions by different scholars about this or not.

In short, "inertial forces" (as well as "fictitious forces" and "inertia forces" and their alike) is one of the most messy areas of mechanics and the mess about it extends to al-

most all aspects related to it (definitions, conceptualizations, instantiations, classifications, terminologies, conventions, views about their nature and origin, etc.).

As indicated already, the confusions and ambiguities caused by the aforementioned differences are exacerbated by the hazy relationship between inertial forces and fictitious forces which have their own sources of confusion and ambiguities (as will be explained next). In fact, what have been said already about inertial forces applies to some extent to "fictitious forces" which are commonly (but seemingly not unanimously) identified with "inertial forces" although these two types of forces should (in our view) be generally distinguished and treated separately at least for the sake of clarity and organization and to avoid more confusion and ambiguities (and even to reflect the apparent conceptual difference between these labels according to what they suggest from their basic language meanings).

To be more specific and clear, we list in the following points a number of definitions of "inertial force" (some or all of which may also be seen as definitions or potential definitions of "fictitious force" and its alike) that can be found in the literature or can be inferred or proposed based on the analysis of the nature of these forces within the commonly-accepted framework of classical mechanics (and contemporary physics in general):

1. A force that is proportional to the (inertial) mass of the object which the force supposedly acts upon.
2. A force that originates from the inertia of material objects.
3. A force that originates from the inertia of material objects as a result of their acceleration.
4. A force that is opposite in direction to an accelerating force acting on a massive object.

Similarly, we list in the following points a number of definitions of "fictitious force" that can be found or suggested in the literature (noting that some or all of these may also be seen by some as definitions or potential definitions of "inertial force"):

1. A force that should be assumed to exist in a non-inertial frame of reference to make Newton's laws of motion applicable in that frame.
2. An apparent force that acts upon a massive object whose motion is observed from a non-inertial frame.
3. A force that should be assumed to exist by an observer whose frame of reference is accelerating relative to an inertial frame.
4. An imaginary force which supposedly acts upon an accelerated body and is equal in magnitude and opposite in direction to a real force.

5. A force which originates from the acceleration of the frame of reference, and not because of interaction between physical objects.
6. A force which is a consequence of the reference frame (i.e. being non-inertial) rather than a physical interaction, and hence this force does not exist in inertial frames.
7. A non-physical force that arises from the kinematics of the situation and is not due to authentic (dynamic) physical interactions or causes.
8. We may also define fictitious force[1] (in the light of causality as related to force; see 2.1.2 of [10]) as a force caused by acceleration (rather than being a cause of acceleration). Accordingly, real forces are causes of acceleration (or change of momentum) while fictitious forces are caused by acceleration.

It is worth noting the following points about these definitions of inertial forces and fictitious forces:

• As we see, the issue of "inertial forces" and "fictitious forces" (as reflected partly in the above definitions) is rather messy and confusing where the mess and confusion arise from various sources and causes such as confusion and mix between "inertial forces" and "fictitious forces", lack of clarity about some essential concepts like inertia, variations in definitions and characterizations, confusion and ambiguity about the instances of these concepts, and so on.

• These definitions (whether of "inertial forces" or "fictitious forces" and whether as classified and contrasted by these labels or within the items of each label) are not equivalent although they are so in some or most cases and instances (within the items of each label). Moreover, they agree on identifying most types and instances of inertial/fictitious forces although there are some types and instances that identified by certain definitions but not other definitions. In short, there may not be an "all-inclusive and all-exclusive" definition among these definitions (although in general each one of these definitions contributes usefully and partly to the characterization and clarification of these concepts and hence they collectively can provide a reasonable characterization if some seeming differences and conflicts are resolved).

