A $3D$ spherical warp drive vector created using a new methodology but with the same physical properties of the Natario warp drive

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Abstract

The Natario warp drive appeared for the first time in 2001.Although the idea of the warp dive as a spacetime distortion that allows a spaceship to travel faster than light predated the Natario work by 7 years Natario introduced in 2001 the new concept of a propulsion vector to define or to generate a warp drive spacetime.Natario defined a warp drive vector for constant speeds in Polar Coordinates using the Hodge Star but remember that a real warp drive must accelerate or de-accelerate in order to be accepted as a physical valid model so it must possesses variable speeds.We developed a new warp drive vector different than the original Natario warp drive vector using a new methodology and without the Hodge Star that encompasses variable speeds.Also Polar Coordinates uses only two dimensions and we know that a real spaceship is a tridimensional $3D$ object inserted inside a tridimensional $3D$ warp bubble that must be defined in real $3D$ Spherical Coordinates.In this work we present the new warp drive vector in tridimensional $3D$ Spherical Coordinates without the Hodge Star for both constant or variable speeds.Our new proposed warp drive vectors also satisfies the Natario requirements for a warp drive spacetime.

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1 Introduction:

The Natario warp drive appeared for the first time in 2001.([1]).Although the idea of the warp dive as a spacetime distortion that allows a spaceship to travel faster than light predated the Natario work by 7 years Natario introduced in 2001 the new concept of a propulsion vector to define or to generate a warp drive spacetime.

This propulsion vector nX uses the form $nX = X^i e_i$ where X^i are the shift vectors responsible for the spaceship propulsion or speed and e_i are the Canonical Basis of the Coordinates System where the shift vectors are based or placed.

Natario (See pg 5 in [1]) defined a warp drive vector $nX = vs * (dx)$ where vs is the constant speed of the warp bubble and $*(dx)$ is the Hodge Star taken over the x-axis of motion in Polar Coordinates(See pg 4 in [1]).(see Appendix D about Polar Coordinates in [2]).(see Appendix A for the complete mathematical demonstration of the Natario calculations for the Hodge Star in [2]).The final form of the original Natario warp drive vector is given by $nX = vs * d(r\cos\theta)$ or better:

$$nX = -2v_s f \cos\theta \mathbf{e}_r + v_s(2f + rf')\sin\theta \mathbf{e}_\theta \tag{1}$$

or

$$nX = 2v_s f \cos\theta \mathbf{e}_r - v_s(2f + rf')\sin\theta \mathbf{e}_\theta \tag{2}$$

However Polar Coordinates are not real tridimensional $3D$ coordinates since it uses only the two dimensional Canonical Basis $\mathbf{e}_r$ and $\mathbf{e}_\theta$.

We adopted the second expression above taken from Natario (pg 5 in [1]) to define a new warp drive vector that do not uses the Hodge Star but retains all the Natario requirements as will be demonstrated in this work.The final form of the new warp drive vector nWD is given by:

$$nWD = 2v_s f \cos\theta \mathbf{e}_r + v_s(2f + rf')\sin\theta \mathbf{e}_\theta \tag{3}$$

Note that this new warp drive vector nWD is symmetrical when compared to the second Natario warp drive vector in the shift vector and Canonical Basis $X^\theta e_\theta$.In the Natario case $X^\theta e_\theta$ is negative $[-v_s(2f + rf')\sin\theta \mathbf{e}_\theta]$ while in the new case $X^\theta e_\theta$ is positive $[+v_s(2f + rf')\sin\theta \mathbf{e}_\theta]$.The symmetry in this case lies over the shift vector X^θ where in the Natario case is $X^\theta = [-v_s(2f + rf')\sin\theta]$ and in our case is $X^\theta = [+v_s(2f + rf')\sin\theta]$

Note also that the new warp drive vector nWD above uses a constant speed because it was derived from the Natario warp drive vector nX also with a constant speed.In this work we examine what happens with the new warp drive vector when the velocity is variable.Remember that a real warp drive must accelerate or de-accelerate in order to be accepted as a physical valid model.

Natario used Polar Coordinates(See pg 4 in [1]) but for a real $3D$ Spherical Coordinates another new warp drive vector must be calculated.Remember that a real spaceship is a tridimensional $3D$ object inserted inside a tridimensional $3D$ warp bubble that must uses all the tridimensional $3D$ Canonical Basis $\mathbf{e}_r$,$\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.(see Appendix E about tridimensional $3D$ Spherical Coordinates in [2]).

