Para -pseudo-derivatives and combinatory analysis

Date: january 27, 2025

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Abstract :

In this paper, I give formulas on the combinatory in $\mathbb{Z}$, this using an equality that I discovered about 40 years ago.(I don't know if this equality already exists, but to my knowledge, no).I use a kind of derivative calculation that I have titled: pseudo derivative calculation.

This allows me to find infinity of formulas on combinatorial analysis. Here are some of them. I hope that the young researchers will deepen this work.

Key words

Derived from a function; derived from a product and a ratio of functions, combinatorial analysis

combinatory analysis

General formula

$$C_n^p = C_{-p-1}^{-n-1}.........(A)$$, $n \geq p$ and $(n,p) \in \mathbb{N}^2$

$$C_{-p}^{-n} = C_{n-1}^{p-1}...........(B)$$

1.0: Para pseudo -derivative of class D_1

$$(C_n^p)' = (n+1)C_n^{p-1}$$

$$(C_n^p)'' = ((n+1)C_n^{p-1})' = (n+1)(n+2)C_n^{p-2}$$

$$(C_n^p)''' = ((n+1)C_n^{p-1})'' = ((n+1)(n+2)C_n^{p-2})' = (n+1)(n+2)(n+3)C_n^{p-3}$$

1.1: First Para pseudo-derivative D_1

$$(C_n^p)' = (C_{-p-1}^{-n-1})'$$

$$(C_n^p)' = (n+1)C_n^{p-1}$$

$(C_{-p-1}^{-n-1})'$ = $\left|-p-1+1\right| C_{-p-1}^{-n-1-1} = pC_{-p-1}^{-n-2}$ = pC_{n+1}^p ; see (B)

Verification:

$(n+1)C_n^{p-1}$ = $\dfrac{(n+1)n\,!}{(p-1)\,!(n-p+1)\,!}$..........(3)

$$pC_{n+1}^p = \frac{p(n+1)!}{p!(n-p+1)!}........(4)$$

$\dfrac{(n+1)n\,!}{(p-1)\,!(n-p+1)\,!} = \dfrac{p(n+1)!}{p!(n-p+1)!}$; verified

1.2: Second Para pseudo-derivative D_1

$$\left((n+1)C_n^{p-1}\right)' = \left(pC_{-p-1}^{-n-2}\right)'$$

$$(n+2)(n+1)C_n^{p-2} = (p-1)pC_{-p-1}^{-n-3}$$

$$(n+2)(n+1)C_n^{p-2} = (p-1)pC_{n+2}^{p}$$ See (B)

Verification:

$$\frac{(n+2)(n+1)n!}{(p-2)!(n-p+2)!} = \frac{p(p-1)(n+2)!}{p!(n-p+2)!}$$

1.3 :Thirst parapseudo-derivative D_1

$$\left((n+2)(n+1)C_n^{p-2}\right)' = \left((p-1)pC_{n+2}^{p}\right)'$$

$$(n+1)(n+2)(n+3)C_n^{p-3} = p(p-1)(p-2)C_{n+3}^{p}$$

1.4: Para pseudo-derivative of order m (D_1)

$$(n+1)(n+2).....(n+m)C_n^{p-m} = p(p-1)(p-2)......(p-m+1)C_{n+m}^{p}$$

In general

$$\frac{(n+m)!}{n!}C_n^{p-m} = \frac{p!}{(p-m)!}C_{n+m}^{p}$$

2.0: Para pseudo-derivative of class D_2

2.1: First Para pseudo-derivative D_2

$(C_n^p)' = (C_{-p-1}^{-n-1})'$. In this kind of Para pseudo-derivatives, the procedure is not the same for the two terms of equality.

$$(C_n^p)' = pC_n^p$$

$$(C_{-p-1}^{-n-1})' = \left|-n-1+1\right| C_{-p-1+1}^{-n-1+1} = nC_{-p}^{-n} = nC_{n-1}^{p-1}$$; see (B)

So: $pC_n^p = nC_{n-1}^{p-1}$

2.2: Second Para pseudo-derivative D_2

$$p(p-1)C_n^p = n(n-1)C_{n-2}^{p-2}$$

2.3 : Para pseudo-derivative D_2 of order (m+1)

$$p(p-1)(p-2)...(p-m)C_n^p = n(n-1)(n-2).....(n-m)C_{n-m-1}^{p-m-1}$$

3.0: Para pseudo-derivative of class D_3

$$(C_n^p)' = (n-p)C_n^p$$

$$(C_{-p-1}^{-n-1})' = (p+1)C_{-p-2}^{-n-1}$$

3.1: First para pseudo-derivative D_3

$$(C_n^p)' = (C_{-p-1}^{-n-1})'$$

$$(n-p)C_n^p = (p+1)C_{-p-2}^{-n-1}$$

$$(n-p)C_n^p = (p+1)C_n^{p+1}$$

3.2 : Second para pseudo-dérivative D_3

$$(C_n^p)'' = (C_{-p-1}^{-n-1})''$$

$$((n-p)C_n^p)' = ((p+1)C_{-p-2}^{-n-1})'$$

$$(n-p)(n-p-1)C_n^p = (p+1)(p+2)C_n^{p+2}$$

3.3: Para pseudo-derivative D_3 of order m; m>0

$$(n-p)(n-p-1).....(n-p-m+1)C_n^p = (p+1)(p+2)....(p+m)C_n^{p+m}$$

Generalization of the concept of Para pseudo-derivatives

4.0: Para pseudo-derivative of class D_4

Definition: the Para pseudo-derivative of

$$(C_n^p)' = (n+1)C_n^{p-1}$$
$$(C_n^p)'' = (n+1)(n+2)C_n^{p-2}$$
$$(C_n^p)''' = (n+1)(n+2)(n+3)C_n^{p-3}$$