[1] We are not aware of such definition in the literature but we include it here as a potential definition (based on our belief in its sensibility, rationality and usefulness). However, it should be noted that this (like at least some other definitions) is not necessarily an "all-inclusive and all-exclusive" definition (noting as well that "acceleration" in this definition requires further attention). In fact, there are some instances and conceptions which do not seem to fit into this definition. For instance, this definition may apply to the reaction forces in Newton's third law which are supposedly real (and hence they are *inertial* but not fictitious as will be indicated later on). Anyway, the scope and size of the present paper do not allow further discussion to this issue.

• Some of these definitions may not be precise (or even correct) in identifying certain types of forces which are commonly classified as "inertial forces" or/and "fictitious forces" although this could be defended by claiming to be a specific convention about this issue or may be avoided by certain modifications or interpretations.
• From the perspective of the literature (as indicated already), these definitions may be considered jointly as definitions of "inertial forces" and "fictitious forces" although we should consider in this context the potential distinction and difference between these two types of forces (i.e. if they are the same or not) which will be discussed later on (see for instance § 6).
• Does the mix between "inertial forces" and "fictitious forces" (as demonstrated by the use of these definitions and their alike interchangeably in the literature) suggest that all fictitious forces originate from inertia? In other words, are there fictitious forces that do not originate from inertia? This issue requires investigation (also see § 6).
• Inertial/fictitious forces have a similarity (according to their instances and some of their definitions) to the reaction force in Newton's third law where the inertial/fictitious force has the same magnitude and opposite direction to the real physical force. Also see § 5 as well as [10, 11].

3 Inertial and Fictitious Forces in Branches of Physics

Classical mechanics (and its Newtonian formulation in particular) is the natural venue for the paradigms of inertial and fictitious forces, and this is indicated directly or indirectly in some of their definitions and characterizations (see § 2). However, classical mechanics is not the only venue although it is the main venue of these paradigms. For example:

1. Lorentz mechanics (which is built on the idea of inertial frames of reference) is an actual or potential venue for the paradigm of inertial forces (and possibly even fictitious forces). However, since Lorentz mechanics is largely kinematical in nature (noting that it is a theory whose essence is the transformation of space and time) these issues are not at the top of the agenda of this subject (unlike classical mechanics; see § 1).
2. The subject of gravitation in general (and the general theory of relativity in particular) is another venue for the paradigms of inertial and fictitious forces (see § 1). It is commonly recognized (and frequently stated in the literature) that gravity in general relativity is a fictitious force (and even an inertial force) where these qualifications are justified by the presumption about the equivalence or resemblance between the effects of gravity

and the effects of acceleration. As far as "inertial forces" is concerned, this should not be restricted to the gravity of general relativity since the gravity of classical physics (according to some definitions of inertial forces; see § 2) should also be an inertial force, i.e. if "inertial forces" means mass-related or mass-based forces then classical gravity should also be regarded as an inertial force (see § 4).[2]

3. Cosmology (as well as astronomy and astrophysics) is a potential venue for the paradigms of inertial and fictitious forces where this is partly due, for instance, to the central role that gravity plays in the development of the Universe and the physical processes that take place in its constituent parts at astronomical level. In fact, the extensive use of general relativity as a theoretical framework for modern cosmology (as well as a number of subjects and fields within astronomy and astrophysics) should make "inertial forces" and "fictitious forces" a natural subject for investigation and consideration in these branches of science (the readers are invited to search for instance "inertial forces in cosmology" on the Internet).
4. Although quantum mechanics may not be a natural venue for the paradigms of inertial and fictitious forces there are some subjects related to quantum mechanics (such as relativistic quantum mechanics and quantum gravity) that can be seen as actual or potential venues for these paradigms. However, there are other reasons for quantum mechanics to be a venue for the paradigms of inertial and fictitious forces as can be observed by inspecting the literature in this regard (the readers are invited to search for instance "inertial forces in quantum mechanics" on the Internet).

