Natural equidistant primes (NEP) and cryptographic coding of the Goldbach's strong conjecture.

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Abstract
For the first time, this article introduces the notion of natural equidistant prime numbers (NEP) which are the only ones to verify the strong Goldbach conjecture naturally in the set of natural integers. From the NEP, we calculate the deducible equidistant prime numbers (DEP) and it is only from NEP + DEP that we obtain all the possible sums of two prime numbers of a given even number. No current algorithm for converting even numbers to the sum of two prime numbers distinguishes NEP from DEP. The natural presence of NEP has been exploited here to set up for the first time a system of coding and deciphering even numbers which allows a calculator to deduce all their possible sums of two prime numbers. This article then has two originalities not published before which will certainly be subject to debate

Keyword. Goldbach's strong conjecture. Equidistant primes. Encoding. Dicephering. Cryptology. Prime numbers. Prime number countig function. Algorithm.

Abbreviations. **GSC** : Goldbach's strong conjecture. **PN** : prime number. **NEP** : natural equidistant primes. **DEP** : deducible equidistant primes.

Note. Figures and tables at pages 5 – 9.

A. Introduction

There are two types of equidistant primes: those that occur naturally and those that are deducible. Recall that for the GSC to be true for any even number denoted E ≥ 4, there must be two primes p and q such that E/2 - p = q - E/2, so that p and q are said to be equidistant. In fact, only natural equidistant primes (NEP) can be used to prove the GSC. We can't know whether there is an even E that doesn't have EPN, which makes it impossible to know whether EPN are strictly necessary to deduce all the others. All other equidistant primes are said to be deducible (DEP for deducible equidistant primes at E/2).

B. Results

B1. The natural equidistant primes (NEP) and the deducible equidistant primes (DEP)

First construct tables 1A-F according to **Figure 1**. Determine π(E) (E is any even 4) by the PN cou ting function, then separate the prime numbers (PN) < E/2 and those > E/2. Then draw the table except that the PN < E/2 are in ascending order and those > E/2 are in descending order because this is how the natural numbers add up to give a value closest to E. The smallest PN which is 3 must be opposite the largest prime number > E/2.

The NEP are colored gray (**Tables 1A-F**). The two NEP p and q appear naturally, so that p + q = E. The line corresponding to the smallest odd PN which is 3 is coloured yellow. As is well known, every even number has a number of possible sums p + q, but we don't see all of them naturally because the density of PN between 0 and E/2 is > that between E/2 and E, which always results in a mismatch between all possible equidistant primes when using the prime counting function. That's why Goldbach's verification must occur naturally with NEP, since they're the only ones we can see in the set of integers. However, by calculation, they will give all the other PED (by deduction). We can see that the number of possible sums p + q is not all natural, but mostly a result of calculation that we deduce. But how are we going to deduce the DEP? I explained this method in a pmore recent article [h**ttps://vixra.org/abs/2501.0066 and https://vixra.org/abs/2501.0117**]. Interested readers can consult it for more details, but very briefly, there are two categories of PN: 6x - 1 or 6x + 5 and those that are 6x + 1. Between two PN 6x - 1 and between two PN 6x + 1 there is a difference of 6n (n ≥ 1). But between PN 6x - 1 and 6x + 1 there are variable gaps of 2n (n ≥ 1).

There are also three categories of even numbers 6x; 6x + 2 and 6x + 4. The 6x are obtained by adding an PN 6x + 1 and another 6x - 1, or vice versa. The 6x + 2 require two 6x + 1 PN. Whereas 6x + 4 are also 6x - 2 and require two 6x - 1 PN. In all cases, the GSC always follows the 6x ± 1 equations, and the sum of the PN is based on the category of the pair.

