

Exploring EEG Dynamics Through Markov Chain Analysis

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ABSTRACT

A Markov chain (MC) is a mathematical model used to describe a system where the probability of moving to the next state depends solely on the current state and not on the sequence of the preceding states. A Markov blanket (MB) for a node includes its parents, children and other parents of its children, capturing the minimal set of nodes required to make the node conditionally independent from the rest of the network. We examined EEG data from healthy individuals to assess MC and MB connectivity patterns associated with two representative electrodes. The electrode FP1, associated with cognitive functions, displayed connections predominantly with frontal and central regions. The electrode C3, located in the primary motor cortex, displayed connections with bilateral motor and parietal regions. The two electrodes had shared connections, highlighting integration between cognitive and motor networks, while also retaining distinct connections that underscored their specialized roles and functions. Temporal analysis demonstrated significant MB fluctuations across time segments, highlighting phases of increased neural reorganization and stability. Entropy analysis showed significant variability in MC and MB dynamics over time. FP1 exhibited greater entropy variability, reflecting its neural flexibility and involvement in cognitive processes, while C3 showed more stable entropy patterns, aligning with its motor-related functionality. We demonstrate the utility of MC and MBs in capturing the dynamic complexity of the nervous activity, underscoring the distinct and overlapping roles of brain regions in balancing dynamic flexibility and functional specialization. Our findings have implications for cognitive neuroscience and brain-computer interface design.

KEYWORDS: Neural State Transitions; Stochastic Modeling; Functional Brain Connectivity; Cognitive State Analysis; Brain Signal Classification.

INTRODUCTION

The human brain is a highly dynamic and interconnected system, characterized by complex interactions between neural regions (Bullmore and Sporns, 2009; Ji et al., 2019; Leitgeb et al., 2020). Electroencephalography (EEG) provides a powerful tool to assess the temporal and spatial patterns of brain activity. Yet, analyzing EEG data requires robust methods capable of capturing both the dynamic changes and the underlying relationships between brain regions. Although EEG analysis techniques like Fourier transforms and time-frequency methods provide valuable insights into the spectral and temporal characteristics of neuronal signals, they may fall short in capturing the dynamic transitions between neural states (Bhardwaj et al., 2021; Morales and Bowers, 2022).

One promising approach involves the application of Markov chains (MC) and Markov blanket (MB) analysis. It provides a mathematical framework for the assessment of conditional dependencies, functional connectivity and temporal evolution of brain activity (Gao and Ji, 2017). This analysis models the probabilistic dynamics of state transitions, where the transition between states is governed by defined probabilities. The likelihood of moving to the next state depends solely on the current state and not on prior sequences (Hajihashemi and Aghababaei Samani, 2022; Luu et al., 2024). Formally, a Markov chain consists of a set of states and a transition probability matrix, each element of the matrix representing the probability of transitioning from one state to another in a single step (Aghababaei Samani, 2022). If the states are denoted as S_1, S_2, \dots, S_n , then the element P_{ij} represents the probability of transitioning from state S_i to state S_j . The sum of probabilities for all transitions from a given state is 1, ensuring a valid probability distribution (Alexeev et al., 2020). The simplicity and versatility of MB and MCs make them a foundational tool in stochastic modeling, providing insights into processes governed by randomness and transition dynamics. Applications of MCs and MBs span diverse fields, including physics, computer science, biology, medicine and economics (Kirchhoff et al., 2017; Wang et al., 2017; Yon and Corlett, 2022). They can be used to model weather patterns, stock market fluctuations, molecular interactions and even human language in text prediction systems (Friston et al., 2021). These methods have also been applied in neuroscience to evaluate interactions between internal and external components of a system such as the brain, providing insights, e.g., into the dynamic interplay between active inference and the environment (Parr 2020; Raja et al., 2020). Based upon the canonical micro-circuitry employed in empirical studies, MB was also used to assess brain architectures at multiple scales, enabling the partitioning of neural systems into single neurons, brain regions and brain-wide networks (Hipólito et al., 2021).

When MCs are applied to EEG dynamics, each state represents a distinct configuration of neural activity and the transitions capture the evolving patterns of brain connectivity over time. In an EEG context, MB analysis identifies the

minimal set of variables (i.e., the minimal number of electrodes) that render a specific variable conditionally independent from the rest of the network. This framework enables researchers to simplify EEG networks by reducing their complexity while retaining the essential dependencies and interactions critical for understanding neural dynamics. This implies that the EEG activity of a specific electrode can be effectively explained by its MB, which encompasses its direct interactions within the network, including its parents, children and co-parents.

