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Beal Conjecture Proved Very Well

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Abstract

By applying basic mathematical principles, the author surely, and instructionally, proves, directly, the Beal conjecture that if $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, where A, B, C, x, y, z are positive integers and $x, y, z = 1, 2, 3, \ldots$, then A, B and C have a common prime factor. One will let r, s and t be prime factors of A, B and C, respectively, such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers. Then, equation $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$ becomes $(Dr)^{(x+2)} + (Es)^{(y+2)} = (Ft)^{(z+2)}$. The proof would be complete after proving that $r^{(x+2)} = t^{(x+2)}$ and $s^{(y+2)} = t^{(y+2)}$, which would imply that $r = s = t$. More formally, the conjectured equality, $r^{(x+2)} = t^{(x+2)}$ would be true if and only if $(r^{(x+2)}/t^{(x+2)}) = 1$; and the conjectured equality, $s^{(y+2)} = t^{(y+2)}$ would be true if and only if $(s^{(y+2)}/t^{(y+2)}) = 1$. These conjectures would be proved in the Beal conjecture proof.

The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the power of each prime factor on the left side of the equation equals the same power of the prime factor on the right side of the equation. High school students can learn and prove this conjecture for a bonus question on a final class exam.

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 "The simplest solution is usually the best solution"---Albert Einstein

Option 1

Introduction

Beal conjecture states that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $\boxed{x, y, z > 2}$, then A, B and C have a common prime factor. The hypothesis of this conjecture is $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$,
Other equivalent hypotheses are as follows:
1. $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $\boxed{x, y, z \geq 3}$.
2. $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, where A, B, C, x, y, z are positive integers and $\boxed{x, y, z = 1, 2, 3, ...}$. by author.

In this paper, the author will use the second equivalent hypothesis. Then, the Beal conjecture states that if $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, where A, B, C, x, y, z are positive integers and $\boxed{x, y, z = 1, 2, 3, ...}$. then A, B and C have a common prime factor.
One will let r, s and t be prime factors of A, B and C, respectively, such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers, Then, the equation, $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$ becomes $D^{(x+2)}r^{(x+2)} + E^{(y+2)}s^{(y+2)} = F^{(z+2)}t^{(z+2)}$. The proof would be complete after showing that $r = s = t$. Since one would like to prove equalities from the equation, $D^{(x+2)}r^{(x+2)} + E^{(y+2)}s^{(y+2)} = F^{(z+2)}t^{(z+2)}$, one will need equalities between the powers of the prime factors on the left side of the equation and the power of the prime factor on the right side of the equation. Two approaches will be covered in finding these equalities.

Approach 1: Common Sense Approach

At a glance, and from the experience gained in solving exponential and logarithmic equations, one can identify the powers involved with respect to the prime factors, r, s, t, as $r^{(x+2)}$, $s^{(y+2)}$, and $t^{(z+2)}$. Thinking like a tenth grader, one would like to have equalities involving $r^{(x+2)}$, $t^{(x+2)}$, $s^{(y+2)}$, $t^{(y+2)}$, $t^{(z+2)}$. The possible equalities between the powers of the prime factors on the left side and the power of the prime factor on the right side of the equation, $D^{(x+2)}r^{(x+2)} + E^{(y+2)}s^{(y+2)} = F^{(z+2)}t^{(z+2)}$, are $r^{(x+2)} = t^{(x+2)}$, $r^{(x+2)} = t^{(z+2)}$, $s^{(y+2)} = t^{(y+2)}$ and $s^{(y+2)} = t^{(z+2)}$. Of these possibilities, only $r^{(x+2)} = t^{(x+2)}$ and $s^{(y+2)} = t^{(y+2)}$, on inspection, would lead to the conclusion, $r = t$, $s = t$, and $r = s = t$. Therefore, one conjectures the equalities, $r^{(x+2)} = t^{(x+2)}$ and $s^{(y+2)} = t^{(y+2)}$. These conjectures would be proved in the Beal conjecture proof.
More formally, the conjectured equality, $r^{(x+2)} = t^{(x+2)}$ would be true if and only if $r^{(x+2)}/t^{(x+2)} = 1$, and the conjectured equality, $s^{(y+2)} = t^{(y+2)}$. would be true if and only if $s^{(y+2)}/t^{(y+2)} = 1$ Two main steps are involved in the proof. In the first step, one will show that $r = t$; and in the second step, one will show that $s = t$.