Example of deduction of DEP from NEP. Let's take the example of the even number E = 44 and so E/2 = 22 *(Table 1B) has practically three possible sums 3 + 41; 7 + 37 and 13 + 31. However, there is ony one pair of NEP visible in Table 1B and it's 7 + 37 from which we deduce the other two. So (7 - 4) + (37 + 4) = 3 + 41. And (7 + 6) + (37 - 6) = 13 + 31. The deduction always follows the same calculation: if an even number E = p + q , the deduction is made according to E = (p - 6n) + (q + 6n) or E = (p + 6n) + (q - 6n). Golablement, the deduction is made according to E = (p - 2n) + (q + 2n) or E = (p + 2n) + (q - 2n).

Another example E = 74 et E/2 = 37 (**Table 1D**) which has practically 4 possibles p + q sums : 3 + 71 ; 7 + 67 ; 13 + 61 ; 31 + 43 ; **37 + 37** (in this paper we only focus on two NEP p and q such that q > p so the latter sum is excluded). The single NEP is 13 + 61 = 74 visible in **Table 1D**. The DEP can all be deduced from the NEP like for exampe 13 + 61 = (13 – 10) + (61 + 10) = 3 + 71 or 13 + 61 = (13 + 18) + (61 - 18) = 31 + 43. This is true for all evens E ≥4.

B2. New Cryptological Coding of GSC

The NEP can also be used to encode even numbers, allowing us to deduce DEP and therefore all possible p + q sums. It seems that every even number E ≥4 in the set N has a unique configuration of NEP (Figures **2A-F**), and even if we find two even numbers E with the same configuration, the NEP and DEP will not be the same. This is a good material for cryptology and all those interested in it, as each number is associated with a specific configuration of its PN and NEP. Mathematically, this coding will enable you to deduce all possible sums p + q by calculation or by using a program that performs E = (p - 6n) + (q + 6n) or E = (p + 6n) + (q - 6n) or E = (p - 2n) + (q + 2n) or E = (p + 2n) + (q - 2n). How does this coding work? Let's take two examples from **Figure 2**. First, the figure is read from the top ; and the NEP line is marked grey with 0, above which the total number of preceding lines is marked, and so on. For example, E = 24 (**Figure 2A**) is associated with the number 1000 because there are three NEP lines preceded by one PN line devoid of NEP .

The NEP and PN of the even numbers E can be used to encode the even number E by associating it with a line and number configuration. Afterwards, a number is associated with it, which, when deciphered, makes it possible to deduce all possible sums p + q = E. Exemple E = 44 et E/2 = 22 (Figure 2B) is coded 203Ø which means that it has a pair of NEPs marked with 0 preceded by two lines of PN and followed by 3 lines of PN devoid of NEP. The Ø sign means that there is a PN < E/2 which has no PN > E/2 in front of it. Let's not forget that first of all we must put the number as explained in Figure 1. The examples given in the figure will help one to understand this encoding and decryption system. The Ø sign always corresponds to single PN close to E/2 on both sides.
For example let's decipher the code 12080706øø (Figure 2F) which means that this number has 12 pairs of PNs (which are not NEPs) followed by a pair of NEPs; then 8 pairs of PN, a NEP line marked by zero; then 7 pairs of PN; a third NEP line; ad finally 6 lines of pairs of PNs, two of which do not have a PN > E/2 opposite marked with the Ø sign. The reader could practice encoding and deciphering numbers. This encoding and decryption system described for the first time in this paper shows its potential usefulness in a cryptological application. Mathematically, it allows you to encode an even number in such a way as to be able to deduce all possible sums p +q.
The reader could practice encoding and deciphering numbers. This encoding and decryption game shows the potential usefulness of this result in a cryptological application. Mathematically, it allows you to encode an even number in such a way as to be able to deduce all possible sums p +q.

C. Discussion

The central question that now arises is the following: do all even numbers E ≥4 have NEP? Is the presence of NEP essential to obtain DEP? Probably an even E that doesn't have a NEP doesn't check GSC, but does it really exist? Current algorithms that provide us with all possible p + q sums in one click confuse NEP and DEP, and this article raises this point for the first time.