In this work we present the new warp drive vector nWD in tridimensional $3D$ Spherical Coordinates for both constant or variable speeds.

In order to fully understand the main idea presented in this work(a new warp drive vector nWD in tridimensional $3D$ Spherical Coordinates obtained independently from the Natario Hodge Star but retaining all the Natario physical features and properties) acquaintance or familiarity with the Natario original warp drive paper in [1] or familiarity with our work in [2] are a previous reading requirement.We provide useful mathematical demonstrations QED(Quod Erad Demonstratum) in the Appendices of [2].

This work is organized as follows:

- A)-Section 2 introduces the new warp drive vector nWD in $2D$ Polar Coordinates for constant speeds.
- B)-Section 3 introduces the new warp drive vector nWD in $2D$ Polar Coordinates for variable speeds.
- C)-Section 4 introduces the new warp drive vector nWD in tridimensional $3D$ Spherical Coordinates for constant speeds.
- D)-Section 5 introduces the new warp drive vector nWD in tridimensional $3D$ Spherical Coordinates for variable speeds.

We adopted in this work a pedagogical language and a presentation style that perhaps will be considered as tedious,monotonous, exhaustive or extensive by experienced or seasoned readers and we designated this work for novices,newcomers,beginners or intermediate students providing in our work all the mathematical references required for the background needed to understand the process we used to generate these new warp drive vectors independently from the Natario Hodge Star but retaining all the Natario physical features and properties.

We hope our paper is suitable to fill this proposed task.

This work was designed as a companion work to our work in [2].

2 The equation of the new warp drive vector nWD in $2D$ polar coordinates with a constant speed vs

The equation of the new warp drive vector nWD is given by:

$$nWD = X^r e_r + X^\theta e_\theta \tag{4}$$

With the contravariant shift vector components X^{rs} and X^θ given by:

$$X^{rs} = 2v_s f(rs)\cos\theta \tag{5}$$

$$X^\theta = +v_s(f(rs) + (rs)f'(rs))\sin\theta \tag{6}$$

Compare the expressions above with the original Natario warp drive vector in $2D$ polar coordinates with a constant speed vs presented in section 2 in [2].The symmetry in this case lies over the shift vector X^θ where in the Natario case is $X^\theta = -v_s(f(rs)+(rs)f'(rs))\sin\theta$ and in our case is $X^\theta = +v_s(f(rs)+(rs)f'(rs))\sin\theta$

Considering a valid $f(rs)$ as a shape function being $f(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $f(rs) = 0$ for small rs(inside the warp bubble) while being $0 < f(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the warped region(see pg 5 in [1]):

We must demonstrate that the new warp drive vector given above satisfies the Natario requirements for a warp bubble defined by:

any new vector nWD generates a warp drive spacetime if $nWD = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nWD = vs(t)$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble.(see pg 4 in [1])(see Appendix G in [2] for an explanation about this statement)

Natario in its warp drive uses the polar coordinates rs and θ.In order to simplify our analysis we consider motion in the $x-axis$ or the equatorial plane rs where $\theta = 0$ $\sin(\theta) = 0$ and $\cos(\theta) = 1$.(see pgs 4,5 and 6 in [1]).

In a $1+1$ spacetime the equatorial plane we get¿:

$$nWD = X^r e_r \tag{7}$$

The contravariant shift vector component X^{rs} is then:

$$X^{rs} = 2v_s f(rs) \tag{8}$$

Remember that Natario(see pg 4 in [1]) defines the x axis as the axis of motion.Inside the bubble $f(rs) = 0$ resulting in a $X^{rs} = 0$ and outside the bubble $f(rs) = \frac{1}{2}$ resulting in a $X^{rs} = vs$ and this illustrates the Natario definition for a warp drive spacetime.See Appendix D in [2].

The difference between the new warp drive vector and the Natario original one occurs if the motion occurs also in $X^\theta e_\theta$

3 The equation of the new warp drive vector nWD in $2D$ polar coordinates with a variable speed vs due to a constant acceleration a

