Geometric functions and surface functions

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Author: Ahcene Ait Saadi

Email: ait_saaadi@yahoo.fr

Abstract:

In this paper, I introduce a new concept (new Frame), which allows me to have functions that have another definition. That is to say, in this frame: a function is an application that associates with each element of the starting set E, zero or several images of the arrival set F.

I study in this frame, the derivability of functions, therefore the equation of a tangent to a curve. The integral calculation, I leave it to the young checkers who, surely, will develop this new and original mathematical tool, this in the interest of science and knowledge.

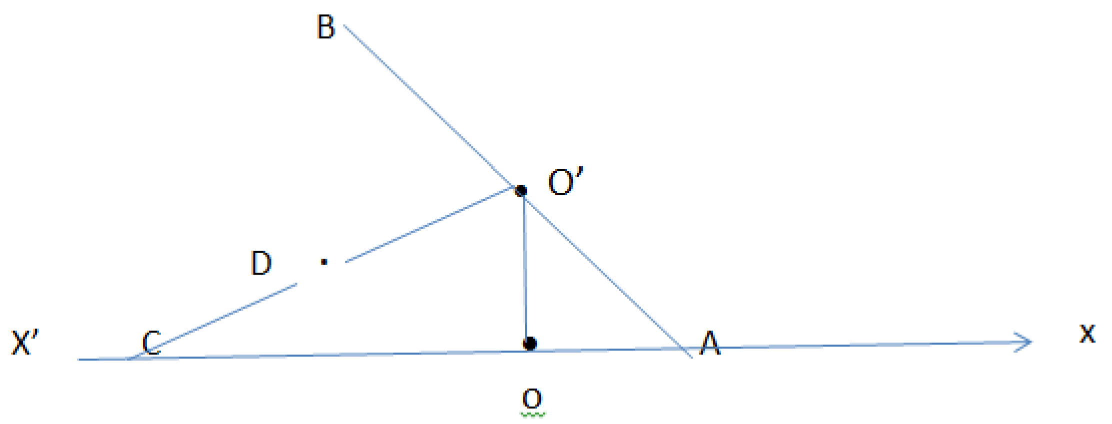
Keywords:

Cartesian frame, derivation, limite of function, l'hospital method

I) <u>Analytical frame in the plan</u>

1.1-Definition:

the analytical frame in the plan, consisting of a horizontal axis (x'ox), of origin (O), called the abscissa axis and a point (O') which is the origin of the ordinates and does not belong to this axis. The distance OO' , is equal to a real value λ $(\lambda \succ 0)$.



Note1.2 : the abscissa of point (A) is OA, has as its image point (B) of the ordinate (O'B)

The abscissa of point (C) is (OC) , has as its image point (D) of the ordinate (O'D)

Sense of the Analytical frame :

-concerning the abscissa, the same process as the Cartesian reference frame.

- concerning the ordinates, the method is as follows:

* If the image is located after the origin O', its ordinate is positive.

Example: $O'B \succ 0$

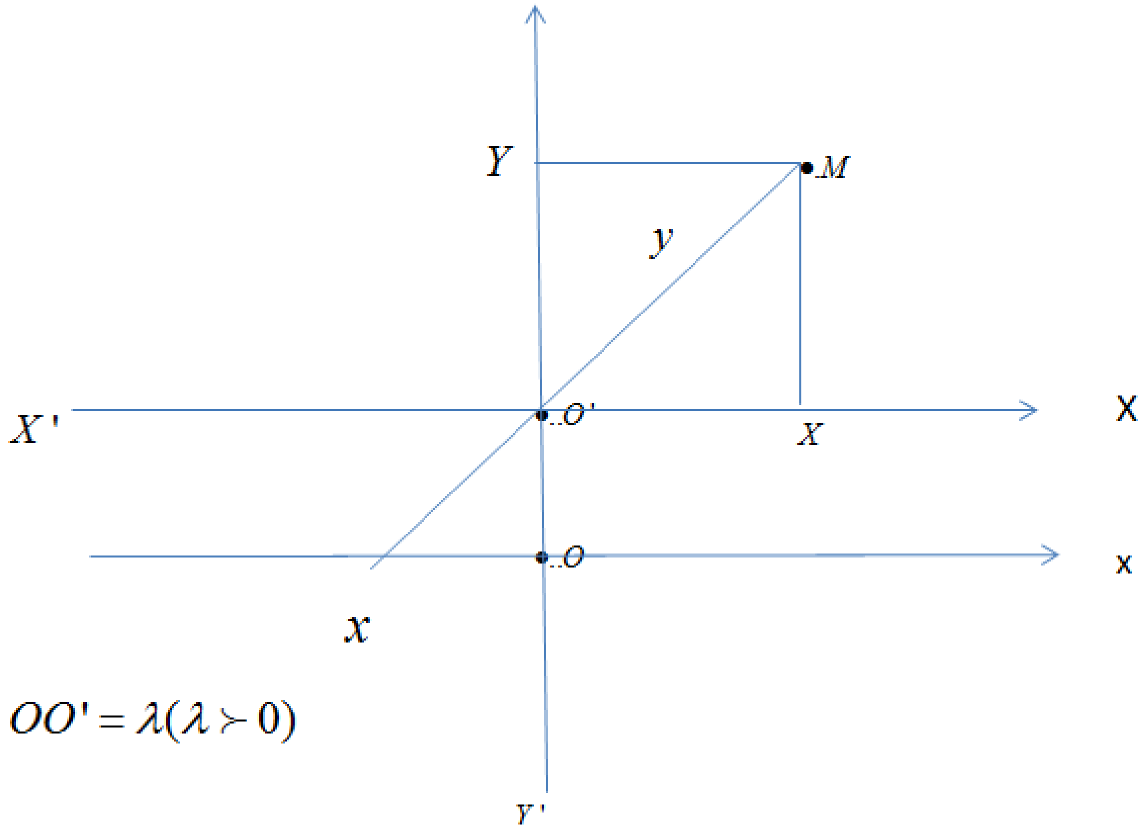
* If the image is located before the origin O', its ordinate is negative.