This study aimed to investigate EEG dynamics by integrating MC analysis and MBs. By focusing on the two specific electrodes FP1 and C3, we explored how connectivity differs between cognitive and motor regions. We showed that, by applying MC analysis to EEG data, it is feasible to investigate patterns of neural state transitions, their temporal dependencies and their potential relevance to cognitive processes. Further, temporal segmentation of EEG data enabled the examination of how microstates evolve over time, uncovering periods of stability interspersed with phases of reorganization. By comparing the average entropy across electrodes, we also identified regional differences in brain dynamics. We conclude that the combination of MB and MC analysis and temporal entropy calculations might provide a novel framework for investigating the complexity and variability of neural interactions.

METHODS

Participants and EEG Data Acquisition. EEG data were collected from ten right-handed healthy volunteers (mean age: 20.1 years; SD = 1.1; range = 18–22 years; males: 5). For details regarding the patients and the EEG techniques used in this study, refer to Jaušovec and Jaušovec (2005) and Tozzi et al. (2021). Signals were recorded using a 64-channel EEG system, following standard electrode placement guidelines (10-20 system). The data were preprocessed to remove artifacts such as eye blinks and muscle movements using independent component analysis and band-pass filtering (0.5-50 Hz).

Markov chain and Markov blanket analysis. The MC analysis of EEG data started with the definition of the state space, which represents the set of all possible configurations of neural activity. States were determined using k-means clustering applied to features extracted from short time windows of the EEG signals, including power spectral density and signal amplitudes. Next, we estimated the state transition matrix to capture the probabilities of transitioning from one state to another. To enhance robustness, the probabilities were computed separately for different datasets and averaged across participants. Functional connectivity among electrodes was examined by analyzing the co-occurrence of states across EEG channels. To identify MBs in EEG data, an analysis of conditional dependencies among the EEG channels was performed, with each electrode treated as a node in a Bayesian network. The MB for a target node included its parents, children and other parents of its children, capturing the minimal set of nodes required to make the target node conditionally independent of the rest. Two selected representative EEG channels, namely FP1 and C3, were chosen for causal analysis. The selection of the two channels was based on their well-established roles in executive functions/decision-making processes for FP1 and in motor planning/execution for C3 (Miasnikova et al., 2022; Hamed et al., 2025). The Graphical Lasso algorithm was then employed, i.e., a regularized covariance estimation technique that infers sparse precision matrices (Grechkin, et al., 2015; Maruhashi et al., 2024). This method computes statistically significant partial correlations among variables, enabling the identification of direct connections.

Dynamics. To compare the MB dynamics for FP1 and C3, three dynamical metrics were analyzed, namely MB size, shared/unique connections and network centrality. The size of the MB, determined by the number of electrodes it encompassed, reflected the degree of localized functional dependencies within the network. A larger MB suggested broader integration with surrounding brain regions, pointing to more extensive involvement in neural processes. Our analysis also quantified the extent of shared connections involving overlapping electrodes, highlighting potential cross-functional integration between regions associated with cognitive and motor functions. Conversely, the examination of unique connections revealed electrodes exclusive to the MB of either FP1 or C3, providing insights into their functional specialization. Network centrality measures such as degree centrality assessed the relative importance of nodes within the MBs. Nodes with high centrality in the FP1 network likely represent core hubs for cognitive control, whereas those in the C3 network may indicate key regions essential for motor coordination.

Temporal changes in connectivity. To analyze temporal changes in connectivity, the MBs evolution over time was explored using segmented EEG recordings. This approach revealed how connectivity patterns shifted dynamically, offering insights into neural processes and adaptability during different time intervals. At first, the EEG data were segmented into discrete time windows. Each dataset was divided into 1-second non-overlapping intervals, providing a temporal resolution that balanced detailed granularity with computational efficiency. Next, MB analysis was conducted on each time segment for the two selected electrodes FP1 and C3.