The equation of the new warp drive vector nWD is given by:

$$nWD = X^t e_t + X^r e_r + X^\theta e_\theta \tag{9}$$

The contravariant shift vector components X^t,X^{rs} and X^θ are defined by:

$$X^t = 2f(rs)(rs)cos\theta a \tag{10}$$

$$X^{rs} = 2[2f(rs)^2 + (rs)f'(rs)](at)cos\theta \tag{11}$$

$$X^\theta = +2f(rs)(at)[2f(rs) + (rs)f'(rs)]\sin\theta \tag{12}$$

Compare the expressions above with the original Natario warp drive vector in $2D$ polar coordinates with a variable speed vs presented in section 3 in [2].The symmetry in this case lies over the shift vector X^θ where in the Natario case is $X^\theta = -2f(rs)(at)[2f(rs) + (rs)f'(rs)]\sin\theta$ and in our case is $X^\theta = +2f(rs)(at)[2f(rs) + (rs)f'(rs)]\sin\theta$

Considering a valid $f(rs)$ as a shape function being $f(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $f(rs) = 0$ for small rs(inside the warp bubble) while being $0 < f(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the warped region(see pg 5 in [1]):

We must demonstrate that the new warp drive vector given above satisfies the Natario requirements for a warp bubble defined by:

any new vector nWD generates a warp drive spacetime if $nWD = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nWD = vs(t)$ with $X = vs(t)$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble.(see pg 4 in [1])(see Appendix G in [2] for an explanation about this statement)

Natario in its warp drive uses the polar coordinates rs and θ.In order to simplify our analysis we consider motion in the $x - axis$ or the equatorial plane rs where $\theta = 0$ $\sin(\theta) = 0$ and $\cos(\theta) = 1$.(see pgs 4,5 and 6 in [1]).

In a $1 + 1$ spacetime the equatorial plane we get¿:

$$nWD = X^t e_t + X^r e_r \tag{13}$$

$$X^t = 2f(rs)(rs)a \tag{14}$$

$$X^{rs} = 2[2f(rs)^2 + (rs)f'(rs)](at) \tag{15}$$

The variable velocity vs due to a constant acceleration a is given by the following equation:

$$vs = 2f(rs)at \tag{16}$$

Remember that Natario(see pg 4 in [1]) defines the x axis as the axis of motion.Inside the bubble $f(rs) = 0$ resulting in a $vs = 0$ and outside the bubble $f(rs) = \frac{1}{2}$ resulting in a $vs = at$ as expected from a variable velocity vs in time t due to a constant acceleration a.Since inside and outside the bubble $f(rs)$ always possesses the same values of 0 or $\frac{1}{2}$ then the derivative $f'(rs)$ of the shape function $f(rs)$ is zero and the shift vector $X^{rs} = 2[2f(rs)^2]at$ with $X^{rs} = 0$ inside the bubble and $X^{rs} = 2[2f(rs)^2]at = 2[2\frac{1}{4}]at = at = vs$ outside the bubble and this illustrates the Natario definition for a warp drive spacetime.See Appendix D in [2]

The difference between the new warp drive vector and the Natario original one occurs if the motion occurs also in X^θ

4 The equation of the new warp drive vector nWD in tridimensional $3D$ spherical coordinates with a constant speed vs

The equation of the new warp drive vector in tridimensional $3D$ spherical coordinates with a constant speed vs nWD is given by:

$$nWD = X^r e_r + X^\theta e_\theta + X^\phi e_\phi \tag{17}$$

With the contravariant shift vector components X^{rs}, X^θ and X^ϕ given by:

$$X^r = vs(t)[\sin\phi][2f(r)\ cos\theta] \tag{18}$$

$$X^\theta = +vs(t)[\sin\phi][2f(r) + rf'(r)]\sin\theta] \tag{19}$$

$$X^\phi = [vs(t)cos\phi][cot\theta[2(f(r)) + (rf'(r))] \tag{20}$$

Compare the expressions above with the warp drive vector in $3D$ spherical coordinates with a constant speed vs presented in section 4 in [2].The symmetry in this case lies over the shift vector X^θ where in the section 4 in [2] case is $X^\theta = -vs(t)[\sin\phi][2f(r) + rf'(r)]\sin\theta]$ and in our case is $X^\theta = +vs(t)[\sin\phi][2f(r) + rf'(r)]\sin\theta]$

Considering a valid $f(r)$ as a shape function being $f(r) = \frac{1}{2}$ for large r(outside the warp bubble) and $f(r) = 0$ for small rs(inside the warp bubble) while being $0 < f(r) < \frac{1}{2}$ in the walls of the warp bubble also known as the warped region(see pg 5 in [1]):

We must demonstrate that our warp drive vector satisfies the Natario criteria for a warp drive defined by:

any warp drive vector nWD generates a warp drive spacetime if $nWD = 0$ and $X = vs = 0$ for a small value of r defined by Natario as the interior of the warp bubble and $nWD = vs(t)$ with $X = vs$ for a large value of r defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble.(see pg 4 in [1])(see Appendix G in [2] for an explanation about this statement)

