Theory and Application of Incomplete Randomness

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Abstract:

Uncertainty is a complex and ubiquitous phenomenon. Randomness is an important concept to describe uncertainty, and its quantitative tool is probability. Through an in-depth study of the distribution law of prime numbers, this paper finds that prime number distribution has both randomness and certainty, which is defined as incomplete randomness. The position of prime numbers in the integer sequence is random, but the number of prime numbers in a certain interval is certain. And there are two trend characteristics of prime number distribution. One big trend is that the density of prime numbers gradually decreases; the other trend is that the probability density in the opposite direction increases slightly. Prime number distribution has a certain randomness, and its distribution is completely controlled by natural laws. There is no accidental cover-up and interference caused by minor factors. The number of prime numbers is fixed. Although there is no accurate function expression, it has a certain degree of certainty. This special type of distribution presents a fixed result and is an incomplete random distribution. The total probability of a particular event is calculated as the cumulative probability: P (total) = ∑ P (n). The total cumulative probability as a quantitative tool for incomplete randomness is a new concept. Unlike classical probability, its value is allowed to be greater than the constant 1.

The discovery of incomplete randomness helps to find the law of prime number distribution, deepen the understanding of the laws of the universe, and broaden the deeper thinking about the nature of nature.

Many conjectures involving prime numbers are unresolved problems, some of which have been around for 300 years. Incomplete randomness can provide a new and unique perspective. This article applies the incomplete random distribution theorem and attempts to give proofs of some of these problems, such as the Mersenne prime conjecture and the Collatz conjecture.

Keywords:

uncertainty, randomness, probability, incomplete randomness, cumulative probability, prime number distribution

Uncertainty is a complex and ubiquitous phenomenon, and also a broad concept. It refers to the fact that people cannot fully determine the possibility of an event or result in the future in experiments or observations, or cannot obtain enough information to accurately predict the outcome of an event.

Randomness is an important concept to describe uncertainty. It can be considered as a type of quantifiable uncertainty, usually referring to the unpredictable characteristics of an event or result under specific conditions. The quantification of randomness usually involves a mathematical description of the uncertainty of random events or processes, including probability and statistics, random processes, etc. Randomness and probability are closely related, and they play a central role in describing and explaining uncertain events. Probability is a mathematical tool used to quantify the probability of a random event. Probability is defined as a measure of a set of events, and its value is between 0 and 1, where 0 means that the event is impossible and 1 means that the event must occur. Randomness involves the probability distribution of an event, that is, the probability of an event, but the specific results of each time are unpredictable. For example, the result of rolling a dice is a random variable, because each time the dice is rolled, people cannot accurately predict which number will appear. [1,2,3,4,5]

Classical probability theory believes that the cause of random distribution must have secondary factors that are not known or controlled by people; random distribution has complete randomness, and the existing probability calculation formula is comprehensive. Therefore, the above randomness is a kind of complete randomness.

However, things in the world are complicated. Observation and analysis show that there are many possible classifications of randomness. For example, the randomness of prime numbers in mathematics in the natural number interval is essentially different from the randomness in the data obtained from physical experimental observations. There is no cover-up and interference of accidental factors caused by secondary factors. The randomness of prime numbers is determined by the inherent laws of natural numbers. Therefore, randomness can be divided into complete randomness and incomplete randomness. As a result, the mathematical tools to describe the two types of randomness will be different.

If determinism and complete randomness are regarded as two extreme cases, incomplete randomness can be regarded as between the two. If the deterministic description can be regarded as infinite, and the probability description of complete randomness is between [0,1], then the probability description of incomplete randomness in the middle can be any non-negative finite real number, that is, its quantitative tool can extend the traditional definition and is not limited by the classical probability space.

The concept of transcending the classical probability model gives us a new perspective and handy tools.

1. Discovery of the law of incomplete random distribution

Through in-depth research on the distribution of prime numbers, it is found that the distribution of prime numbers has two characteristics: randomness and determinism. The position of prime numbers in the integer sequence is random, but the number of prime numbers in a certain interval is deterministic. In addition, there are two trend characteristics of prime number distribution [6,7]: there is a large trend that the prime number density gradually decreases; there is also a reverse small trend of probability density change. Therefore, the distribution of prime numbers has some special characteristics. The prime number distribution type does not belong to a completely random distribution, so the distribution type of prime numbers needs to be redefined.

Incompletely random distribution: The random events that occur are completely controlled by natural laws and the results are fixed. They cannot be normalized, and the total sum of probabilities can be greater than the value 1.

Completely random distribution: The results of random events that occur are not fixed and can be normalized. The total sum of probabilities is 1.