If it can be shown that every even number ≥4 has at least one pair of NEP, this means the proof of the GSC. It could even be the case that an even number E does not have a NEP but has DEP, which the current GSC algorithms do not allow us to know. In fact, all the NEP of large numbers could be deduced from those which precede them but this subject has not been addressed here for the moment. But in fact if an even number does not have a NEP, this means that it does not naturally verify the GSC. Does GSC need to be demonstrated with NEP or NEP+DEP or with one of the two? In fact, if no NEP, no natural GSC, and in this case, the GSC would be deduced by the calculation by looking for the DEPs. But deduction by the calculation will never be proof of its veracity.

In addition, this article presents for the first time a coding of even numbers having NEP which makes it possible to deduce all possible sums either by a calculation or by a computer program. This debate deserves close attention in the future.

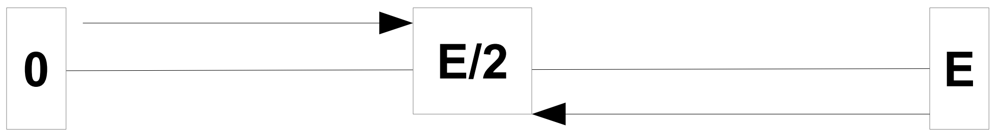

Figure 1 : Primes numbers (PN) < E/2 are in ascending order while those > E/2 are in a descending order from the closest PN to E to E/2. The results obtained with this system are shown in tables 1A-F.

<u>Tables 1A-F</u> : Positions of **natural equidistant primes (grey)** which form the basis of the calculation to find the other equidistant primes deducible by the equations 6x ± 1 by gaps of 6 or by variable gaps of 2n (n ≥ 1).

Table 1A

p	E/2	q
3	12	23
5	**12**	**19**
7	**12**	**17**
11	**12**	**13**

Table 1B

p	E/2	q
3	22	43
5	22	41
7	**22**	**37**
11	22	31
13	22	29
17	22	23
19	22	

Table 1C

p	E/2	q
3	24	47
5	**24**	**43**
7	**24**	**41**
11	**24**	**37**
13	24	31
17	24	29
19	24	ø
23	24	ø

Table 1D

p	E/2	q
3	37	79
5	37	73
7	37	71
11	37	67
13	**37**	**61**
17	37	59
19	37	53
23	37	47
29	37	43
31	37	41
37	**37**	**37**

Table 1E

p	E/2	q
3	80	179
5	80	173
7	80	167
11	80	163
13	80	157
17	80	151
19	80	149
23	80	139
29	80	137
31	80	131
37	80	127
41	80	113
43	80	109
47	80	107
53	80	103
59	**80**	**101**
61	80	97
67	80	89
71		
73		
79		

Table 1F

p	E/2	q
3	180	397
5	180	389
7	180	383
11	180	379
13	180	373
17	180	367
19	180	359
23	180	353
29	180	349
31	180	347
37	180	337
41	180	331
43	180	317
47	**180**	**313**
53	180	311
59	180	307
61	180	293
67	180	283
71	180	281
73	180	277
79	180	271
89	180	269
97	180	**263**
101	180	257
103	180	251
107	180	241
109	180	239
113	180	233
127	180	229
131	180	227
137	**180**	**223**
139	180	211
149	180	199
151	180	197
157	180	193
163	180	191
167	180	181
173	180	
179	180	