Entropy. Conditional dependencies, i.e., MB membership, were combined with information-theoretic measures, i.e., entropy (Smart et al., 2024). The entropy within a MB quantified the uncertainty or diversity of information exchanged

between a target electrode and the local network of its immediate neighbors within localized brain regions (Steiner and Tuljapurkar, 2020). The reduction in uncertainty represents how efficiently the activity of a node is explained by its MB, quantified as

$$H(X) - H(X|MB),$$

where $H(X)$ is the entropy of the target node and $H(X|MB)$ is the conditional entropy given its MB.

The entropy calculation was derived from the variability of activity over time among the electrodes within the MB. The EEG time-series data were segmented into fixed-length time windows, each segment consisting of 1000 data points with no overlap between successive windows. The activity (signal amplitude) of the constituent electrodes for each identified MB was extracted for the specified time segment. Subsequently, the variability of each electrode's activity within the MB was analysed to estimate its relative contribution to the overall variability. Variance was calculated for each electrode and a probability distribution was created by normalizing these variances. The normalized variance for each electrode was determined by dividing its individual variance by the total variance summed across all electrodes within the MB. Then, Shannon entropy was computed from this probability distribution to measure the uncertainty or diversity of variability within the MB.

Higher entropy within a MB indicated more evenly distributed activity among the electrodes, suggesting dynamic and flexible interactions. In turn, lower entropy suggested predictable interactions, pointing towards stable or rigid connectivity patterns, dominance by fewer electrodes, more specialized or stable interactions.

Statistics. Chi-squared tests and Fisher's exact tests were used to assess statistically significant differences in frequencies among electrodes. Statistical tests such as t-tests or Mann-Whitney U tests were used to analyze transition matrices, connectivity patterns and interaction strengths within the MBs of FP1 and C3, offering insights into their distinct and overlapping roles in neural networks.

Graphical rendering. The direct connections between the two electrodes FP1 and C3 were illustrated by network graphs visualized over a 2D schematic of the standard 10–20 EEG electrode system. Line plots, bar plots or heatmaps provided insights in connectivity changes, transient connections and stable patterns persisting across time.

Software and Tools. All analyses were conducted using Python (v3.8) with libraries including NumPy, Pandas, SciPy and NetworkX for numerical computations, data manipulation, statistical testing and network visualization. Preprocessing of EEG data was performed using MNE-Python. Graphical Lasso was implemented using the `sklearn.covariance` module. Visualizations of electrode mountings and MB networks were generated using Matplotlib and NetworkX.

RESULTS

The Markov blankets identified through Graphical Lasso unveiled distinct connectivity patterns for the target electrodes FP1 and C3, underscoring the distinct functional networks associated with the two electrodes and the specificity of MB connectivity in reflecting neural processes (**Figure 1**). For FP1, a frontal electrode associated with cognitive functions, MB predominantly included connections with neighboring frontal and central regions such as FP2, F3 and FZ, which aligned with the functional significance of FP1 in executive functions and decision-making processes. In contrast, C3, located in the primary motor cortex, exhibited a MB encompassing bilateral motor regions such as C4 and CZ and parietal areas such as P3, consistent with the well-established role of C3 in motor planning and execution. These findings reflect the functional specialization of these two regions, with FP1 supporting cognitive integration and C3 focusing on sensorimotor coordination.