 "The simplest solution is usually the best solution"---Albert Einstein

Approach 2: Factorization Approach

In approach 2, one would be guided by the properties of factored numerical Beal equations.

A. Illustration of the equality $r^{(x+2)} = t^{(x+2)}$ of factored Beal equation

For the factorization with respect to $r^{(x+2)}$:

$\boxed{r^{(x+2)} = t^{(x+2)}}$ $\quad D^{(x+2)}r^{(x+2)} + E^{(y+2)}s^{(y+2)} = F^{(z+2)}t^{(z+2)}$

$$\underbrace{r^{(x+2)}}_{K}[\underbrace{D^{(x+2)} + E^{(y+2)}s^{(y+2)} \bullet r^{-(x+2)}}_{L}] = \underbrace{t^{(x+2)}}_{M}\underbrace{t^{[z-x]}F^{(z+2)}}_{P}$$

$(K = M)$

Example 1

$33^5 + 66^5 = 33^6$

$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$

$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$

B. Illustration of the equality $s^{(y+2)} = t^{(y+2)}$ of factored Beal equation

For the factorization with respect to $s^{(y+2)}$:

$\boxed{s^{(y+2)} = t^{(y+2)}}$ $\quad D^{(x+2)}r^{(x+2)} + E^{(y+2)}s^{(y+2)} = F^{(z+2)}t^{(z+2)}$

$$\underbrace{s^{(y+2)}}_{K}[\underbrace{E^{(y+2)} + D^{(x+2)}r^{(x+2)} \bullet s^{-(y+2)}}_{L}] = \underbrace{t^{(y+2)}}_{M}\underbrace{t^{(z-y)}F^{(z+2)}}_{P}$$

(K=M)

Example 2

$34^5 + 51^4 = 85^4$

$17^5 \bullet 2^5 + 17^4 \bullet 3^4 = 17^4 \bullet 5^4$

$\underbrace{17^4}_{k}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$

From either Approach 1 or Approach 2, one will next prove the equalities

$\boxed{r^{(x+2)} = t^{(x+2)}}$ and $\boxed{s^{(y+2)} = t^{(y+2)}}$, and deduce $\boxed{r = s = t}$.

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Option 2

Beal Conjecture Proved Very Well

Given: $A^{(x+2)}+B^{(y+2)}=C^{(z+2)}$, A,B,C,x,y,z are positive integers and $x,y,z=1,2,3,....$

Required: To prove that A, B and C have a common prime factor.