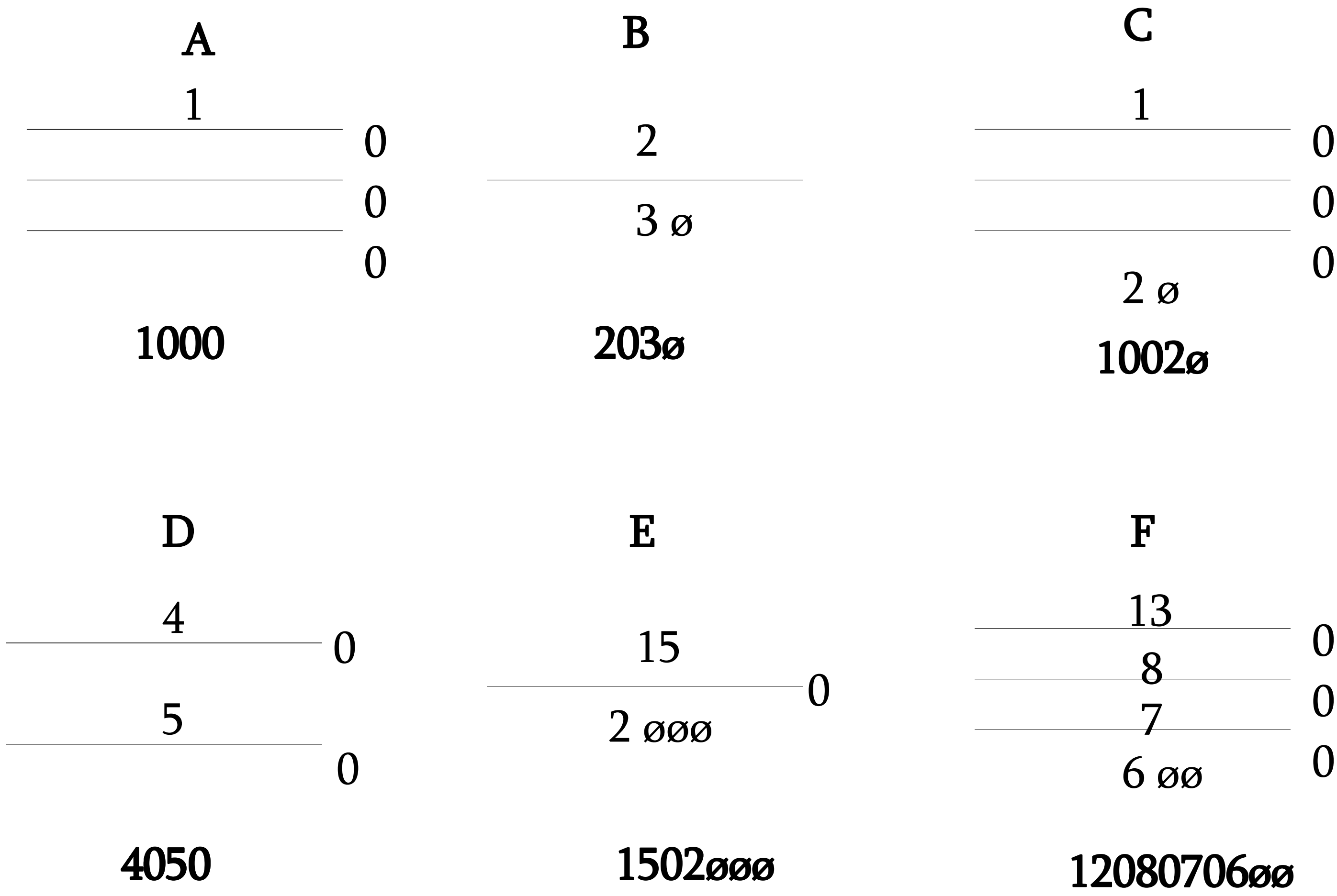

Figure 2 : **Coding and deciphering of even numbers based on GSC**.
The figures correspond in order to Tables 1A-F. It is read from top to bottom.
Each line marked with 0 corresponds to a NEP pair. The number at the bottom or top of the line gives the number of PN pairs that precede or follow the NEP pair. The x sign means that there is no PN on the right, i.e. > E/2. The coded number at the bottom brings together all the information about the even number. We speak of coding because with the coded number an independent calculator can deduce all the possible sums p + q satisfying the GSC. The encoding number is obtained by reading the figure from top to bottom.

References.

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