On Lorentz invariant non-locality described by differential equations with derivatives of infinite order

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Abstract

The paper considers Lorentz invariant equations with infinite-order derivatives with soliton solutions. Within the framework of the Lagrangian formalism for fields description and when describing point particles in the form of probability amplitudes, such equations are not considered. If the axiomatic nature of these approaches is abandoned and we limit ourselves only to the requirement of Lorentz invariance of differential equations, then the consideration of such equations allows for non-locality. The paper discusses some general features of nonlocality described by such equations and their differences from the description of nonlocality in the Copenhagen interpretation of the quantum mechanical description. In particular, it is shown that in such a model the question of paradoxes of the Einstein - Podolsky - Rosen type is removed.

Keywords:non-locality, nonlinear differential equations, soliton.

Soliton-like solutions of nonlinear equations are often considered to be extended particle models [1-6]. An example is the Skyrme model [7-9] which describes the internal structure of baryons and light nuclei. The soliton approach to describing particles, compared to describing particles as point objects, has greater freedom in description, since it adds at least one parameter - the spatial dimensions of the particle. Within the extended particle model, the concept of the proper angular momentum of particles also seems more natural. Moreover, in contrast to the point model of particles and their description in the form of probability amplitudes that limit the order of the derivative in the equations, the soliton model does not have such limitations. As will be shown below, this permits writing Lorentz invariant equations that allow non-locality. Usually, non-locality in the equations of motion for a field is described by integro-differential equations [10]. When considering non-locality, in particular in non-local quantum theories, a certain fundamental length or space-time region is also introduced in which the metric relations used in the special theory of relativity do not apply. At distances greater than this fundamental length, the action of such relations is assumed. In this case, questions arise when determining the value of such a length and when transforming 1) the integral when moving to another reference system. If the restriction of the order ($n = \infty$) of the derivative in Lorentz invariant equations is abandoned, then there is no need to specify this length. Let us write equations with infinite-order derivatives that assume the presence of particle-like solitons, for example, of the form

$$(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2})^{(\infty)}(u_{xx} - u_{tt} - sin(u)) = 0 \quad (1)$$

or

$$(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2})(u_{xx} - u_{tt} - sin(u)) + (\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2})^{(\infty)}(u_{xx} - u_{tt} - sin(u)) = 0 \quad (2)$$

These equations will be Lorentz invariant, since each differential operator in them is Lorentz invariant. Equations (1), (2) will have both solutions of the Sin-Gordon equation and solutions in the form of waves with unit speed. That is, such equations do not exclude interaction with unit speed. Then, the resulting spatial derivative of infinite order will give a potential opportunity to describe the non-locality of a finite radius with an accuracy of some constant. Indeed, since the derivative of finite even order $2m$ at a point depends on the field at another point at a distance m of infinitely small $m/N, N \to \infty$, then by tending $m \to \infty$ a finite radius of dependence is obtained. But such nonlocality will not result in instantaneous transmission of information about the change in the field at any point, since in addition to the spatial derivatives of infinite order in equations of type (1), it is also necessary to take into account the contribution from the time derivatives of the same orders. Thus, the change in the field at a point will also depend on the prehistory of the field's behavior in some neighborhood. The spatial radius of this neighborhood L and the time of the current prehistory T will be related as , where c is the speed of light (in the equations described here $c = 1$). If we assume that in order to observe the spatial nonlocality L, it is necessary to conduct, in order to accumulate data, an experiment of duration $T \simeq L/c$, then such a model will not have consequences such as the EinsteinвЂ“PodolskyвЂ“Rosen paradox. Also, within the $3D + 1$ soliton model similar to (1), (2) in which the soliton particle will "feel"some finite spatio-temporal neighborhood, as a kind of a pilot wave, one can try to explain the double-slit interference of particles and similar effects. Thus, the presented model is closer to the de Broglie-Bohm theory [11]. The property of equality of spatial and temporal derivatives of equations of type (1), (2) is of interest and relevance, given the latest experiments with interference in the time domain [12]. Given this equality, the consequence of this model will be the termination of the observation of nonlocality effects when a certain time limit between observations of individual particles is exceeded, since the spatial radius of nonlocality L is finite here. Besides, the model of the form

$$(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2})^{(n)}(u_{xx} - u_{tt} - sin(u)) = 0 \quad (3)$$

is also of interest for a finite order of the derivative n, since it has solutions both in the form of solitons of the Sin-Gordon equation and in the form of "light-like"waves with a speed of 1, within one scalar field. Further study of the interaction of these solutions from the point of view of the authors is interesting from a methodological point of view.

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