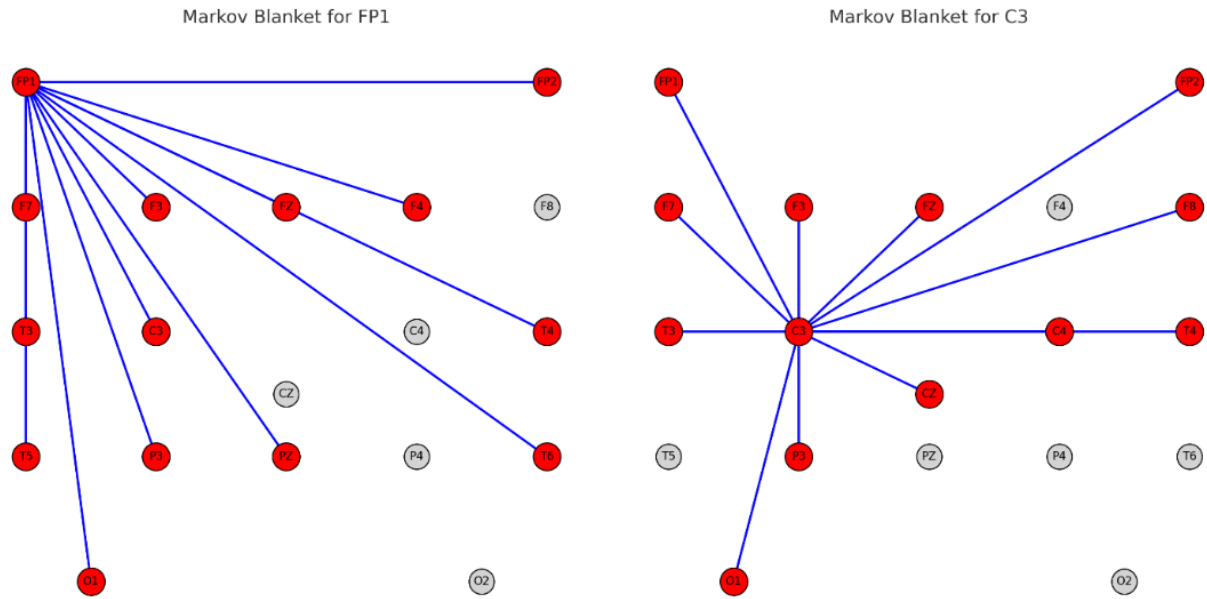


Figure 1. Markov blanket connectivity patterns for the electrodes FP1 and C3. The red nodes represent the EEG electrodes that are part of the respective Markov blankets for FP1 and C3. The blue edges denote conditional dependencies between FP1, C3 and their associated electrodes, depicted as significant partial correlations derived from the precision matrix.

Dynamics, temporal analysis, entropy. A comprehensive analysis of the three dynamical metrics (i.e., MB size, shared/unique connections and network centrality) provided a detailed understanding of functional dynamics and integration within broader brain systems, demonstrating a balance between integration and specialization (**Figure 2A**). FP1's unique connections were linked to cognitive processes such as executive functioning and decision-making, while C3's unique connections emphasized its role in motor planning and execution. The size of the MB showed that FP1 had 13 connections, and C3 had 12 connections, pointing towards comparable levels of localized functional dependencies. Eight electrodes were shared between FP1 and C3, namely FP2, T4, FZ, O1, P3, F7, T3 and F3. This overlap highlighted functional integration between cognitive regions in the frontal lobe and motor regions in the central areas, suggesting interconnected processes. The analysis also revealed unique connections for each electrode. FP1 had five unique connections, namely C3, F4, T5, PZ and T6, which pointed to its role in cognitive functions. In contrast, C3 had four unique connections, namely C4, F8, CZ and FP1, emphasizing its specialization in motor planning and execution.

Notable variability in the number of connections was found over time for both FP1 and C3, reflecting dynamic shifts in localized functional dependencies (**Figure 2B**). This variability suggested that the neural activity associated with the two electrodes was influenced by fluctuations in neural states. Additionally, certain time windows exhibited consistent connectivity patterns reflecting stable neural activity, while other time windows displayed significant variability, potentially corresponding to periods of transitions between distinct neuronal states.

Temporal entropy analysis showed significant variability in MB dynamics over time, underscoring the specialized functional roles of these electrodes (**Figure 2C**). For FP1, entropy demonstrated higher variability, with peaks indicating periods of dynamic reorganization. In contrast, C3 exhibited more stable entropy patterns, reflecting its consistent role in motor output. Over time, certain segments showed peaks in entropy, indicating periods of increased variability, potential network reorganization and adaptive changes in the underlying neural processes. During the initial time segments, both electrodes exhibited lower entropy values, indicating a more stable and predictable neural network configuration with limited variability in their MB activity.