Example 1.3: $O'D \prec 0$

Coordinates of the points: $O; A; B; C; D$

$O(0,-\lambda)$; $A(0A,-O'A)$; $B(OA,O'B)$; $C(-OC,-O'C)$; $D(-OC,-O'D)$

2.1: Relationship between the Cartesian frame (O',X,Y) and the analytical frame :



The coordinates of the point M in the analytical frame: $M(x, y)$

In this case $x \prec 0; y \succ 0$

The coordinates of the point M in the Cartesian coordinate system: $M(X,Y)$

$$X = \frac{-xy}{\sqrt{\lambda^2 + x^2}} \; ; \; Y = \frac{\lambda y}{\sqrt{\lambda^2 + x^2}}$$

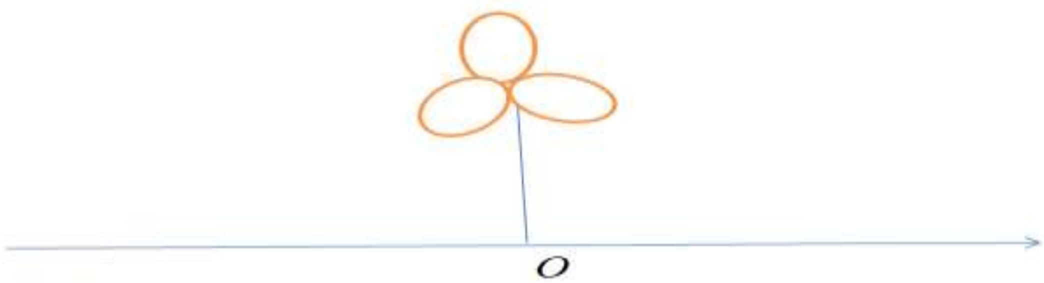
3.1-Equation of the line in the analytical frame :

$Y = aX + b$, is the equation of the line in the Cartesian frame.

Substituting Y and X by their values, we obtain:

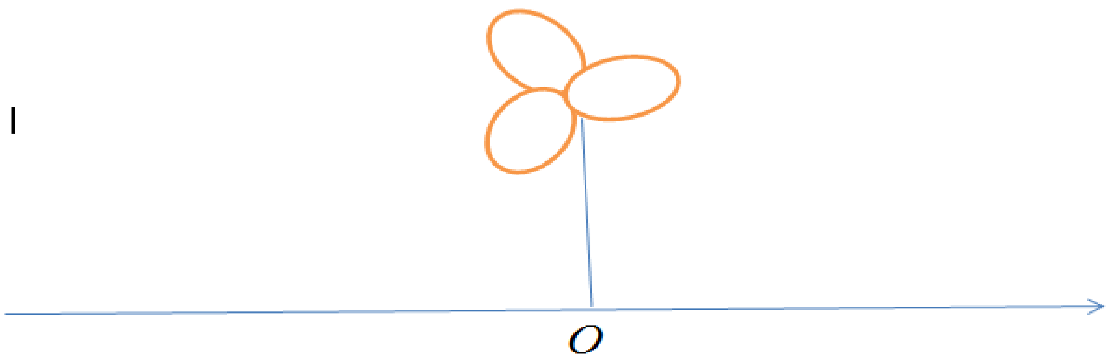
$$\frac{\lambda y}{\sqrt{\lambda^2+x^2}}=\frac{-axy}{\sqrt{\lambda^2+x^2}}+b \Rightarrow y=\frac{b\sqrt{\lambda^2+x^2}}{\lambda+ax}$$ equation of the line in the analytical frame.

3.2: Curve of the function $x \longrightarrow \cos x$ in the analytical frame, $x \in [0;2\pi]$



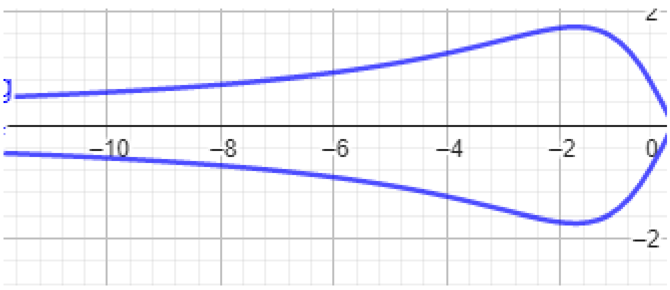
3.3: Curve of the function $x \longrightarrow \sin x$ in the analytical frame, $x \in [0;2\pi]$

I



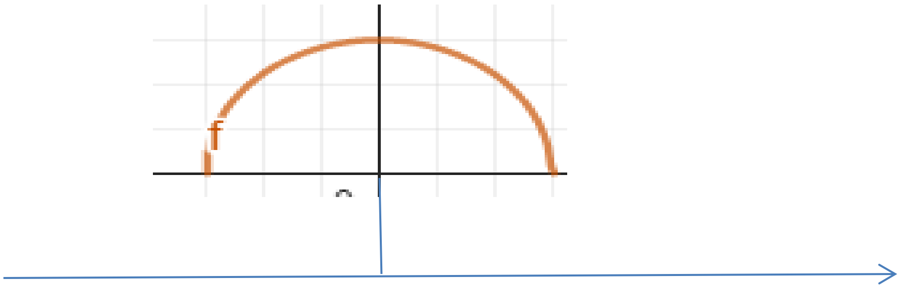
There is a rotation.