………..etc

Let's consider the following equality

$$C_{n+1}^{p+1} = C_n^{p+1} + C_n^p$$

4.1 : First para pseudo-dérivative D_4 :

$$(n+1)C_{n+1}^p = (n+1)C_n^p + (n+1)C_n^{p-1} + C_{n+1}^p + \left[C_{n+1}^p\right]$$

Note: In this kind of derivatives we always add the term C_{n+1}^p

4.2: Second Para pseudo-derivative D_4 : :

$$(n+2)(n+1)C_{n+1}^{p-1} = (n+2)(n+1)C_n^{p-1} + (n+2)(n+1)C_n^{p-2} + C_{n+1}^p + \left[2(n+1)C_{n+1}^{p-1}\right]$$

For the second Para pseudo-derivative we add the term $2(n+1)C_{n+1}^{p-1}$

4.3: n'ieme Para pseudo-derivative D_5 : $\forall m \geq 2$, $m \in \mathbb{N}$

$$(n+m)(n+m-1).....(n+2)C_{n+1}^{p-m+2} = (n+m-1).....(n+1)C_n^{p-m+2} +$$
$$(n+m-1).....(n+1)C_n^{p-m+1} + \left[(m-1)(n+m-1).....(n+2)C_{n+1}^{p-m+2}\right]$$

A formula, hence I forgot the demonstration:

$$(\alpha n+1)(\alpha n+2+....(\alpha n+\alpha-1)C_{\alpha n}^{\alpha p} = (\alpha p+\alpha-1)(\alpha p+\alpha-2)....(\alpha p+1)C_{\alpha n+\alpha-1}^{\alpha n+\alpha-1}$$

5.0: Para pseudo-derivative of class D_5

Definition: the Para pseudo-derivative of

$$(C_n^p)' = nC_{n-1}^p$$
$$(C_n^p)'' = (nC_{n-1}^p)' = n(n-1)C_{n-2}^p$$
$$(C_n^p)''' = n(n-1)(n-2)C_{n-3}^p$$

Let's consider the following equality

5.1 : $C_{n+1}^{p+1} = C_n^{p+1} + C_n^p$

The first Para pseudo –derivative: $(C_{n+1}^{p+1})' = (C_n^{p+1} + C_n^p)'$

5.2: $(n+1)C_n^{p+1} = nC_{n-1}^{p+1} + nC_{n-1}^p + \left[C_n^{p+1} \right]$

The second Para pseudo –derivative: $(C_{n+1}^{p+1})'' = (C_n^{p+1} + C_n^p)''$

5.3: $(n+1)nC_{n-1}^{p+1} = n(n-1)C_{n-2}^{p+1} + n(n-1)C_{n-2}^p + \left[2nC_{n-1}^{p+1} \right]$

The third Para pseudo –derivative: $(C_{n+1}^{p+1})''' = (C_n^{p+1} + C_n^p)'''$

5.4 : $(n+1)n(n-1)C_{n-2}^{p+1} = n(n-1)(n-2)C_{n-3}^{p+1} + n(n-1)(n-2)C_{n-3}^p + \left[3n(n-1)C_{n-2}^{p+1} \right]$

Para pseudo-derivative of order m ; m ≥ 2

5.5
$$(n+1)n(n-1)....(n-m+2)C_{n-m+1}^{p+1} = n(n-1)....(n-m+1)C_{n-m}^{p+1} +$$
$$n(n-1)....(n-m+1)C_{n-m}^p + mn(n-1)(n-2)......(n-m+2)C_{n-m+1}^{p+1}$$

6.0: Para pseudo-derivative of class D_6

$$(C_n^p)' = (n+1)C_n^{p-1}$$
$$(C_n^p)'' = ((n+1)C_n^{p-1})' = (n+2)(n+1)C_n^{p-2})$$
$$(C_n^p)''' = ((n+2)(n+1)C_n^{p-2}))' = (n+3)(n+2)(n+1)C_n^{p-3})$$

Let's consider the following equality

$$C_n^p = C_{n-2}^{p-2} + 2C_{n-2}^{p-1} + +C_{n-2}^p$$

The first Para pseudo –derivative

6.1: $(n+1)C_n^{p-1} = (n-1)C_{n-2}^{p-3} + 2(n-1)C_{n-2}^{p-2} + (n-1)C_{n-2}^{p-1} + \left[2C_n^{p-1}\right]$