Plan: Let r, s and t be prime factors of A, B and C, respectively, such that $A=Dr$, $B=Es$, and $C=Ft$, where D, E and F are positive integers, Then, the equation $A^{(x+2)}+B^{(y+2)}=C^{(z+2)}$ becomes $(Dr)^{(x+2)}+(Es)^{(y+2)}=(Ft)^{(z+2)}$
The proof would be complete after showing that $r=s=t$. Two conjectured equalities, $r^{(x+2)}=t^{(x+2)}$ and $s^{(y+2)}=t^{(y+2)}$, which would imply that $r=s=t$, will be proved. More formally, $r^{(x+2)}=t^{(x+2)}$ if and only if $(r^{(x+2)}/t^{(x+2)})=1$. This **biconditional** statement $r^{(x+2)}=t^{(x+2)}$ if and only if $r^{(x+2)}/t^{(x+2)}=1$. would be split up into two **conditional** statements as follows: 1. If $r^{(x+2)}=t^{(x+2)}$, then $r^{(x+2)}/t^{(x+2)}=1$ and 2. If $r^{(x+2)}/t^{(x+2)}=1$, then $r^{(x+2)}=t^{(x+2)}$. For the first statement, one will, assume that $r^{(x+2)}=t^{(x+2)}$, and show that $r^{(x+2)}/t^{(x+2)}=1$. For the second statement, one will assume that $(r^{(x+2)}/t^{(x+2)})=1$, and show that $r^{(x+2)}=t^{(x+2)}$. After showing that both conditional statements are true, one would have proved that $r^{(x+2)}=t^{(x+2)}$ if and only if $r^{(x+2)}/t^{(x+2)}=1$. The second conjectured equality, $s^{(y+2)}=t^{(y+2)}$ would be true if and only if $s^{(y+2)}/t^{(y+2)}=1$. This biconditional statement would be split up into two **conditional** statements, as follows: 1. If $s^{(y+2)}=t^{(y+2)}$, then $s^{(y+2)}/t^{(y+2)})=1$, and 2. If $s^{(y+2)}/t^{(y+2)}=1$, then $s^{(y+2)}=t^{(y+2)}$. For the first statement, one will assume that $s^{(y+2)}=t^{(y+2)}$, and show that $s^{(y+2)}/t^{(y+2)}=1$. For the second statement, one will assume that $s^{(y+2)}/t^{(y+2)}=1$, and show that $s^{(y+2)}=t^{(y+2)}$. After showing that both conditional statements are true, one would have proved that $s^{(y+2)}=t^{(y+2)}$ if and only if $s^{(y+2)}/t^{(y+2)}=1$.

 "The simplest solution is usually the best solution"---Albert Einstein

Proof:

Step 1: The conjectured equality, $r^{(x+2)} = t^{(x+2)}$ would be true if and only if $r^{(x+2)}/t^{(x+2)} = 1$.

Statements	**Reasons**
1. $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}, x, y, z = 1,2,3,...(1)$ Let r, s and t be prime factors of A, B and C, respectively, such that $A = Dr$, $B = Es$, and $C = Ft$, where D, E and F are positive integers, Then, the equation $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$ becomes $(Dr)^{(x+2)} + (Es)^{(y+2)} = (Ft)^{(z+2)}$ (2)	**1.** Given (Hypothesis)
2. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$ (3)	**2.** Dividing both sides of Equation (2) by $(Ft)^{(z+2)}$. (Division axiom)

Because of the equality, $r^{(x+2)} = t^{(x+2)}$, a $t^{(x+2)}$ factor is needed on the right side of equation (2)

3. $(Dr)^{(x+2)} + (Es)^{(y+2)} = t^{(x+2)}t^{-(x+2)}(Ft)^{(z+2)}$ (4)	**3.** $t^{(x+2)} \bullet t^{-(x+2)} = 1, (n \times 1 = 1 \times n = n)$ Multiplicative property of 1.
4. Replace $t^{(x+2)}$ by $r^{(x+2)}$ in Equation (4) to obtain $(Dr)^{(x+2)} + (Es)^{(y+2)} = r^{(x+2)} \bullet t^{-(x+2)}(Ft)^{(z+2)}$	**4.** Hypothesis of the first conditional statement is $r^{(x+2)} = t^{(x+2)}$. $x, y, z = 1,2,3,...$
5. $(Dr)^{(x+2)} + (Es)^{(y+2)} = \dfrac{r^{(x+2)}}{t^{(x+2)}}(Ft)^{(z+2)}$	**5.** Positive exponents only $x, y, z = 1,2,3,...$
6. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = \dfrac{r^{(x+2)}}{t^{(x+2)}}$	**6.** Solving for $\dfrac{r^{(x+2)}}{t^{(x+2)}}$ $x, y, z = 1,2,3,...$
7. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$	**7.** From equation (3),
8. Therefore, $1 = \dfrac{r^{(x+2)}}{t^{(x+2)}}$	**8.** Transitive property of equality (Quantities equal to the same quantity are equal to each other) $x, y, z = 1,2,3,...$

Therefore, if $r^{(x+2)} = t^{(x+2)}$, $r^{(x+2)}/t^{(x+2)} = 1$; and one has shown that the first conditional statement is true.