4 Characteristics of Inertial Forces

In this section we briefly discuss the characteristics or properties of inertial forces mainly from the perspective of their definitions and characterizations and hence the discussion here can be regarded as a continuation to the discussion of § 2. Referring to the aforementioned (see § 2) mess and confusion about inertial forces (and related themes such as fictitious forces which are linked to inertial forces) it is difficult to characterize inertial forces (to reflect what is presented in the literature) by anything other than being forces related to the inertia of the object that is supposed to be under the influence of these forces. Now, if

[2] In fact, there is another reason for the gravitation to be a venue for the paradigms of inertial and fictitious forces that is the connection between inertia and gravity through Mach's proposal. We discussed this issue previously (see for instance point 3 of § 4.4 of [12]) and hence we do not repeat. This issue is also discussed extensively in the literature of physics (see for instance [7, 13–16]).

we note that the inertia of an object is proportional to its (inertial) mass then we can say that the characteristic property that is shared by all types of inertial forces is that they are proportional to (inertial) mass.[3]

Now, if we note the proportionality between the inertial and gravitational mass (assuming a conceptual distinction between them; see [8]) and accept the equivalence principle (essentially in its classical form which means the equality or identicality between inertial mass and gravitational mass; see [8]) then we should extend "inertial forces" (and possibly "fictitious forces" and their alike) even to gravity. As it is well known, this represents the stand of general relativity although the justification of this in general relativity is based on a more elaborate (and rather artificial) theoretical framework. However, this should demonstrate that considering gravity as an inertial force (and possibly even fictitious force) may not be limited to general relativity if we accept the aforementioned definitions, assumptions and considerations.

Accordingly, "inertial forces" should then mean "mass-related forces", i.e. any force that is linked to "mass" (whether the mass being a cause and origin of this force or being affected by this force or ... etc.). So, "inertial forces" becomes a label for distinguishing these forces from "non mass-related forces" (or "non-inertial forces") such as electric forces or magnetic forces which are not related to mass (since electric and magnetic forces are related to the statics and dynamics of electric charge rather than mass). This should necessitate (or at least indicate or motivate) the distinction between "inertial forces" (which is very generic and general) and "fictitious forces" (which is rather distinctive and specific). This issue will be pursued further in § 6 (noting that this issue is also related to the issue of the physical reality of fictitious forces which will be investigated next; see § 5).

5 The Physical Reality of Fictitious Forces

The views in the literature about the physical reality of fictitious forces (i.e. whether these forces are real or not)[4] are different and even contradictory and they usually depend on the disciplines (in which these forces are treated) and conceptualizations as well as personal choices and preferences (see for instance [9]). In [10] we discussed briefly the reality of

[3] We note that to be more inclusive and general, as well as more concise, we may define and characterize inertial forces as "mass-related forces" to distinguish them, for instance, from charge-related forces such as electric and magnetic forces which are caused by electric charges. In fact, this is what we generally follow in this paper (as indicated earlier and will be repeated later).

[4] The reader is referred to [10] for the meaning of "fictitious" and "real" in this context. Also see footnote [5] .

fictitious forces where we indicated that these forces are real. Although this is not incorrect in general, we need to elaborate and discuss some details about this issue in this paper due first to its importance in itself (as part of the investigation of this paper which, according to its tile, is about fictitious forces) and due second to its connection to the issue of whether or not "inertial force" and "fictitious force" are (or should be) the same (as will be discussed in § 6).

In brief, let assume that there is a non-inertial observer O who watches two material objects: object A which is free (i.e. it is not subject to any force[5] and hence it is accelerating relative to the frame of observer O) and object B which is clamped to the frame of observer O (and hence it is at rest with respect to the frame of observer O). Now, we have the following situation:

• Object A is accelerating in the eye of observer O and hence it must be subject (according to observer O) to a force (noting that this is justified by the presumption that observer O is assuming the validity of Newton's laws of motion in his frame of reference which implicitly implies that he assumes the inertiality of his frame). This force is *really fictitious* because it does not exist at all except in the eye of observer O due to the non-inertial nature of his frame (in contrast to his presumption of the inertiality of his frame).