3.4: Curve of the function $x \longrightarrow x$ in the analytical frame

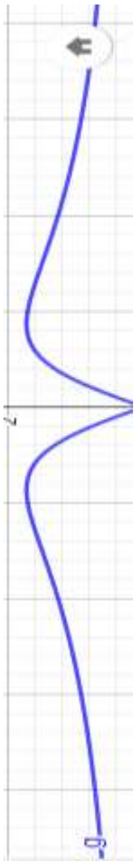


3.5: Curve of the function $f(x)=1$ in the analytical frame

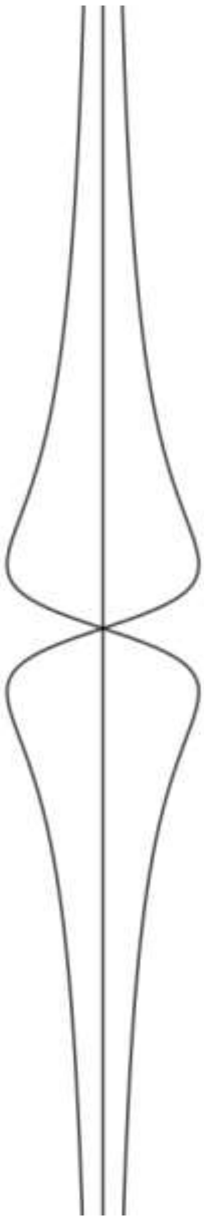
it's a semi-circle of radius 1



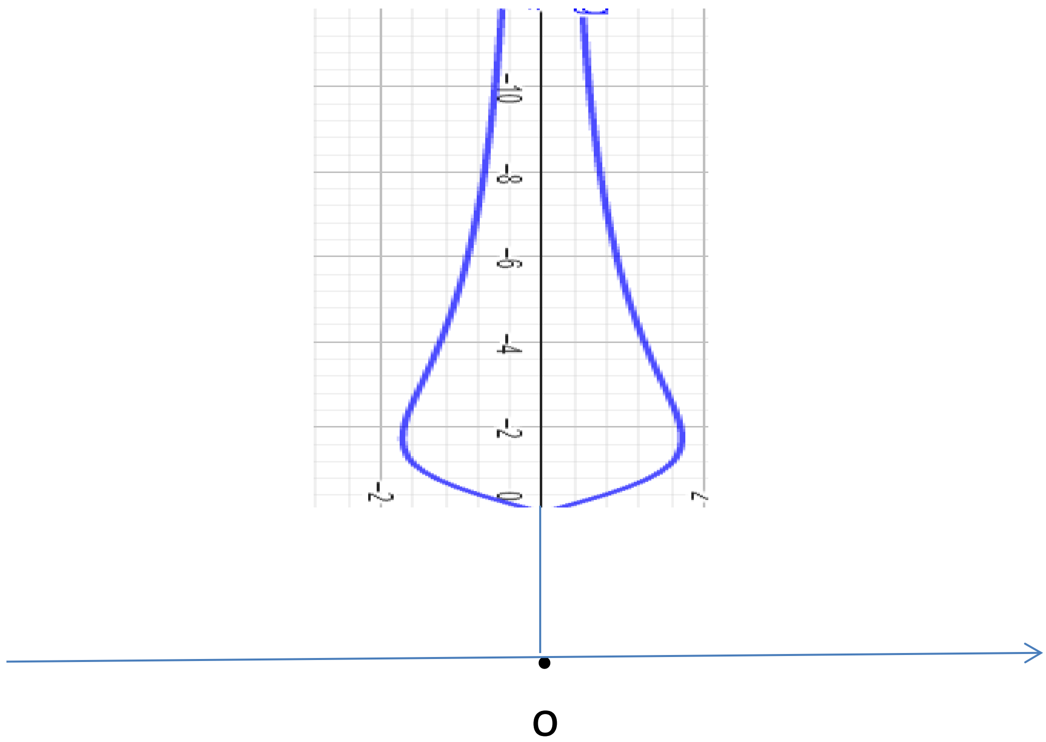
3.6:curve $x \to \frac{1}{x}$ in the analytical frame



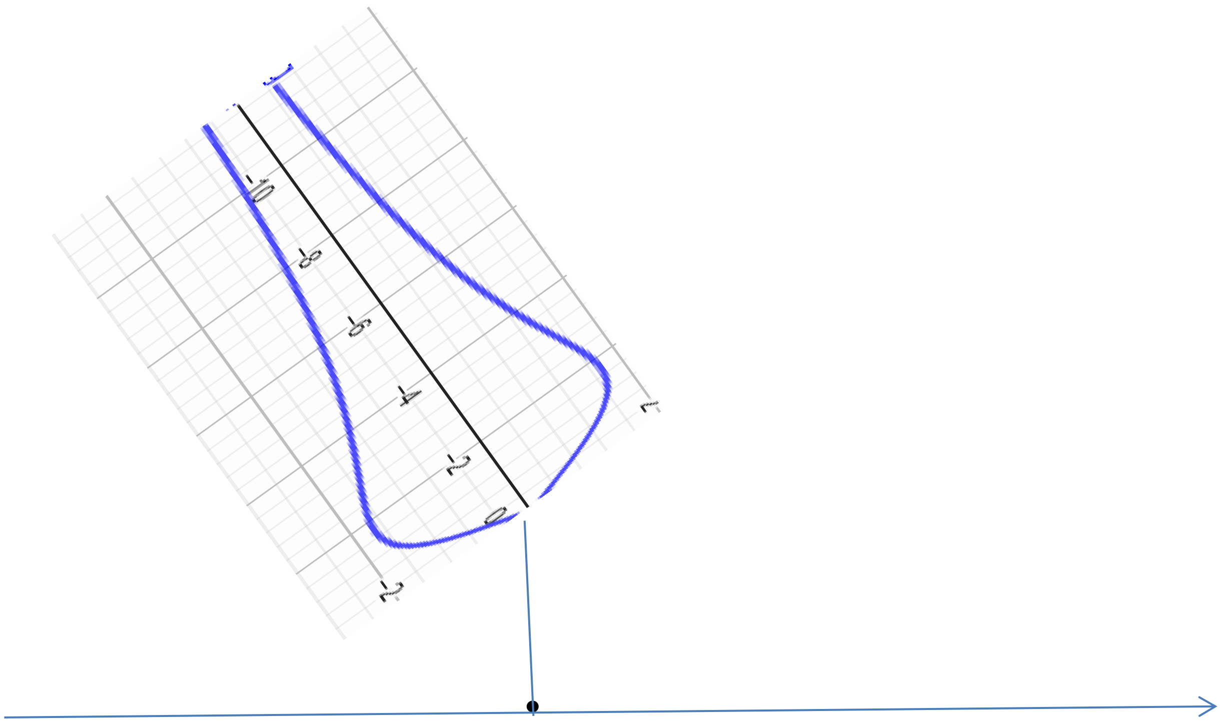
3.7:Curve of the function $x \to \frac{1}{|x|}$ in the analytical frame