Now, one will show that the second conditional statement is also true,

9. $\frac{r^{(x+2)}}{t^{(x+2)}} = 1$,

9. Hypothesis of the second conditional statement is $r^{(x+2)}/t^{(x+2)} = 1$ $x,y,z = 1,2,3,...$

10. $\frac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$

10. From equation (3),

11. $\frac{r^{(x+2)}}{t^{(x+2)}} = \frac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}}$ (5)

11. Transitive property of equality (Quantities equal to the same quantity are equal to each other)

12. $r^{(x+2)}(Ft)^{(z+2)} = t^{(x+2)}\left[(Dr)^{(x+2)} + (Es)^{(y+2)}\right]$ (6)

12 Cross-multiplying equation (5)

13. $(Ft)^{(z+2)} = (Dr)^{(x+2)} + (Es)^{(y+2)}$ (2b)

13. From (2). Symmetric property of equality $x,y,z = 1,2,3,...$

14. Divide left side of (6) by $(Ft)^{(z+2)}$ and the right side by $(Dr)^{(x+2)} + (Es)^{(y+2)}$, since
$(Ft)^{(z+2)} = (Dr)^{(x+2)} + (Es)^{(y+2)}$ (from eqn, 1)
Then $r^{(x+2)} = t^{(x+2)}$

14. Division axiom

Therefore, if $r^{(x+2)}/t^{(x+2)} = 1$, $r^{(x+2)} = t^{(x+2)}$, and one has shown that the second conditional statement is true..

Since the two **conditional** statements, above, have been proved, the **biconditional** statement, $r^{(x+2)} = t^{(x+2)}$ if and only if $r^{(x+2)}/t^{(x+2)} = 1$. has been proved.

Continuing, if $r^{(x+2)} = t^{(x+2)}$, $r = t$

$$\left[(\log r^{(x+2)} = \log t^{(x+2)};\ (x+2)\log r = (x+2)\log t;\ \log r = \log t;\ r = t)\right]$$

Step 2: The conjectured equality, $s^{(y+2)} = t^{(y+2)}$ would be true if and only if $s^{(y+2)}/t^{(y+2)} = 1$, would similarly be proved as in Step 1.

15. $(Dr)^{(x+2)} + (Es)^{(y+2)} = (Ft)^{(z+2)}$ (2)

15. From Step1

16. $\frac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$ (3)

16. From equation (3) of Step 1

Because of the equality, $s^{(y+2)} = t^{(y+2)}$, a $t^{(y+2)}$ factor is needed on the right side of equation (2)

17. $(Dr)^{(x+2)} + (Es)^{(y+2)} = t^{(y+2)}t^{-(y+2)}(Ft)^{(z+2)}$ (7)

17. $t^{(y+2)} \bullet t^{-(y+2)} = 1$, $(n \times 1 = 1 \times n = n)$ Multiplicative property of 1.

18. Replace $t^{(y+2)}$ by $s^{(y+2)}$ in Equation (7) to obtain

$$(Dr)^{(x+2)} + (Es)^{(y+2)} = s^{(y+2)} \bullet t^{-(y+2)}(Ft)^{(z+2)}$$

18. Hypothesis of the first conditional statement of Step 2 is $s^{(y+2)} = t^{(y+2)}$

19. $(Dr)^{(x+2)} + (Es)^{(y+2)} = \dfrac{s^{(y+2)}}{t^{(y+2)}}(Ft)^{(z+2)}$

19. Positive exponents only

20.. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = \dfrac{s^{(y+2)}}{t^{(y+2)}}$

20. Solving for $\dfrac{s^{(y+2)}}{t^{(y+2)}}$

21. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$

21. From equation (3),

22. $s^{(y+2)}/t^{(y+2)} = 1$

22. Transitive property of equality (Quantities equal to the same quantity are equal to each other)

..