• Object B is at rest in the eye of observer O and hence it must be subject (according to observer O) to zero net force (assuming that observer O is not aware of the status of his frame as non-inertial and hence he assume that his frame is inertial). However, observer O feels that he needs to exert a force to keep object B at rest and hence observer O needs to assume that there is a counteracting (or balancing) force that annuls his force. This annulling force is *apparently fictitious* because it does exist since it is counteracting a real force and it is not due *entirely* to the non-inertial nature of his frame although the non-inertial nature plays a role in the need for exerting a force to keep object B at rest in that frame. In fact, the reality of this annulling force is based on and justified by the reality of the opposite force that observer O needs to exert to keep object B at rest in his frame (noting the link of this to Newton's third law which will be indicated later).

Accordingly, we need to distinguish between *really fictitious* forces and *apparently fictitious* forces and this distinction may be based on the following conceptualizations (or characterizations or criteria or ... etc.) which can be gathered or inferred from this example:[6]

[5] When we say "not subject to any force" it should mean not subject to any *real* force. In other words, not subject to any force as seen from an inertial frame (or equivalently from the absolute frame of classical mechanics; see [12]).

[6] We note that these conceptualizations are related to this example and may not necessarily apply exactly

1. *Really fictitious* forces are those forces that do not balance (or counteract or annul) any real force (see footnote [5]), while *apparently fictitious* forces are those forces that balance real forces.
2. *Really fictitious* forces are those forces which are needed (for the validity of Newton's laws of motion) only because of the non-inertial nature of the frame, while *apparently fictitious* forces are those forces which are needed (for the validity of Newton's laws of motion) because of the non-inertial nature of the frame plus another physical factor (which in our example is the force that observer O needs to exert to keep object B at rest in his frame).
3. *Apparently fictitious* forces are those forces whose effects are felt directly in the non-inertial frame (e.g. by push or pull), while *really fictitious* forces are those forces whose effects are not felt directly in the non-inertial frame.
4. *Apparently fictitious* forces are those forces that demonstrate Newton's first law (i.e. from the viewpoint of non-inertial frame), while *really fictitious* forces are those forces that demonstrate Newton's second law.
5. *Apparently fictitious* forces are those forces that act on static objects (i.e. objects at rest as seen from the non-inertial frame), while *really fictitious* forces are those forces that act on dynamic objects (i.e. objects in motion or rather accelerating).
6. As seen by an inertial observer O' who watches observer O and his objects A and B, *really fictitious* forces do not exist at all, while *apparently fictitious* forces do exist *but as reaction forces* (acting on observer O in our example).

We conclude this section with the following important remarks:

• When we say[7] "fictitious forces are real" it should be understood within the context of denying the fictitious nature of these forces intrinsically (as suggested by the "fictitious" label) although we accept their fictitious nature in some cases and circumstances. In other words, we can say that we have fictitious forces which are real (i.e. *apparently fictitious* forces) and fictitious forces which are illusory (i.e. *really fictitious* forces).

• It is important to note (as hinted earlier) the difference in the conceptualization of the annulling (or balancing or counteracting) force since it is a force acting on object B according to observer O while it is a force acting on observer O according to observer O'

or/and entirely to other examples (although similar conceptualizations are expected to apply). We should also note that when we talk about fictitious forces then it should be obvious that their fictitious nature is related to non-inertial frames and as seen from these frames because "fictitious forces" are attributes of non-inertial frames.

[7] We are referring to [10] where we avoided there this issue due to its rather complicated nature and our intention to discuss it later on in detail (as we do in the present paper).