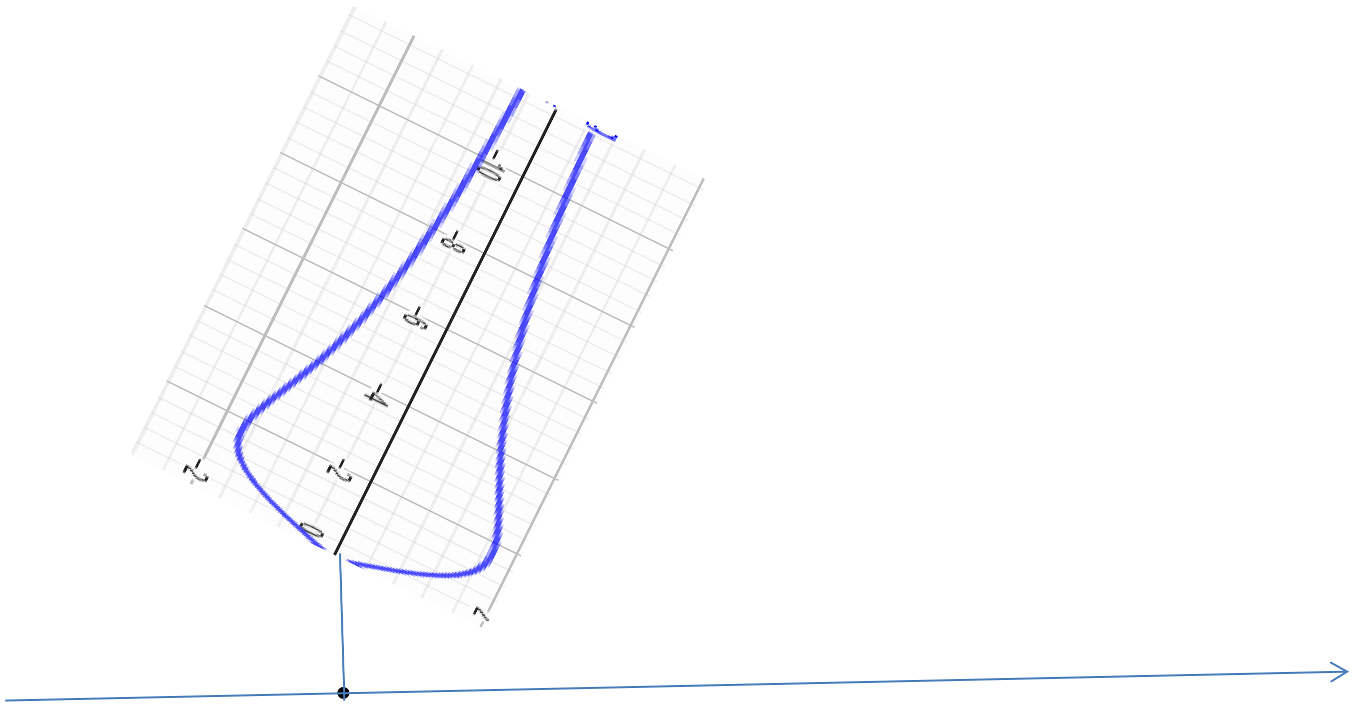
3.8: Curve of the function: $x \to \frac{1}{x^2}$



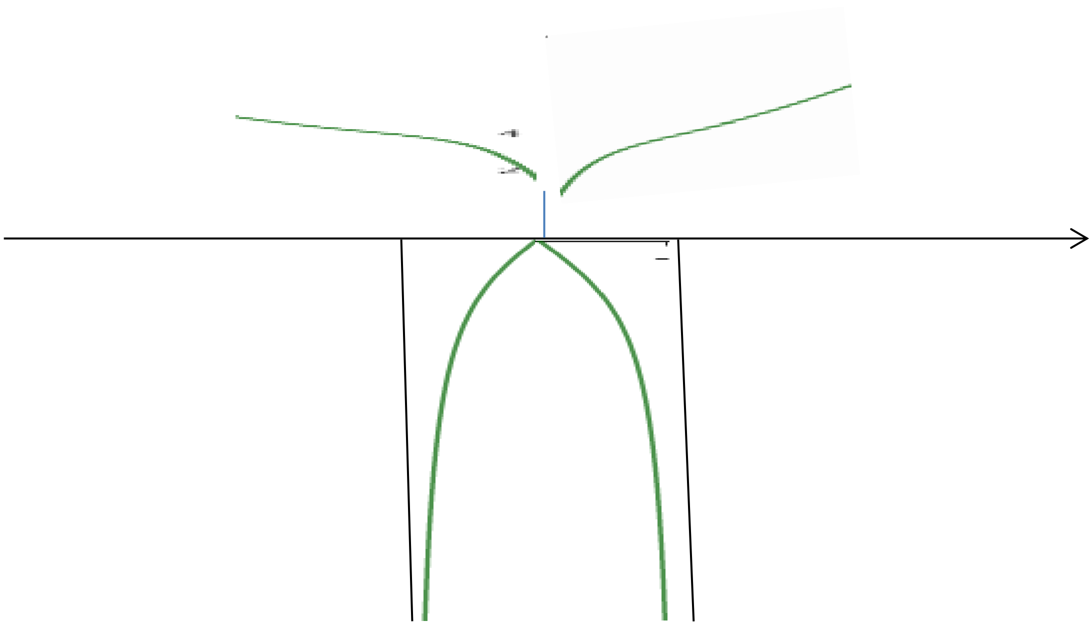
3.9: Curve of the function: $x \to \frac{1}{x-1}$



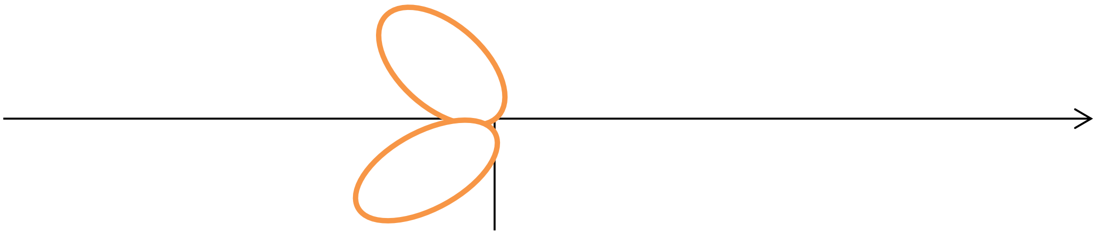
3.10: Curve of the function: $x \to \frac{1}{x+1}$