Therefore, if $s^{(y+2)} = t^{(y+2)}$, $s^{(y+2)}/t^{(y+2)} = 1$; and one has shown that the first conditional statement of Step 2 is true.

Now, one will show that the second conditional statement of Step 2 is also true,

23 $s^{(y+2)}/t^{(y+2)} = 1$

23. Hypothesis of the second conditional statement in Step 2 is $s^{(y+2)}/t^{(y+2)} = 1$

24. $\dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}} = 1$

24. Equation (3) in Step 1

25. $\dfrac{s^{(y+2)}}{t^{(y+2)}} = \dfrac{(Dr)^{(x+2)} + (Es)^{(y+2)}}{(Ft)^{(z+2)}}$ (8)

25. Transitive property of equality (Quantities equal to the same quantity are eqoal to each other)

26. $s^{(y+2)}(Ft)^{(z+2)} = t^{(y+2)}\left[(Dr)^{(x+2)} + (Es)^{(y+2)}\right]$ (9)

26. Cross-multiplying equation (8)

27. $(Ft)^{(z+2)} = (Dr)^{(x+2)} + (Es)^{(y+2)}$ (2b)

27. Symmetric property of (2)

28. $s^{(y+2)} = t^{(y+2)}$

(Divide the left side of (9) by $(Ft)^{(z+2)}$ and the right side by $(Dr)^{(x+2)} + (Es)^{(y+2)}$, since $(Ft)^{(z+2)} = (Dr)^{(x+2)} + (Es)^{(y+2)}$)

28. Division axiom

Therefore, if $\dfrac{s^{(y+2)}}{t^{(y+2)}} = 1$, $s^{(y+2)} = t^{(y+2)}$, and the second conditional statement of Step 2 is true.

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Since the two **conditional** statements in Step 2, above, have been proved, the **biconditional** statement, $s^{(y+2)} = t^{(y+2)}$ if and only if $\frac{s^{(y+2)}}{t^{(y+2)}} = 1$. has been proved. Continuing,

if $s^{(y+2)} = t^{(y+2)}$, $s = t$. $\left[(\log s^{(y+2)} = \log t^{(y+2)};\ (y+2)\log s = (y+2)\log t; = \log s = \log t;\ s = t)\right]$

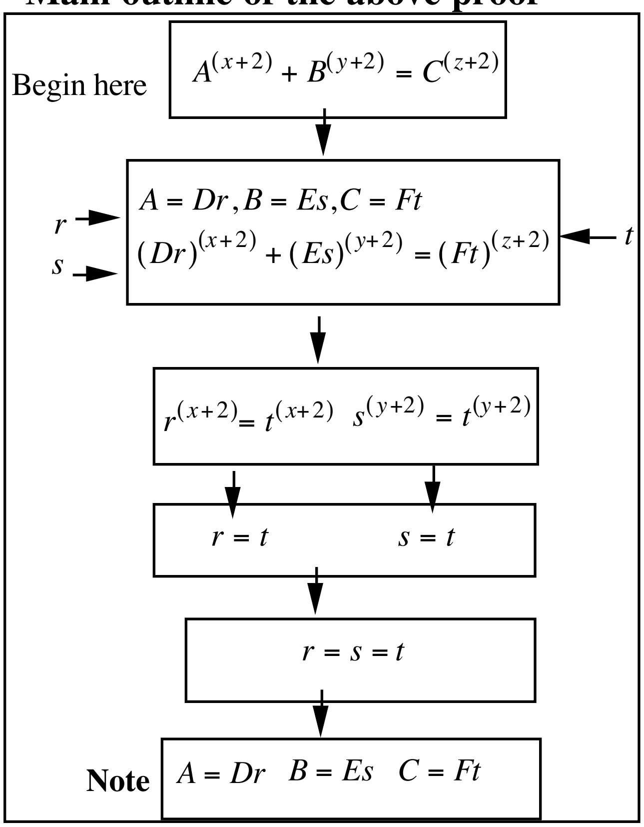
Step 3:

29. $r = t$	**29.** From Step 1
30. $s = t$	**30.** From Step 2
31**.** $\therefore\ r = s$	**31.**Transitive property of equality
32. $r = s = t$	**32.** Transitive property of equality

Since $A = Dr$ **,** $B = Es$ **,** $C = Ft$ (from Step 1) and $r = s = t$ (statement **32**), A, B and C have a common prime factor, and the proof is complete.