(noting that it is a reaction force according to the terminology of Newton's third law). So, although this "real" force is the same physically in both frames and relative to both observers it is not the same conceptually since in the eye of observer O it is a fictitious force acting on object B while in the eye of observer O' it is a real force acting on observer O (noting that being inertial or non-inertial in the eyes of these observers depends on the definitions, conventions and classifications of these forces considering that it is actually a reaction force).[8]

• The physical reality of fictitious forces cannot be denied in some cases and instances since they are classified as such (i.e. being fictitious) on purely theoretical and conceptual bases rather than actual and realistic bases. In fact, the above example (with regard to the force on object B) should demonstrate this. However, to be more clear about this issue let's give another example that is: let have a material particle that is rotating uniformly in a circle around a fixed axis (because the particle is bound to an inextensible string that keeps it at a constant distant from that axis). Now, from the viewpoint of an inertial observer (say standing in the frame of the ground to which the axis is fixed) we have a *real* centripetal force $\mathbf{F}_1$ exerted by the string (or axis) on the particle, and a *real* reaction force $\mathbf{F}_2$ (because of Newton's third law) exerted by the particle on the string (or axis). However, from the viewpoint of a non-inertial observer who is rotating uniformly with the particle (by standing in the frame of the rotating particle) we have a *fictitious* centrifugal force $\mathbf{F}_3$ "from an unknown source" acting on the particle, and a *real* force $\mathbf{F}_4$ exerted by the string (or axis) on the particle (where $\mathbf{F}_3$ and $\mathbf{F}_4$ balance each other and hence they keep the particle at rest in this frame). As we see (based on rationality and common sense), $\mathbf{F}_2$ and $\mathbf{F}_3$ are physically the same; the only difference between them is in their conceptualizations which originates from the difference in the frames of reference to which these conceptualizations belong. Now, since $\mathbf{F}_2$ is real (according to classical mechanics or at least this is what it should be since $\mathbf{F}_1$ is real) then $\mathbf{F}_3$ must also be real (i.e. it has a physical reality even though this physical reality can be seen and detected in the inertial frame but not in the non-inertial frame). In fact, this should confirm and further clarify what have been said above, that is some fictitious forces are real while others are not and this difference is based on the reality or not of these forces from the viewpoint of corresponding inertial frames, i.e. the fictitious force is real/not if its essence[9] is real/not

[8] It is a reaction force in the eye of observer O' and hence when we say "actually a reaction force" this is a "biased" view (i.e. we are considering the view of observer O' as the real and correct one) and this should be justified only by accepting the existence of absolute frame as the ultimate and real reference to all inertial frames (as well as non-inertial frames). See [12].

[9] "Essence" is because of the difference in the conceptualization and labeling as seen from inertial and

in a corresponding inertial frame.

• The above discussion (as well as similar discussions, in our writings as well as in the general literature, about the nature of fictitious forces) should indicate that "force" is not an absolute[10] physical entity but it is frame dependent. In fact, the very labeling and characterization of the concept of "fictitious force" should indicate or suggest that force is not absolute and hence it is frame dependent (at least in some of its types and instances). This is seemingly in conflict with the implicit understanding (at least in classical mechanics) that "force" is an absolute and frame-independent physical entity. In fact, this could add more sources of ambiguity and problems to the paradigm of "force" especially within the framework of classical mechanics and from epistemological perspective (see § [10]). However, some of these problematic issues could be addressed and solved by accepting the existence of absolute frame (see [12]).

So far, it seems that we obtained a better understanding of the nature of fictitious forces and their classification from the perspective of their physical reality. However, the situation is more complex than this. In fact, to deepen and widen our understanding of the nature and types of fictitious forces (as well as their relationship to "inertial forces" in a given sense) we need to investigate the fictitious nature and types of fictitious forces as seen from different non-inertial frames (noting that in the above discussions we considered these forces from the perspectives of an inertial frame and a non-inertial frame). There are other aspects that require further investigation and attention to improve our understanding of the nature and types of fictitious forces. However, these issues will not be pursued in the present paper.