3.11 : curve $x \to \frac{x}{x^2 - 1}$

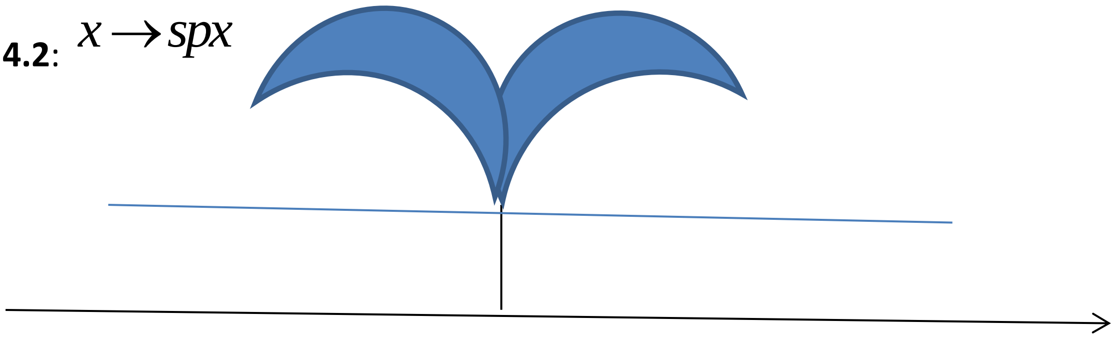


3.12: curve $x \to \frac{x}{x^2+1}$

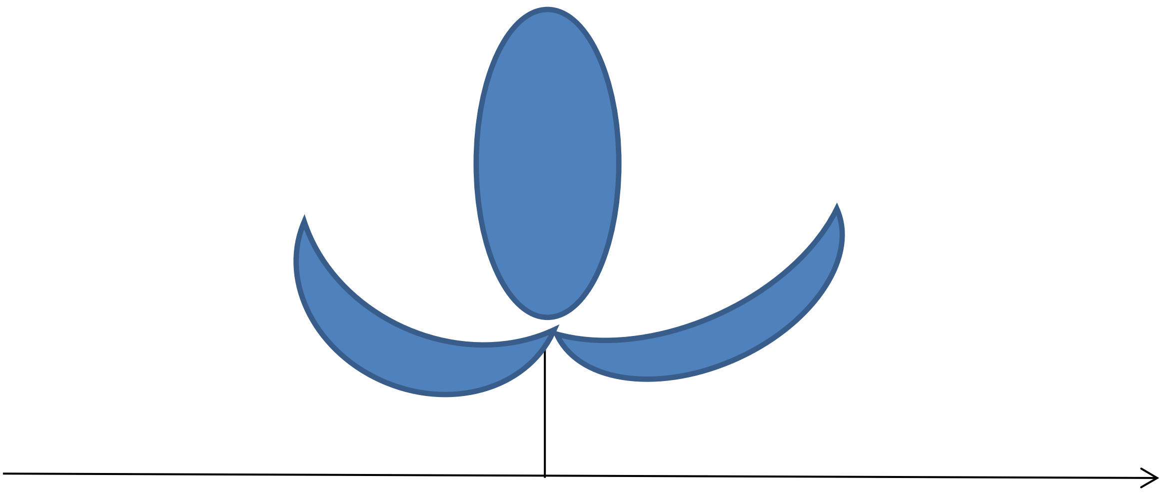


4.1:Surface functions : these are surface curves

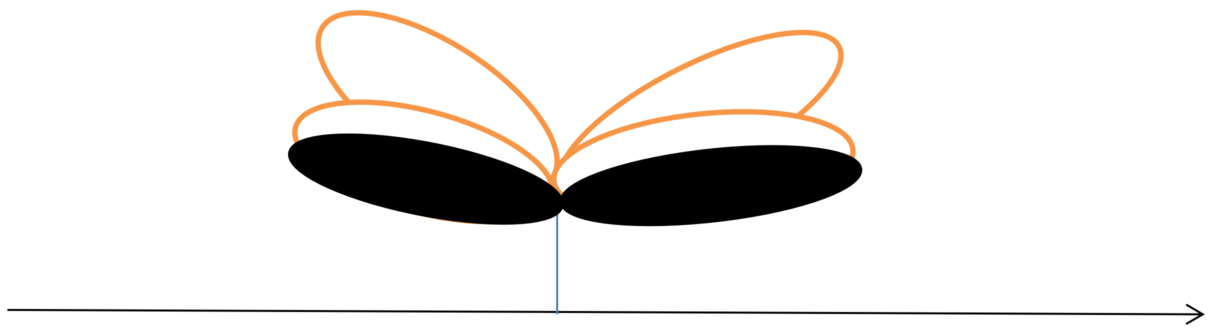
4.2: $x \to spx$



4.3: $x \to cpx$



4.4: $x \rightarrow spxcpx$



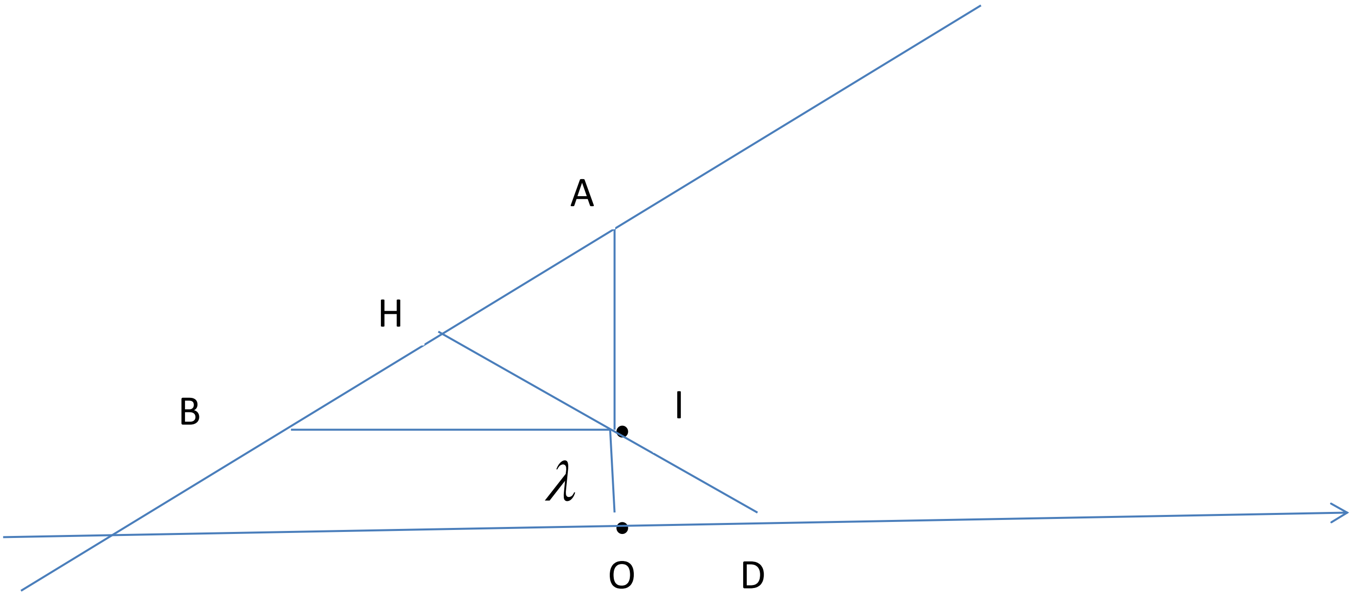
<u>5-1: Equation of the circle</u> of center (a,b) and radius $\sqrt{a^2+b^2}$ in the Cartesian frame.