Natario in its warp drive uses the polar coordinates r and θ.In order to simplify our analysis we consider motion in the $x-axis$(like Natario did) or the equatorial plane $x-y$ in r where $\theta = 0$ $\sin(\theta) = 0$ and $\cos(\theta) = 1$.(see pgs 4,5 and 6 in [1]).Also the equatorial plane $x-y$ makes an angle of 90 degrees with the $z-axis$ so $\sin\phi = 1$ and $cos\phi = 0$.

Then the contravariant components reduces to:

$$X^r = vs(t)[\sin\phi][2f(r)\ cos\theta] \rightarrow X^r = vs(t)[2f(r)] \rightarrow \sin\phi = 1 \rightarrow\ cos\theta = 1 \tag{21}$$

$$X^\theta = +vs(t)[\sin\phi][2f(r) + rf'(r)]\sin\theta] = 0 \rightarrow \sin\phi = 1 \rightarrow \sin\theta = 0 \tag{22}$$

$$X^\phi = [vs(t)cos\phi][cot\theta[2(f(r)) + (rf'(r))] = 0 \rightarrow cos\phi = 0 \tag{23}$$

Remember that Natario(see pg 4 in [1]) defines the x axis as the axis of motion.Inside the bubble $f(r) = 0$ resulting in a $X^r = 0$ and outside the bubble $f(r) = \frac{1}{2}$ resulting in a $X^r = vs$ and this illustrates the Natario definition for a warp drive spacetime.See Appendix E in [2].

Only in tridimensional motion the results becomes different.

The difference between the new warp drive vector and the warp drive presented in section 4 in [2] occurs only if the motion occurs also in X^θ.

5 The equation of the new warp drive vector nWD in tridimensional $3D$ spherical coordinates with a variable speed vs due to a constant acceleration a

The equation of the new warp drive vector in tridimensional $3D$ spherical coordinates with a variable speed vs due to a constant acceleration a nWD is given by:

$$nWD = X^t e_t + X^r e_r + X^\theta e_\theta + X^\phi e_\phi \tag{24}$$

With the contravariant shift vector components X^t,X^{rs}, X^θ and X^ϕ given by:

$$X^t = 2(rf(r)a))(\sin\phi)(\cos\theta) \tag{25}$$

$$X^r = (2at)[2f(r)^2 + (rf'(r))](\sin\phi)(\cos\theta) \tag{26}$$

$$X^\theta = +(2f(r)at)[2f(r) + rf'(r)](\sin\phi)(\sin\theta) \tag{27}$$

$$X^\phi = (2f(r)at)[2f(r) + (rf'(r))](cos\phi)(cot\theta) \tag{28}$$

Compare the expressions above with the warp drive vector in $3D$ spherical coordinates with a variable speed vs presented in section 5 in [2].The symmetry in this case lies over the shift vector X^θ where in the section 5 in [2] case is $X^\theta = -(2f(r)at)[2f(r) + rf'(r)](\sin\phi)(\sin\theta)$ and in our case is $X^\theta = +(2f(r)at)[2f(r) + rf'(r)](\sin\phi)(\sin\theta)$

Considering a valid $f(r)$ as a shape function being $f(r) = \frac{1}{2}$ for large r(outside the warp bubble) and $f(r) = 0$ for small rs(inside the warp bubble) while being $0 < f(r) < \frac{1}{2}$ in the walls of the warp bubble also known as the warped region(see pg 5 in [1]):

We must demonstrate that our warp drive vector satisfies the Natario criteria for a warp drive defined by:

any warp drive vector nWD generates a warp drive spacetime if $nWD = 0$ and $X = vs = 0$ for a small value of r defined by Natario as the interior of the warp bubble and $nWD = vs(t)$ with $X = vs$ for a large value of r defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble.(see pg 4 in [1])(see Appendix G in [2] for an explanation about this statement)