The results presented by the random distribution of prime numbers are fixed, but there is no accurate function expression. In any specified interval, the results of prime number detection experiments are certain. Although it is impossible to predict the exact location of prime numbers, it is possible to predict the lower bound of the number of prime numbers in a certain interval; the random distribution of dice and coin tossing experiments can be affected by uncontrollable secondary factors, and the results presented are not fixed. It is impossible to predict a specific result in a certain experiment, nor is it possible to predict the minimum number of times a specific result will appear in a certain experimental interval. The results of the dice rolling experiment are uncertain in any range of experimental times. Prime numbers are a special type of distribution, and the results presented are fixed as incomplete random distribution. In an incomplete random distribution, the total probability of a particular event is calculated as the cumulative probability: $P\ (total) = \sum P\ (n)$. The total cumulative probability is a new and special concept different from simple probability, and its value is allowed to be greater than the constant 1.

2. Proof of the Incomplete Random Distribution Theorem

For incomplete random distribution, the total probability of a particular event is calculated as the cumulative probability: $P\ (total) = \sum P\ (n)$.

Proof method:

Assume that the prime number distribution type belongs to completely random distribution.

According to the prime number theorem, the average probability that an integer less than the integer x is a prime number is 1/Ln(x). Now calculate the probability that there is a prime number among the integers less than the integer 100.

Calculate according to the probability theory of completely random distribution:

The probability that each integer less than 100 is not a prime number is 1-1/Ln(100), and the probability that there is a prime number among the integers less than the integer 100 is $1-(1-1/Ln(100))^{100}$=0.9999999999977.

The result shows that the probability of a prime number is less than the value 1, that is, a prime number does not necessarily exist.

Use the incomplete random distribution theorem to calculate:

The total cumulative probability P (total) = $\sum$P (n).

Because the prime number theorem shows that 1/Ln(x) is the lower bound of the average probability that an integer less than x is prime, the lower bound probability that there is a prime number among integers less than 100 is 100 1/Ln(100) added together to 21.7.

As a result, the event of the existence of prime numbers has occurred at least 21 times, that is, prime numbers must exist.

The real situation: There are 25 prime numbers among integers less than 100. Anyone who verifies the existence of 25 prime numbers is a fact, and prime numbers are a must. The assumption contradicts the real situation, so it is proved that the prime number distribution type is a new type of distribution, that is, it belongs to an incomplete random distribution.

3. Application of the Incomplete Random Distribution Theorem in Mathematics

Incomplete randomness and its cumulative probability calculation tools can be used to effectively solve many difficult problems in number theory. Here are some examples.

Example 1: Proof of the Mersenne prime conjecture[8]

The Mersenne prime conjecture was proposed in 1644. It refers to whether there are infinite Mersenne primes among positive integers of the form 2^n-1.

Proof method:

Let n>1 be any positive integer. According to the prime number theorem, the probability that 2^n-1 is a prime number is conservatively estimated to be approximately $1/Ln(2^n-1)$, so the lower bound function (cumulative probability) of the number of Mersenne primes less than 2^n is conservatively estimated to be approximately

$$\sum_{x=2}^{n} \frac{1}{ln(2^x - 1)}$$

Because when x increases, $Ln(2^x-1)$ approaches $Ln(2^x)$, so the above formula can be simplified to

$$\frac{1}{\ln(2)} \sum_{x=2}^{n} \frac{1}{x}$$

Because when n approaches infinity, the harmonic series in the above formula diverges. The function value, that is, the number of Mersenne primes, also tends to infinity. This proves the Mersenne prime conjecture, that is, there are infinite Mersenne primes.

Because the last digits of a positive integer of type 2^n-1 can only be 1, 3, 5, and 7; and because the probability that a positive integer m is prime when the last digits are 1, 3, 7, and 9 is 10/4*1/Ln(m), so the probability that a positive integer m is prime when the last digits are 1, 3, 5, and 7 is 10/4*1/Ln(m)*3/4. After simplification, the more accurate probability that a positive integer of type 2^n-1 is prime is approximately $15/8*/Ln(2^n-1)$. Therefore, when n>1 is an arbitrary positive integer, the number of Mersenne primes less than 2^n (cumulative probability) is more accurately estimated to be approximately

$$\frac{15}{8\ln(2)} \sum_{x=2}^{n} \frac{1}{x}$$

Example 2: Proof of the Collatz conjecture[9]

The Collatz conjecture, also known as the 3x+1 conjecture, states that for a natural number x, if it is an odd number, it is multiplied by 3 and then added to 1. If it is an even number, an even factor 2^n is extracted. After several operations, it will always return to 1. This conjecture was proposed by German mathematician Lothar Collatz in 1937 and has not yet been thoroughly proved.