In the analytic frame: $$y = \frac{2(\lambda b - ax)}{\sqrt{\lambda^2 + x^2}}$$

<u>5- 2: equation of the line in the analytical frame</u> :

$$y = \frac{b\sqrt{\lambda^2 + x^2}}{\lambda + ax}$$

6.1:Applications:_ relationship in a triangle



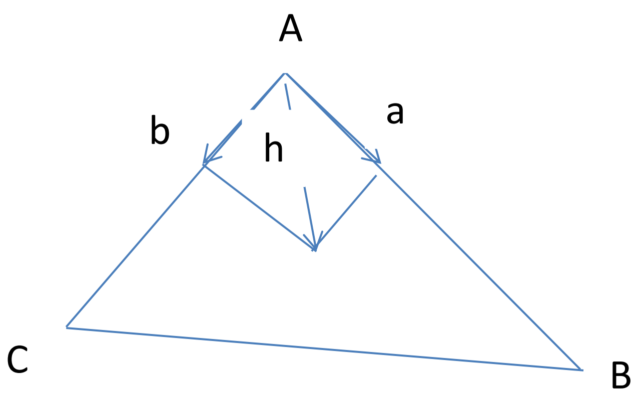
If $x=0$, $IA=\frac{b\sqrt{\lambda^2}}{\lambda}=b$

If $x=\infty$, $y=IB=\frac{b}{a}$

If $x=OD$, $y=\frac{bID}{\lambda+aOD}=IH$

Which gives the relation: $\frac{OI}{IA}+\frac{OD}{IB}=\frac{ID}{IH}$

6.2: Generalization for any triangle

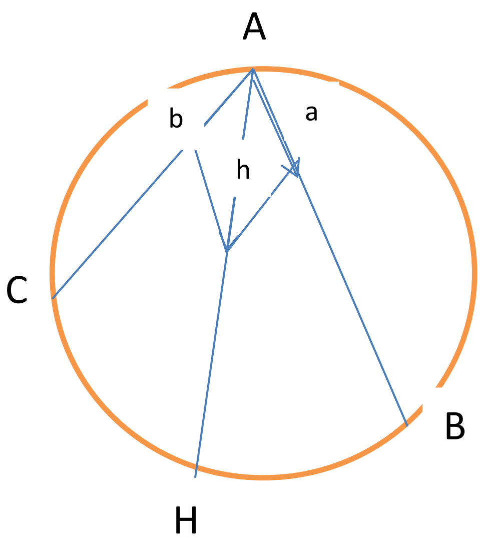


$$\vec{a}+\vec{b}=\vec{h}$$

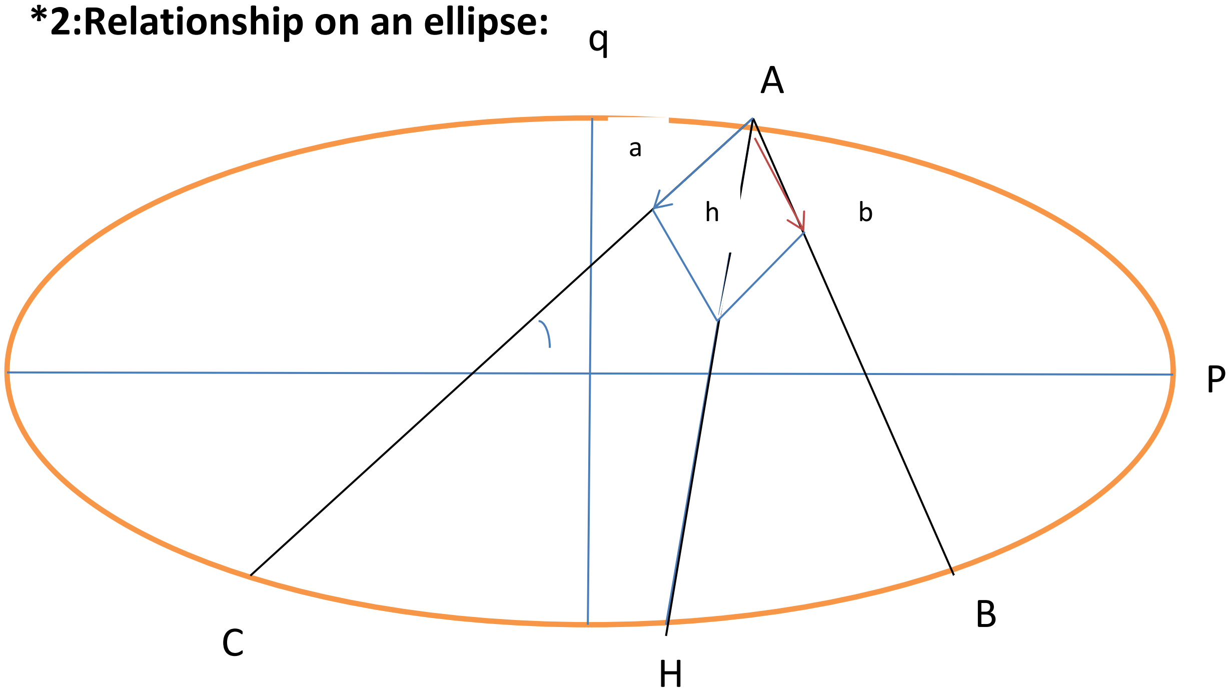
$$\frac{\overline{h}}{\overline{AH}}=\frac{\overline{a}}{\overline{AB}}+\frac{\overline{b}}{\overline{AC}}$$

6.3 -relations on the circle and on the ellipse

*** 1: Relation on the circle:** $\bar{b}\,\overline{AH} = \bar{a}\,\overline{AB} + \bar{b}\,\overline{AC}$



***2:Relationship on an ellipse:**



let be an ellipse of axes p and q.

let be α the angle that the major axis p, makes with AB

β The angle that makes $\vec{a}$ with $\vec{b}$ and $\vec{a}+\vec{b}=\vec{h}$ and $m^2=\frac{p^2}{q^2}$