Natario in its warp drive uses the polar coordinates r and θ.In order to simplify our analysis we consider motion in the $x - axis$(like Natario did) or the equatorial plane $x - y$ in r where $\theta = 0$ $\sin(\theta) = 0$ and $\cos(\theta) = 1$.(see pgs 4,5 and 6 in [1]).Also the equatorial plane $x - y$ makes an angle of 90 degrees with the $z - axis$ so $\sin\phi = 1$ and $cos\phi = 0$.Then the contravariant components reduces to:

$$X^t = 2(rf(r)a))(\sin\phi)(\cos\theta) \rightarrow X^t = 2(rf(r)a)) \rightarrow \sin\phi = 1 \rightarrow \cos\theta = 1 \tag{29}$$

$$X^r = (2at)[2f(r)^2 + (rf'(r))](\sin\phi)(\cos\theta) \rightarrow X^r = (2at)[2f(r)^2 + (rf'(r))] \rightarrow \sin\phi = 1 \rightarrow \cos\theta = 1 \tag{30}$$

$$X^{\theta} = +(2f(r)at)[2f(r) + rf'(r)](\sin\phi)(\sin\theta) = 0 \rightarrow \sin\phi = 1 \rightarrow \sin\theta = 0 \tag{31}$$

$$X^{\phi} = (2f(r)at)[2f(r) + (rf'(r))](cos\phi)(cot\theta) = 0 \rightarrow \cos\phi = 0 \tag{32}$$

The remaining contravariant components are:

$$X^{t} = 2(rf(r)a))(\sin\phi)(\cos\theta) \rightarrow X^{t} = 2(rf(r)a)) \rightarrow \sin\phi = 1 \rightarrow \cos\theta = 1 \tag{33}$$

$$X^{r} = (2at)[2f(r)^{2} + (rf'(r))](\sin\phi)(\cos\theta) \rightarrow X^{r} = (2at)[2f(r)^{2} + (rf'(r))] \rightarrow \sin\phi = 1 \rightarrow \cos\theta = 1 \tag{34}$$

$$nWD = X^{t}e_{t} + X^{r}e_{r} \tag{35}$$

$$X^{t} = 2rf(r)a \tag{36}$$

$$X^{rs} = 2[2f(r)^{2} + rf'(r)]at \tag{37}$$

The variable velocity vs due to a constant acceleration a is given by the following equation:

$$vs = 2f(r)at \tag{38}$$

Remember that Natario(pg 4 in [1]) defines the x axis as the axis of motion.Inside the bubble $f = 0$ resulting in a $vs = 0$ and outside the bubble $f = \frac{1}{2}$ resulting in a $vs = at$ as expected from a variable velocity vs in time t due to a constant acceleration a.Since inside and outside the bubble $f(r)$ always possesses the same values of 0 or $\frac{1}{2}$ then the derivative $f'(r)$ of the shape function $f(r)$ is zero and the shift vector $X^{rs} = 2[2f(r)^{2}]at$ with $X^{r} = 0$ inside the bubble and $X^{rs} = 2[2f(r)^{2}]at = 2[2\frac{1}{4}]at = at = vs$ outside the bubble and this illustrates the Natario definition for a warp drive spacetime.See Appendix E in [2]

Only in tridimensional motion the results becomes different.

The difference between the new warp drive vector and the warp drive presented in section 5 in [2] occurs only if the motion occurs also in X^{θ}.

6 Conclusion

In this work we introduced new warp drive vectors using independent mathematical techniques.We focused ourselves in the $3D$ spherical coordinates for both constant and variable speeds.

Remember that a real spaceship is a tridimensional $3D$ object inserted inside a tridimensional $3D$ warp bubble that must uses all the tridimensional $3D$ Canonical Basis $\mathbf{e}_r$,$\mathbf{e}_\theta$ and $\mathbf{e}_\phi$.

Our focus were concentrated in the new methods to obtain warp drive vectors with the same physical properties of the Natario warp drive.As a matter of fact our warp drives are symmetrical when compared with the Natario warp drive and also when compared with the warp drives presented in [2].

The Natario warp drive is probably the best candidate(known until now) for an interstellar space travel considering the fact that a spaceship in a real superluminal spaceflight will encounter(or collide against) hazardous objects(asteroids,comets,interstellar dust and debris etc) and the Natario spacetime offers an excellent protection to the crew members as depicted in the works [7],[8],[9] and [10].However since these works were based in the original Natario 2001 paper this line of reason must be extended to encompass the new tridimensional $3D$ spherical coordinates symmetrical warp drive vector

The application of the new tridimensional $3D$ spherical coordinates symmetric warp drive vector wether in constant or variable speeds to the ADM(Arnowitt-Dresner-Misner) formalism equations in General Relativity using the approach of MTW(Misner-Thorne-Wheeler)resembling the works [3],[4],[5] and [6] will appear in a future work.

References

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