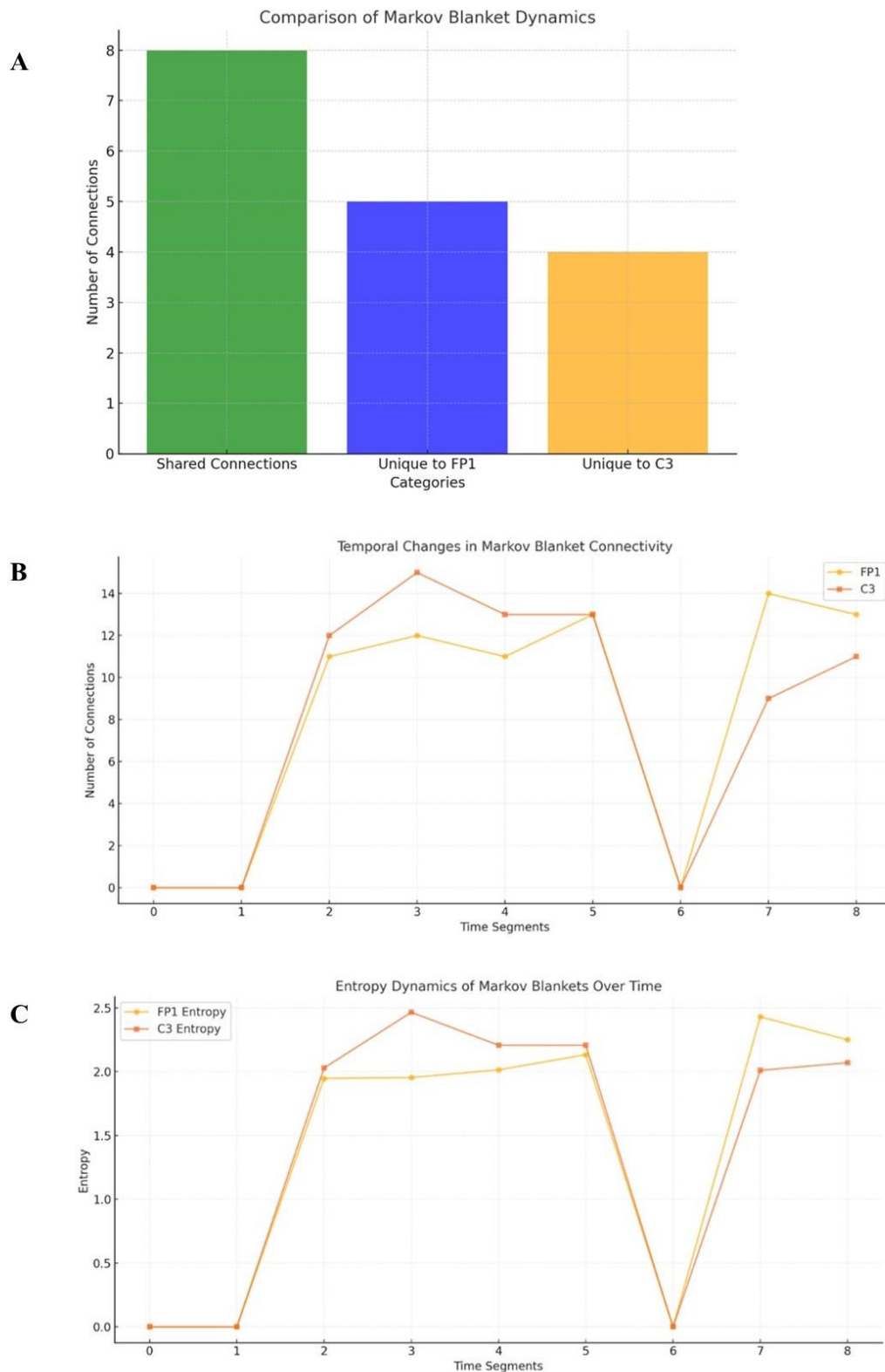


Figure 2. Markov Blanket analysis of the two electrodes FP1 and C3. **Figure 2A.** Comparison of the Markov Blanket dynamics of FP1 and C3. The shared connections underscore the collaborative interaction between cognitive and motor regions, while the unique connections reflect the distinct functional roles of FP1 in cognitive processing and C3 in motor coordination. **Figure 2B.** Temporal changes in Markov blankets connectivity. The line plot illustrates the temporal variations in the number of connections, representing the size of the Markov blankets, across different time segments. **Figure 2C.** Entropy dynamics of Markov blankets over time. The line plot illustrates the entropy dynamics of FP1 and C3 across various time segments. Frontal regions displayed higher average entropy compared to occipital and motor regions, suggesting greater neural flexibility and integration in cognitive areas.