Discussion

Main outline of the above proof



The above proof is beautiful mathematics because of the symmetric structure of the proof, One can observe that Step 2 could be viewed as a duplication of Step 1 with $r^{(x+2)}$ replaced by $s^{(y+2)}$, and $t^{(x+2)}$ replaced by $t^{(y+2)}$. The beauty continues when $r^{(x+2)} = t^{(x+2)}$ and $s^{(y+2)} = t^{(y+2)}$ imply that $r = t$ and $s = t$, respectively, resulting in the conclusion, $r = s = t$, In the previous papers, viXra:2001.0694, viXra:2012.0041), the conjecture of these equalities was based on only the properties of the factored numerical Beal equations. In the present paper, a common sense approach as well as a factoring approach was the basis.

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Comparison of the solutions of

A: $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$ (Present paper) and **B:** $A^x + B^y = C^z$ (viXra:2012.0120)

A. Given: $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, A,B,C,x,y,z are positive integers with $\boxed{x,y,z = 1,2,3,...}$

In Step 14. one applies logarithms to solve the exponential equation $\boxed{r^{(x+2)} = t^{(x+2)}}$

$\log r^{(x+2)} = \log t^{(x+2)}$; $(x+2)\log r = (x+2)\log t$; $\log r = \log t$, $\boxed{r = t}$

B. From a previous paper, viXra:2403.0109

Given: $A^x + B^y = C^z$, A,B,C,x,y,z are positive integers with $\boxed{x,y,z > 2}$.

Here also, one similarly applies logarithms to solve the exponenntial equation $\boxed{r^x = t^x}$

$\log r^x = \log t^x$; $x\log r = x\log t$; $\log r = \log t$; $\boxed{r = t}$

Even though the exponents in $r^{(x+2)} = t^{(x+2)}$ in box A are different from the exponents in $r^x = t^x$. $x,y > 2$ in box **B,** the application of logarithms produced the same result $r = t$ **.**

Option 3

Conclusion

The author has surely proved the Beal conjecture that if $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, where A,B,C,x,y,z are positive integers and $x,y,z = 1,2,3,...$, then A, B and C have a common prime factor. The proof was based on the two equalities, $r^{(x+2)} = t^{(x+2)}$ and $s^{(y+2)} = t^{(y+2)}$. which were conjectured and proved. These equalities were conjectured using common sense as well as the factorization properties of the factored numerical Beal equations. To prove these equalities, one began with the hypothesis, $A^{(x+2)} + B^{(y+2)} = C^{(z+2)}$, A,B,C,x,y,z being positive integers and $x,y,z = 1,2,3,...$, and proceeded logically, combining axioms, and definitions to reach the conclusion that A, B and C have a common prime factor. In a previous paper (viXra:2403.0109), the hypothesis was $A^x + B^y = C^z$, with A,B,C,x,y,z being positive integers and $x,y,z > 2$, However, the same procedure in this paper was used to obtain the same conclusion.
High school students can learn and prove this conjecture as a bonus question on a final class exam.

Extra: Fermat's Last Theorem can be proved by modifying the above proof as follows: For the hypothesis, let $(x+2),(y+2),(z+2) = n+2$, $n = 1,2,3,...$, $r \neq s \neq t$ and prove by contradiction (see viXra:2003.0303).

PS: Other proofs of Beal Conjecture by the author are at viXra:2012.0120; viXra:2001.0694, viXra:1702.0331; viXra:1609.0383; viXra:1609.0157; viXra:2012.0041

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