An Algebraic relationship between primes and How it can be used to prove Goldbach conjecture

Samuel Bonaya Buya

30/1/2025

Contents

Abstract

In this research a general algebraic relationship is established between a pair of primes. This relationship shown to be a useful tool to prove the Binary Goldbach conjecture and other conjectures involving prime gap.

Keywords Algebraic relationship between primes; proof of binary Goldbach conjecture; Quadratic equation for solving the prime gap problem

Introduction

In the paper reference [1] an algebraic relationship between an integer greater than 1 and prime numbers was established given by:

$$m = \sqrt{n^2 + p_1 p_2} \tag{1}$$

It was consequently shown that

$$2m = 2\sqrt{n^2 + p_1 p_2} = p_1 + p_2 \tag{2}$$

In the paper Reference [2] it was shown that the number of primes in the interval $[m, 2m]$ is given by

$$\pi[m, 2m] \geq R(2m) \geq 1 \tag{3}$$

Where $R(2m)$ represents the number of Goldbach partitions of the composite even number $2m$. In the paper reference [3] an algebraic relationship between consecutive primes was established given by:

$$p_{i+1} = p_i + \sqrt{p_i \pm (2k_i - 1)} \tag{4}$$

In the paper reference [4] neccessary and sufficient condition for prime number pair qualfy for Goldbach partition of a composite even number we established. It was shown that if a composite even number is partitioned as below:

$$2m = 2p_1 + k_1 \tag{5}$$

with p_1 prime, for p_1 to qyalifiy to be Goldbach partition prime of $2m$ the

$$k_1 + p_1 = p_2 \tag{6}$$

In other words

$$k_1 = g_{2,1} = p_2 - p_1 \tag{7}$$

This paper will examine the algebraic relationship between in more detail with the aim of getting an efficient tool for solving prime gap problems.

Algebraic relationship between primes and proof of Goldbach conjecture

Consider two primes p_1 and p_2. They are connected together by the following algebraic relationship

$$p_2 = p_1 + \sqrt{p_1^2 - p_2(p_2 - 2g_{2,1})} || : p_2 \geq p_1 \tag{8}$$

$g_{2,1} = p_2 - p_1$

Example

$$11 = 7 + \sqrt{7^2 - 11 \times 3} || g = \frac{11 - 3}{2} = 4$$

$$23 = 19 + \sqrt{19^2 - 23 \times 15} || g = \frac{23 - 15}{2} = 4$$

Thus

$$p_2 + p_1 = 2p_1 + \sqrt{p_1^2 - p_2(p_2 - 2g_{2,1})} || : p_2 \geq p_1 \tag{9}$$

or

$$2m = 2p_1 + \sqrt{p_1^2 - p_2(p_2 - 2g_{2,1})} = p_1 + p_2 || : p_2 \geq p_1 \tag{10}$$

or

$$2m = 2p_1 + \sqrt{p_1^2 - (p_1 + g_{2,1})(p_2 - 2g_{2,1})} = p_1 + p_2 || : p_2 \geq p_1 \tag{11}$$

or

$$2m = 2p_1 + \sqrt{p_1^2 - (p_1 + g_{2,1})(p_1 - g_{2,1})} = p_1 + p_2 || : p_2 \geq p_1 \tag{12}$$

Now

$$m = \frac{2p_1 + \sqrt{p_1^2 - (p_1 + g_{2,1})(p_1 - g_{2,1})}}{2} = \frac{p_1 + p_2}{2} \geq 2|| : p_2 \geq p_1 \tag{13}$$

All semiprime even numbers have at least one Goldbach partition. We need to come up with a proof that all nonsemiprime even numbers have at least one Goldbach partition.

For all nonsemiprime even numbers $p_2 > p_1$ meaning that there exists a prine p_2 in between m and $2m$ such that:

$$m = \frac{2p_1 + \sqrt{p_1^2 - (p_1 + g_{2,1})(p_1 - g_{2,1})}}{2} < p_2 < 2m = 2p_1 + \sqrt{p_1^2 - (p_1 + g_{2,1})(p_1 - g_{2,1})} \tag{14}$$

The quadraric inequality (7) above implies that for every semiprime even number $2m$ there exists at least one Goldbach partition prime p_2 in the interval between m and $2m$, meaning that all nonsemiprime even numbers have at least one Goldbach partition. This result is in Agreement with Bertrand's postulate. It should be noted that for even numbers greater than 6 inequality (7) can simply written in the form (8) below consistent to the Bertrand's postulate:

$$m = \frac{p_1 + p_2}{2} < p_2 < 2m = p_1 + p_2 || p_2 > p_1 || m > 3 \tag{15}$$

Thus

$$m = 4 = \frac{3+5}{2} < 5 < 2m = 8 = 3 + 5$$

$$m = 5 = \frac{3+7}{2} < 7 < 2m = 10 = 3 + 7$$

$$m = 6 = \frac{5+7}{2} < 7 < 2m = 12 = 5 + 7$$

$$m = 7 = \frac{3+11}{2} < 11 < 2m = 14 = 3 + 11$$

$$m = 8 = \frac{3+13}{2} < 13 < 2m = 16 = 3 + 13$$

$$m = 8 = \frac{5+11}{2} < 11 < 2m = 16 = 5 + 11$$

Formulae (8) and (12) mean that there is a quadratic equation connecting two primes given by

$$p_2^2 - 2p_1p_2 + (p_1^2 - g_{1,2}^2) = 0 \tag{16}$$

From equation (16) we establish that:

$$s_g = p_1p_2 = \frac{p_2^2 + (p_1^2 - g_{1,2}^2)}{2} \tag{17}$$

and also:

$$p_1^2 + p_2^2 = 2p_1p_2 + g_{1,2}^2 \tag{18}$$

Thus

$$2m = 2p_1 + \sqrt{p_1^2 + p_2^2 - 2p_1p_2} = p_1 + p_2$$

By equation (16) twin primes are subject to the quadratic equation:

$$x^2 - 2p_1x + p_1^2 - 4 = 0 \tag{19}$$

Primes with a gap of $2n$ are subject to the quadratic equation:

$$x^2 - 2p_1x + p_1^2 - 4n^2 = 0 \tag{20}$$

Thus the solution of

$$x^2 - 6x + (3^2 - 50^2) = 0$$

is $(x_1, x_2) = (53, -47)$. The gap the elements of the solution is 100. However the gap between 3 and 53 is 50.

References

[1] Samuel Bonaya Buya and John Bezaleel Nchima (2024). A Nessary and Sufficient Condition for Proof of the Binary Goldbach Conjecture. Proofs of Binary Goldbach, Andrica and Legendre Conjectures. Notes on the Riemann Hypothesis. International Journal of Pure and Applied Mathematics Research, 4(1), 12-27. doi: 10.51483/IJPAMR.4.1.2024.12-27 . [2] Bonaya, Samuel Buya, Confirming Buya's and Bezaleel's proof of the Binary Goldbach conjecture using Bertrand's postulate[Edition 8] (January 25, 2024). Available at SSRN: https://ssrn.com/ abstract=4868948 or http://dx.doi.org/10.2139/ssrn.4868948 [3] Bonaya Samuel Buya, Quadratic Inequality for Solving the Prime Gap Problem and Proving the Binary Goldbach Conjecture Available at http://vixra.org/abs/2501.0161

[4]Bonaya Samuel Buya, A neccessary and sufficient condition that must be met by primes making up the Goldbach partition of a composite even numbers Available at http://vixra.org/abs/2501.0164