Proof method:

Each operation of the Collatz conjecture, whether it is an odd number or an even number, can be represented by the mathematical model $(k*2^n)*3+2^n$, which can be simplified to $(3k+1)*2^n$. The distribution of the odd factor k is controlled by natural laws and is not completely random. Because after any positive integer is determined and different people perform operations, the odd factor after each operation is exactly the same, and there is determinism.

Let 2t be a perfect square number just less than 3k, then $2^t < (3k+1) \leqslant 2*2^t$. Since k is an arbitrary odd number, the value of (3k+1) fluctuates between greater than 2^t and less than or equal to $2*2^t$, and is randomly distributed in the area close to $2*2^t$. Since (3k+1) does not exist in the area greater than 2^t and less than 3k, and (3k+1) is an even number, the probability that (3k+1) is equal to $2*2^t$ after each operation is conservatively estimated to be $1/(2*2^t-2^t)=1/2^t$. Function $\sum 1/2^t$ is the lower bound function of the number of times (3k+1) is a perfect square number.

In fact, k is an arbitrary odd number, and (3k+1) must be an even number after each operation. Even numbers exist in many forms, because n is an arbitrary positive integer in the even number $k*2^n$, and the number of operations to extract an even number is different. As a result, the k value at the next operation fluctuates, and there is no monotonically increasing trend. Therefore, the probability value $1/2^t$ that (3k+1) is a perfect square number fluctuates up and down in the initial stage. When (3k+1) is a perfect square, each item of the probability value is 1/2, because the odd factor k is 1 at this time. The probability value of (3k+1) being a perfect square is $1/(2^2-2^1)=1/2$ when the operation continues, and there is no trend of gradually decreasing with the increase of the number of operations. The value of the cumulative probability function $\sum 1/2^t$ that (3k+1) is a perfect square is monotonically increasing, but the rising slope of the function curve is sometimes high and sometimes low in the initial stage. When (3k+1) is a perfect square, the rising slope of the function curve is fixed and there is no trend of gradually decreasing.

Comparing the cumulative probability function $\sum 1/2^t$ with the harmonic function $\sum 1/n$, each item 1/n of the harmonic function $\sum 1/n$ has a trend of gradually decreasing value as the number of items n increases. This causes the rising slope of the harmonic function $\sum 1/n$ curve to gradually decrease.

For any positive integer $k*2^n$, no matter how large it is, the odd factor k in it is a fixed value. Therefore, the value of t in the perfect square number 2^t just less than 3k is a finite value. When the value of n in the harmonic function $\sum 1/n$ is greater than 2^t, the value of the corresponding function $1/2^t$ will gradually become greater than the value of the function 1/n. That is, when the value of n is less than 2^t, the value of the function 1/n is greater than the value of the function $1/2^t$; when the value of n is greater than 2^t, the value of the function 1/n is less than the value of the function $1/2^t$.

The number 2^t is a finite value, and the number of times the value of function 1/n is greater than the function $1/2^t$ is finite; the number of times the value of function 1/n is less than the function $1/2^t$ is infinite.

Therefore, when the number of operations approaches infinity, the value of the cumulative probability function $\sum 1/2^t$ is greater than the value of the harmonic function $\sum 1/n$ of the corresponding number. Because the value of the harmonic function $\sum 1/n$ tends to infinity when n tends to infinity, when the number of operations gradually increases, the cumulative probability function value $\sum 1/2^t >> 1$ that (3k+1) is a perfect square number, and it also tends to infinity when the number of operations is infinite, and does not converge.

The cumulative probability function value $\sum 1/2^t$ (lower bound function) represents the lower bound of the number of times (3k+1) is a perfect square number, and is much larger than the value 1 when the number of operations gradually increases. When the number of operations approaches infinity, the cumulative probability function value $\sum 1/2^t$ is infinite and does not converge. This shows that the event that (3k+1) is a perfect square number is inevitable, which proves the Collatz conjecture.

4. Conclusion and Discussion

Starting from observation and analysis, this paper gives a classification of randomness and defines incomplete randomness, which is between determinism and complete randomness. The quantitative tool used is cumulative probability, whose value is allowed to be greater than the constant 1.

The discovery of incomplete randomness has deepened the understanding of the laws of the universe and broadened the deeper thinking about the nature of nature.

This discovery helps to find new laws of prime number distribution and create a new paradigm for mathematical research. Many conjectures involving prime numbers are unresolved problems, some of which have been around for 300 years. Incomplete randomness can provide a new and unique perspective. This paper applies the incomplete random distribution theorem and tries to give proofs of some of these problems, such as the Mersenne prime conjecture and the Collatz conjecture.

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