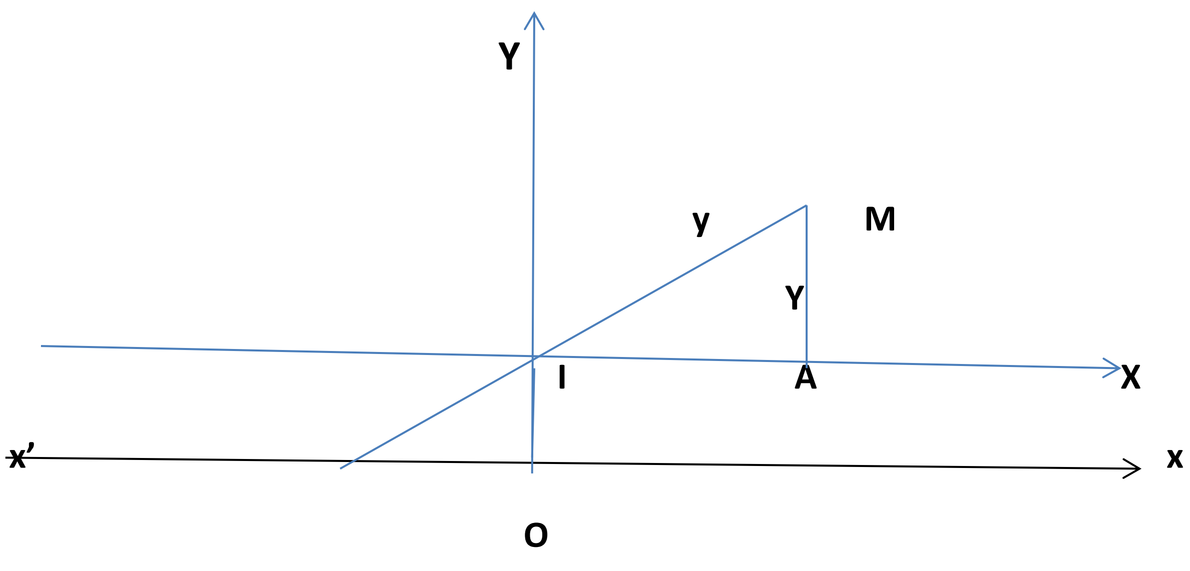
$$\frac{\overline{AH}}{\bar{h}}=\frac{\bar{a}\,\overline{AC}(\cos^2\alpha+m^2\sin^2\alpha)+\bar{b}\,\overline{AB}(\sin^2\alpha+m^2\cos^2\alpha)}{(a^2-2\bar{a}\bar{b}\cos\beta)(\cos^2\alpha+m^2\sin^2\alpha)-b(1-m^2)\sin^2\alpha+b^2(\sin^2\alpha+m^2\cos^2\alpha)}$$

If $\alpha=0$ $\Rightarrow\frac{\overline{AH}}{\bar{h}}=\frac{\bar{a}\,\overline{AC}+m^2\,\bar{b}\,\overline{AB}}{a^2+m^2b^2-2\bar{a}\bar{b}\cos\beta}$

If $m^2=1$ it is the circle $\Rightarrow a^2+m^2b^2-2\vec{a}\vec{b}\cos\beta=\vec{h}$

7.1 Pseudo derivatives in the analytic frame

$oI=\lambda$



Let be an orthonormal frame $(IX.IY)$ and the analytical frame x'ox.

$Y=\frac{\lambda y}{\sqrt{1+x^2}}$; $X=\frac{-xy}{\sqrt{1+x^2}}$; $y=IM=f(x)$ in the analytic frame

$$Y' = \frac{(1+x^2) f'(x) - xf(x)}{(1+x^2)\sqrt{1+x^2}} \; ; X' = -\frac{x(1+x^2) f'(x) + f'x)}{(1+x^2)\sqrt{1+x^2}}$$

$\frac{Y'}{X'} = f_p'(x)$ who is the pseudo derivative

for any function of the form:

$$\lim_{x \to x_0} \frac{\frac{xf(x)}{\sqrt{1+x^2}} - \frac{x_0 f(x_0)}{\sqrt{1+x_0^2}}}{\frac{f(x)}{\sqrt{1+x^2}} \; \frac{f(x_0)}{\sqrt{1+x_0^2}}} = f_p'(x_0)$$

$$f_p'(x) = -\frac{(1+x^2) f'(x) - xf(x)}{x(1+x^2) f'(x) + f(x)}$$

Note 7.2: these pseudo derivatives represent the method of the hospital, with regard to the limits.

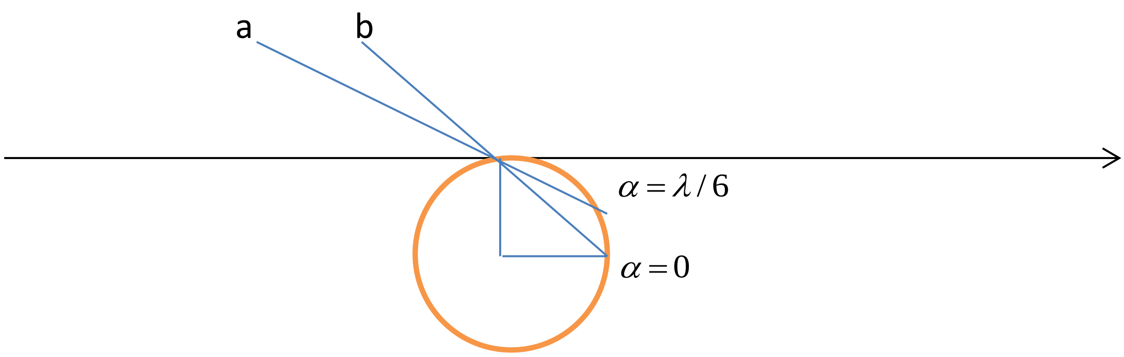
Example when $x \to e$

$$\lim \frac{x^2\sqrt{\ln x} - e^2}{x\sqrt{\ln x} - e} = \frac{2}{3} \; ; \qquad \lim \frac{x-e}{x\sqrt{\ln x} - e} = \frac{5e}{3}$$

II) Trigonometric Frame

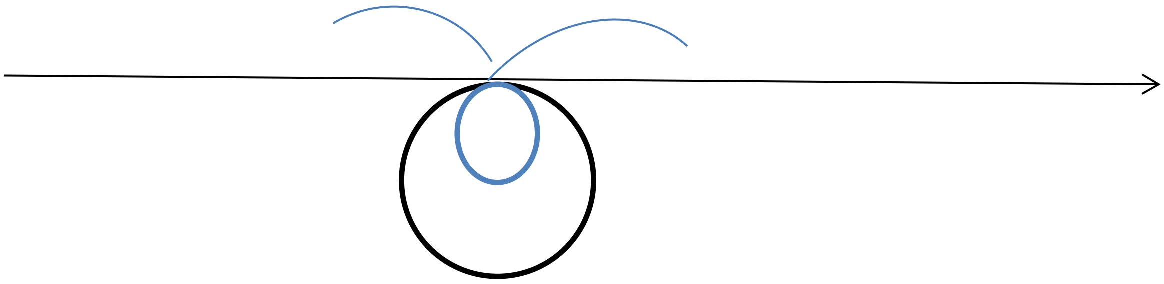
8.1: in this frame the variable is an angle. We preserve all the properties of the analytical frame.