6 Relationship between Inertial and Fictitious Forces

The issue of whether or not "inertial force" and "fictitious force" are the same (and hence these two terms represent the same concept or refer to the same instances in reality) seems to be controversial (noting that the literature is not clear about this issue although these terms are commonly used interchangeably). In our view, this issue depends on the definitions and characterizations that we adopt for "inertial force" and "fictitious force" (see § 2 and § 4) as well as on the issue of the reality of "fictitious forces" (see § 5).

non-inertial frames.

[10] "Absolute" here should mean "frame-independent in its physical reality and existence". For example, the "mass" of a material object (in its basic and generic meaning and regardless of any specific quantitative consideration) is absolute in this sense.

As suggested earlier (see the last paragraph of § 4) we think we have at least two reasons for making distinction between inertial forces and fictitious forces. The first reason is the generality of "inertial forces" versus the specificity of "fictitious forces", while the second reason is the physical reality (or non-reality) of only some types of "fictitious forces" (as established in § 5).[11] In fact, making such a distinction should be inline with the obvious conceptual difference between "inertial forces" and "fictitious forces" and what they suggest according to their basic language meanings.

In short, our position about these issues is outlined in the following points:

• With regard to the issue of whether or not "inertial forces" and "fictitious forces" are (or should be) the same, we propose that they are not (or should not be) the same and hence fictitious forces (whether they are *really fictitious* or *apparently fictitious* or both) represent a subset of inertial forces (where "inertial forces" means mass-related forces). However, it should be noted that in this proposal we are assuming that fictitious forces are exclusively *inertial* (i.e. mass-related) and hence we have no fictitious forces of non-inertial nature (see § 2).

• With regard to the issue of the physical reality of fictitious forces we propose making a clear distinction between "really fictitious forces" (which are fictitious forces) and "apparently fictitious forces" (which are real forces although they look fictitious from a non-inertial frame of reference). This should endorse the distinction between inertial forces and fictitious forces.

7 Conclusions

We outline in the following points the main conclusions that we can obtain from the investigation of the present paper:

1. The subject of "inertial forces" and "fictitious forces" (and related terms and concepts such as "inertia forces" and "false forces") is one of the most messy and confusing areas and aspects of classical mechanics (and actually mechanics in general and even beyond mechanics where the mess and confusion extend to other branches of physics and disciplines such as gravitation and cosmology). The mess and confusion about this is wide and deep and takes almost all shapes and forms such as definitions, conceptualizations, instantiations, classifications, terminologies, conventions, views about their nature and origin, and so on. This mess and confusion create deep confusion and uncertainty about

[11] Actually, the second reason should be an endorsement to the first reason.

the epistemology of classical mechanics (and even beyond) in addition to the mess and confusion from other sources which we discussed elsewhere (see for instance [10, 11]), and should be regarded as one of the most embarrassing things in classical mechanics (and even beyond).

2. A particularly important source of mess and confusion (as indicated in the previous point) is the confusion about the relationship between "inertial forces" and "fictitious forces" and whether they refer to the same thing (conceptually or/and realistically) or not. Our view is that it is more appropriate to make a clear distinction between them where fictitious forces should be characterized and classified as a subset of inertial forces (assuming that all fictitious forces are inertial in nature, i.e. they are mass-related). Moreover, we propose making another distinction between "really fictitious forces" and "apparently fictitious forces" where this distinction is based on our view about the physical reality of the so-called "fictitious forces", i.e. we think that some types of "fictitious forces" are physically real while other types of "fictitious forces" are illusory in the sense that they have no authentic physical origin or existence although they have an (imaginary) existence from the viewpoint of a non-inertial observer (because this observer assumes, wrongly, the validity of Newton's laws of motion in his frame of reference imagining that his frame is inertial). In fact, we identified and proposed in this paper a number of physical characterizations and distinctions between these two types of "fictitious forces" (i.e. "really fictitious forces" and "apparently fictitious forces").

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