CONCLUSIONS

The application of MC and MB analysis to EEG traces provides a unique perspective to assess the temporal dynamics of the brain activity. Its adaptability extends to multimodal data, enhancing its utility across diverse research and clinical contexts. Graphical Lasso, used to estimate conditional dependencies and differences in connectivity patterns, is computationally efficient for high-dimensional data, making it well-suited for EEG datasets. By reducing the complexity of EEG networks to a minimal set of functionally relevant connections, the MC and MB approach enables focused analyses of localized dynamics. Temporal segmentation allows tracking changes in connectivity and entropy over time, capturing state transitions and moments of neural reorganization. Connectivity variability in EEG MBs may reveal critical aspects of neural dynamics. High variability points towards a flexible system capable of shifting between states or configurations like task engagement and cognitive load, whereas low variability suggests stability, beneficial for sustained attention or repetitive tasks. These patterns also underscore the interplay between functional segregation and integration in the brain. Peaks in connectivity variability likely represent moments of heightened integration, while troughs indicate functional segregation where regions operate independently to perform specialized tasks.

Entropy measures within MBs quantify variability and uncertainty, providing insights into neural flexibility and stability. Higher entropy in frontal regions suggests diverse interactions supporting cognitive tasks like attention and decision-making, while lower entropy in motor areas suggests specialized, predictable interactions essential for precise motor control. Entropy also highlights transitions between cognitive states, with increasing entropy signaling reorganization pointing towards sustained task-specific processing. Contributions of individual MBs to global network entropy link local dynamics to broader organizational principles, enhancing understanding of the brain's information-theoretic properties. MB provides a way to study limited areas of the brain. For instance, the MB of electrode C3, critical for sensorimotor functions, provides significant insights into the network dynamics and functional connectivity of the primary motor cortex. The MB of the electrode C3 encompasses nodes like C4, F3 and CZ, underscoring interactions related to motor planning, execution and bilateral coordination. Also, connections to regions like P3 and O1 suggest sensory feedback integration and visuomotor processing. Enhanced connectivity within the MB may relate to advanced motor skills, whereas aberrant patterns could suggest impairments.

The localized dynamics detected by the graph-based nature of MC and MBs provide a foundation for targeted analyses, facilitating applications in neuroscience and clinical contexts. For brain-computer interfaces (BCIs) this is particularly valuable, as C3 often serves as a critical node for decoding motor intentions (Zanona et al., 2023). MBs play a pivotal role in BCIs by structuring and optimizing feature selection from high-dimensional EEG data (Ivanov et al., 2023). A MB represents the minimal set of variables needed to describe the target electrode's activity, enabling BCIs to focus on the most relevant connections while avoiding redundant or noisy inputs. This efficiency enhances robustness, training data requirements, computational performance and decoding accuracy, particularly in motor BCIs, where electrodes like C3 and C4 are crucial for predicting motor intentions, and in cognitive BCIs, where electrodes such as FP1 and FZ are key for monitoring attention or mental workload (Oliver et al., 2012; Torre Tresols et al., 2024). By focusing on the most informative features, BCIs can monitor changes in C3's MB during motor tasks to refine control or provide neurofeedback to train users in modulating connectivity. Beyond EEG, MB analysis might extend to multimodal BCIs, integrating EEG with signals like fMRI or EMG to identify cross-modal dependencies.

Despite its advantages, the MC and MB analysis has limitations. The accuracy of identifying MBs depends on statistical models like Graphical Lasso which are sensitive to noise and parameter selection (Lingjærde et al., 2021). EEG's inherent noisiness and susceptibility to artifacts can affect results. The method primarily relies on linear relationships, potentially overlooking nonlinear dynamics crucial to brain connectivity. Still, temporal segmentation introduces challenges in selecting appropriate window sizes, influencing resolution and interpretation. Individual variability in brain anatomy and network organization also complicates generalizations. With a sample size of ten participants, our findings may lack broader applicability. Further, entropy calculations, while insightful, may not fully capture higher-order network properties or complex information transfer. Addressing these limitations through nonlinear modeling, larger samples and multimodal data integration could enhance robustness and interpretability.

In conclusion, Markov chains and Markov blankets provide a powerful framework for investigating EEG dynamics, uncovering patterns of state transitions and their modulation by specific demands. This approach reveals the dynamic complexity of brain activity, offering localized and temporal insights into neural connectivity. Our findings have implications for the assessment of functional specialization, cognitive differences and clinical requirements, advancing both neuroscience research and technologies like BCIs.

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DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.

Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.

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