The second Para pseudo –derivative: (Δ) value to be discovered

6.2: $(n+2)(n+1)C_n^{p-2} = n(n-1)C_{n-2}^{p-4} + 2n(n-1)C_{n-2}^{p-3} + n(n-1)C_{n-2}^{p-2} + [\Delta]$

The third Para pseudo –derivative: (Δ') value to be discovered

$$(n+3)(n+2)(n+1)C_n^{p-3} = (n+1)n(n-1)C_{n-2}^{p-5} + 2(n+1)n(n-1)C_{n-2}^{p-4} +$$
$$(n+1)n(n-1)C_{n-2}^3 + [\Delta']$$

Another example: let be the following para pseudo-derivative

$$(C_n^p)' = (n+1)C_n^{p-1}$$
$$(C_{-p-1}^{-n-1})' = \left|-p-1+1\right| C_{-p-1}^{-n-2} = pC_{n+1}^p$$

We have equality: $C_n^p C_{n+1}^{p+1} = C_{-p-1}^{-n-1} C_{-p-2}^{-n-2}$

$$(C_n^p C_{n+1}^{p+1})' = (C_{-p-1}^{-n-1} C_{-p-2}^{-n-2})' \Rightarrow (uv)' = u'v + v'u$$

$$(n+1)C_n^{p-1}C_{n+1}^{p+1} + (n+2)C_{n+1}^p C_n^p = pC_{n+1}^p C_{n+1}^{p+1} + (p+1)C_{n+2}^{p+1} C_n^p$$

We can make successive pseudo-derivatives

Likewise for: $\left(\frac{C_{n+1}^{p+1}}{C_n^p}\right)' = \left(\frac{C_{n+1}^{p+1}}{C_n^p}\right)' \Rightarrow (\frac{u}{v})' = \frac{u'v - v'u}{v^2}$

$$(n+2)C_{n+1}^p C_n^p - (n+1)C_n^{p-1}C_{n+1}^{p+1} = (p+1)C_{n+2}^{p+1}C_n^p - pC_{n+1}^p C_{n+1}^{p+1}$$

We can make successive pseudo-derivatives

the same for :

$$(C_n^p C_m^k C_p^q)' = (C_{-p-1}^{-n-1}C_{-k-1}^{-m-1}C_{-q-1}^{-p-1})'$$
$$(n+1)C_n^{p-1}C_m^k C_p^q + (m+1)C_n^p C_m^{k-1}C_p^q + (p+1)C_n^p C_m^k C_p^{q-1} = pC_{n+1}^p C_m^k C_p^q + kC_n^p C_m^k C_{p+1}^q)$$

other formulas

7.1 : $(n-p)(n+p+1)C_{n+p}^{n-p} = (2p+2)(2p+1)C_{n+p+1}^{n-p-1}$

7.2 : $p(n+1)C_n^p = (n-p+2)(n-p+1)C_{n+1}^{p-1}$

7.3 : $(np+1)C_{np}^{n+p} = (n+p+1)C_{np+1}^{n+p+1}$

7.4: $(np+\alpha)C_{np+\alpha-1}^{n+p} = (n+p+1)C_{np+\alpha}^{n+p+1}$

7.5: $(np+2)(np+1)C_{np}^{n+p-1} = (n+p+1)(n+p)C_{np+2}^{n+p+1}$

7.6: $(np+m)(np+m-1).....(np+1)C_{np}^{n+p-m+2} = (n+p+1)(n+p)....(n+p-m+2)C_{np+m}^{n+p+1}$

7.7: $p!n!C_{n-1}^p = (p+1)!(n-1)!C_n^{p+1}$

7.8: $n^2(n-1)C_{n-1}^p C_{n-2}^p = (p+1)^2(p+2)C_n^{p+1}C_n^{p+2}$

7.9: $\frac{(n+\alpha)!}{(n-\alpha)!}C_{n-\alpha}^p C_n^{p-\alpha} = \frac{(p+\alpha)!}{(p-\alpha)!}C_{n+\alpha}^p C_n^{p+\alpha}$

7.10: $\left(\frac{C_n^{p-1}}{p}\right)^2 = \frac{C_n^p C_{n+1}^p}{(n+1)(n-p+1)}$

7.11: $\frac{2(n+1)}{p(p-1)}C_n^{p-2}C_n^{p-1} = \frac{pC_{n+1}^p C_{n+1}^p}{(n+1)(n-p+2)} + \frac{pC_{n+2}^p C_n^p}{(n+2)(n-p+1)}$

7.12: $\frac{2(n+1)}{p}C_n^{p-2}C_n^{p-1}=\frac{p(p-1)C_{n+1}^{p}C_{n+1}^{p}}{(n+1)(n-p+1)}+\frac{p(p-1)C_{n+2}^{p}C_n^{p}}{(n+2)(n-p+1)}$

7.13: $p(p-1)(n+p)!C_{n+p+2}^{p}=(n+p+2)!C_{n+p}^{p-2}$

7.14: $p(p-1)n!C_{n+2}^{p}=(n+2)!C_n^{p-2}$