a is the image of (0); b is image of ($\lambda/6$)

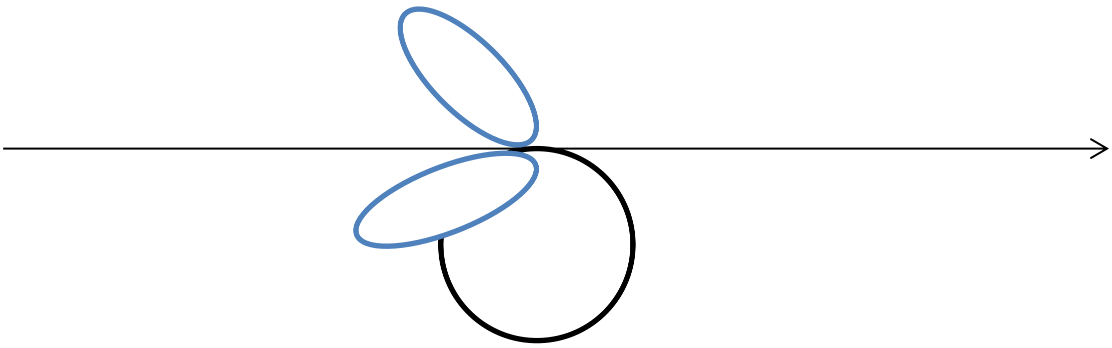


8.2: Curve of function $x \to \sin x$

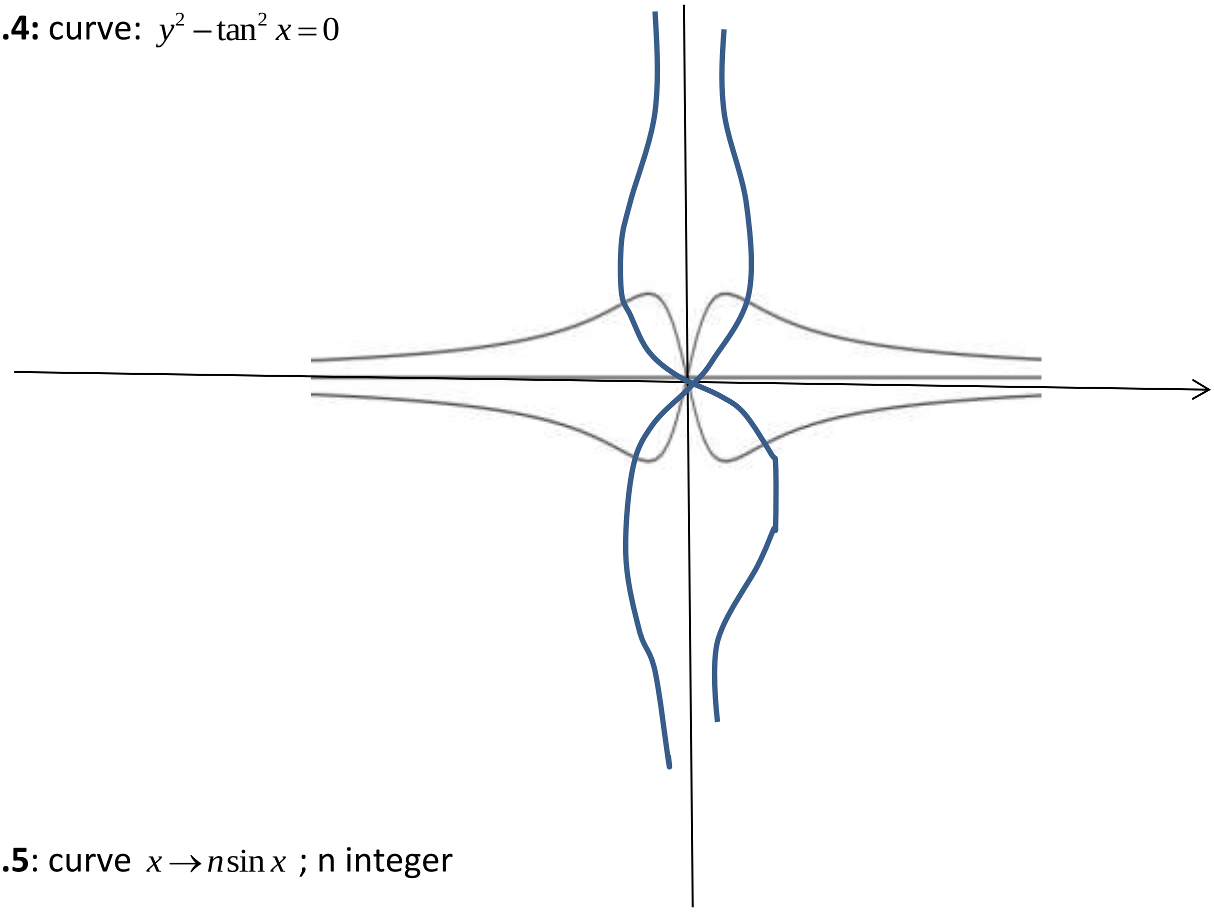
the color of the curve is blue



8.3: curve of function $x \to \cos x$

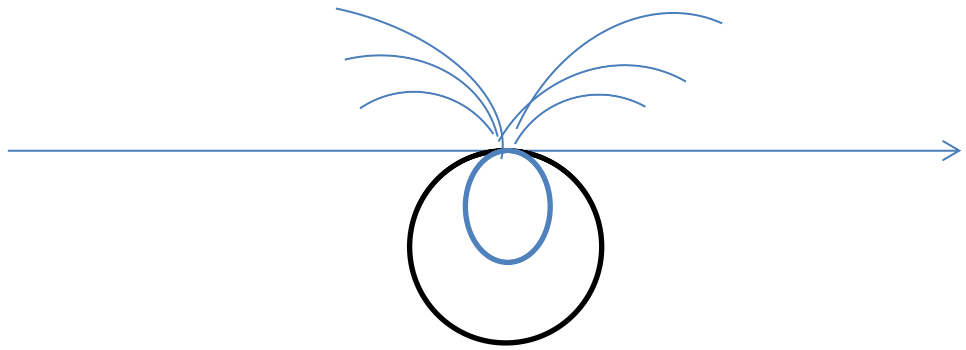


8.4: curve: $y^2 - \tan^2 x = 0$



8.5: curve $x \to n \sin x$; n integer

Here n=3



8.6: $x \to x \sin x$; x real