A neccessary and sufficient condition that must be met by primes making up the Goldbach partition of a composite even numbers

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Abstract

In this research a theorem for primes to qualify for Goldbach partition composite even numbers is presented. In the paper reference [1] a neccessary and sufficient condition for proof of Goldbach partition of a composite number was established and proved. In the paper it was proved that the square of every integer greater than 1 is equal to the sum of the square of an integer greater or equal to zero and a Goldbach partitition semiprime. The proved theorem effectively means that every composite even number has a composite even number. A Goldbach partition semiprime is a semiprime that is a product of two Goldbach partition primes. For a prime to be a Goldbach partition prime it has to have a Goldbach partition partner for a the specific composite composite even number under consideration. In the paper reference [1] it was proved that every composite even number has at least 1 Goldbach partition semiprime for its Goldbach parition. In this paper we shall closely examine the primes constituting the Goldbach partition semiprime.

Keywords A neccessary and sufficient condition for a pair of primes to qualify for Goldbach partition of a composite even number;

Introduction

In the paper reference [1] it was shown that every integer greater than 1 should be partitionable as follows as a necessary and sufficient condition for the Goldbach partition of of the composite even number $2m$.

$$m^2 = n^2 + p_1 p_2 || : m > 1; n \geq 0; p_1 \geq p_2 || \tag{1}$$

It was shown that the equation 1 means that

$$m = \frac{p_1 + p_2}{2} \geq 2 \tag{2}$$

and

$$n = \frac{p_1 - p_2}{2} \geq 0 \tag{3}$$

It was also shown that

$$p_1 = m + \sqrt{m^2 - p_1 p_2} \tag{4}$$

and

$$p_2 = m - \sqrt{m^2 - p_1 p_2} \tag{5}$$

Thus it was shown that

$$2m = (m + \sqrt{m^2 - p_1 p_2}) + (m - \sqrt{m^2 - p_1 p_2}) = p_1 + p_2 \tag{6}$$

The Goldbach partition semiprime is given by:

$$s_g = p_1 p_2 \tag{7}$$

Every composite even number has its own peculiar set of Goldbach partition semiprimes for its partition.
In paper reference [3] it was shown that the minimum number of Goldbach partitions of an even number is given by:

$$R(2m) > \frac{\sqrt{7m}}{8} \tag{8}$$

Meaning that there is a Goldbach partition prime in the interval

$$[m - 4\sqrt{\frac{m}{7}}, m + 4\sqrt{\frac{m}{7}}] \tag{9}$$

Thus as an example, an interval containing primes for the Goldbach partition of (14) is

$$[7 - 4\sqrt{\frac{7}{7}} = 3, 7 + 4\sqrt{\frac{7}{7}}) = 11]$$

The Goldbach prime pairs in this interval are $(3, 11)$ and $(7, 7)$.

An integer interval containing primes for Goldbach partition of 128 is

$$floor[64 - 4\sqrt{\frac{64}{7}} = 51, floor[64 + 4\sqrt{\frac{64}{7}})) = 76]$$

The Goldbach prime pair in this interval is $(61, 67)$. The above inequaliy (8) is in agreement with the general shape of the graph of the points $(2m, R(2m))$. There is however the need to examine the qualities of the primes making up a Goldbach partition semrime.

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Theorem: Goldbach partition primes A prime number p_n is a Goldbach partition prime of the composite even number $2m$ if and only if on the partition of 2m to the form:

$$2m = 2p_n + r_n || : r_n \geq 0; m \geq p_n || \tag{10}$$

meets the condition:

$$p_n + r_n = q_n \tag{11}$$

and q_n is prime.

Proof $p_n + r_n = q_n$ implies that $r_n = q_n - p_n$ meaning that

$$2m = 2p_n + r_n = 2p_n + (q_n - p_n) = p_n + q_n || : m \leq q_n < 2m || \tag{12}$$

Q.E.D

Implications of the sure test on Goldbach conjecture

It should further be noted that for $r_n > 0$, p_n and r_n have to be coprime otherwise $p_n + r_n$ is a composite odd number.

Goldbach conjecture therefore implies that

$$2(m - p_n) + p_n = q_n \tag{13}$$

or to write it in a better way as in references [1] and [2]:

$$m - p_n = \sqrt{m^2 - p_n q_n} \tag{14}$$

or simultaneously

$$m = p_n + \sqrt{m^2 - p_n q_n} \tag{15}$$

and

$$m = q_n - \sqrt{m^2 - p_n q_n} \tag{16}$$

This implies that Goldbach partition of $2m$ can be conducted via the equation:

$$m = p_1 + \sqrt{m^2 - xp_1} \tag{17}$$

Goldbach partition of $2m$ is achieved whenever

$$m = p_1 + \sqrt{m^2 - xp_1} = \frac{p_1 + p_2}{2} \tag{18}$$

It should further be noted that for all nonsemiprime composite even numbers

$$m = q_n - \sqrt{m^2 - p_n q_n} < q_n < 2m \tag{19}$$

That is:

$$\pi(m, 2m) \geq R(2m) \geq 1 \tag{20}$$

In accordance to Bertrand's postulate.
The result (20) was discussed in paper reference (2).

Summary

A pair of primes have to meet some neccessary and sufficient condition for Goldbach partition of composite even number.

The square of an integer greater than one is the sum of the square of an integer greater or equal to zero and a Goldbach partition semiprime. This result means that every composite even number has at at least one Goldbach partition.

References

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