Simple Proof of the Riemann hypothesis

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1. Abstract

The Riemann hypothesis states that the real part of all non-trivial zeros of the Riemann zeta function is $\frac{1}{2}$ in the critical strip. In this paper we proved the hypothesis by using the properties of the Riemann zeta functional equation at $\zeta\,(\mathrm{s}) = 0$ and also using the integral representation (Mellin transformation) of the zeta function.

2. Proof of the Riemann Hypothesis

The functional equation of the analytically continued Riemann Zeta function shows that;

$$\zeta\,(\mathrm{s}) = \; 2^{s}\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\zeta\,(1-\mathrm{s})\Gamma\,(1-\mathrm{s}).$$

For $\zeta\,(\mathrm{s}) = 0$ whereby $s = a + \boldsymbol{i}b$,

$$\zeta\,(\mathrm{s}) = \; \zeta\,(1-\mathrm{s}).$$

From the Mellin transformation of the Riemann zeta function;

$$\zeta(\mathrm{s}) = \frac{1}{\Gamma\,(\mathrm{s})}\int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt.$$

When $\zeta\,(\mathrm{s}) = 0$,

$$\int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt = 0.\,(equation\ 1)$$

Let's consider $\zeta\,(1-\bar{s}) = \zeta\,(\bar{s}) = 0$ for the conjugate non-trivial zero $\bar{s} = a - \boldsymbol{i}b$.

$$\zeta\,(1-\bar{s}) = \frac{1}{\Gamma\,(1-\bar{s})}\int_0^\infty \frac{t^{-\bar{s}}}{e^t - 1}\,dt$$

$$\int_0^\infty \frac{t^{-\bar{s}}}{e^t - 1}\,dt = 0.\,(equation\ 2)$$

But $(equation\ 1) = (equation\ 2)$. Implying that,

$$\int_0^\infty \frac{t^{s-1}}{e^t - 1}\,dt = \int_0^\infty \frac{t^{-\bar{s}}}{e^t - 1}\,dt.$$

Comparing both sides of the equation just above gives;

$$s - 1 = -\bar{s}.$$

$$(a + \boldsymbol{i}b) - 1 = -(a - \boldsymbol{i}b)$$

$$a = \frac{\mathbf{1}}{\mathbf{2}}.$$

Hence the real part of all the non-trivial zeros of the Riemann zeta function is $\frac{1}{2}$.

Conclusion

The Riemann hypothesis is correct since we have proven that the real part of all the non-trivial zeros of the Riemann zeta function must be $\frac{\mathbf{1}}{\mathbf{2}}$.