Complex variables of the Riemann zeta function and disproof of the Riemann hypothesis [v2]

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Abstract

The Riemann zeta function is a function of a complex variable $s = \sigma + it$ where σ and t as real real numbers. In this research we will examine the different ways in which this complex variable can be constructed. We will examine one possible case of violation of the Riemann hypothesis. We will look at functions that caprure distribution of primes through their zeroes

Keywords Disproof of Riemann hypothesis; complex variables for the Riemann zeta function; Alternative functions with zeroes capturing distribution of primes

Construction of a complex variable s with the real part being a constant and an example of disproof of the Riemann hypothesis

In the paper reference [1] a complex variable was constructed given by:

$$s = \frac{\ln(-\sqrt{k})}{\ln k} = \frac{1}{2} + \frac{i\pi}{\ln k} \tag{1}$$

In the same paper a complex variable of the form (2) below was introduced

$$s = \frac{\ln(-\sqrt[n]{k})}{\ln k} = \frac{1}{n} + \frac{i\pi}{\ln k} \tag{2}$$

In a similar vein a complex of the forms (3) and (5) below are can also be derived.

$$s = \frac{\ln(-k^n)}{\ln k} = n + \frac{i\pi}{\ln k} \tag{3}$$

$$s = \frac{\ln(-k^n)}{m \ln k} = \frac{n}{m} + \frac{i\pi}{m \ln k} \tag{4}$$

In the paper reference [1] it was shown Example Result that Contradicts the Riemann Hypothesis is

$$\zeta(-1000 - i\frac{1000\pi}{\ln 2}) = 0$$

We also have complex variables like

$$s = \frac{\ln(-(\tan x)^n)}{\tan x} = n + i\frac{\pi}{\tan x} \tag{5}$$

and so on.

A complex variable with the real part being variable

A complex variable of the forms (6) , (7) and (8) below can also be derived

$$s = \frac{\ln(-k^n)}{e^x \ln k} = \frac{n}{e^x} + \frac{i\pi}{e^x \ln k} \tag{6}$$

$$s = \frac{\ln(-k^n)}{e^x \ln k} = \frac{n}{\pi^x} + \frac{i}{\pi^{x-1} \ln k} \tag{7}$$

$$s = \frac{\ln(-k^m)}{\sqrt[n]{x} \ln k} = \frac{m}{\sqrt[n]{x}} + \frac{i\pi}{\sqrt[n]{x} \ln k} \tag{8}$$

It should also be noted that

$$s = \frac{\ln(-k^{\sqrt[n]{x}})}{\sqrt[n]{x} \ln k} = 1 + \frac{i\pi}{\sqrt[n]{x} \ln k} \tag{9}$$

Also

$$s = \frac{\ln(-k^{x^n})}{x^n \ln k} = 1 + \frac{i\pi}{x^n \ln k} \tag{10}$$

Other complex variables with real part variable include

$$s = \frac{\ln(-k^{\sin x})}{\ln k} = \sin x + \frac{i\pi}{\ln k} \tag{11}$$

and

$$s = \frac{\ln(-x^{\sin x})}{\ln x} = \sin x + \frac{i\pi}{\ln x} \tag{12}$$

In equation (9) if we put $n = -1$ and $k = x^{i\pi}$ we obtain the equation

$$s = -i\frac{\ln(-(x^{i\pi})^{x^n})}{\pi x^n \ln x} = 1 + \frac{x}{\ln x} \tag{13}$$

Again

$$\frac{\ln(-x)}{\ln y} = \frac{\ln x}{\ln y} + i\frac{\pi}{\ln y} \tag{14}$$

$$\frac{\ln(-x)}{x} = \frac{\ln x}{x} + i\frac{\pi}{x} \tag{15}$$

$$\frac{x}{\ln(-x)} = \frac{x}{\ln x + i\pi} = \frac{x}{\ln x + i\pi} = \frac{x(\ln x - i\pi)}{\ln^2 x + \pi^2} \tag{16}$$

so that

$$\frac{\ln(-x)(\ln x - i\pi)}{\ln^2 x + \pi^2} = 1 \tag{17}$$

So that

$$\pi^2 = \ln(-x)(\ln x - i\pi) - \ln^2 x \tag{18}$$

So that

$$\ln x = \sqrt{\ln(-x)(\ln x - i\pi) - \pi^2} \tag{19}$$

If we set

$$z = \ln(-x)(\ln x - i\pi) \tag{20}$$

Then

$$\ln x = \sqrt{z - \pi^2} \tag{21}$$

So that

$$x = e^{\sqrt{\ln(-x)(\ln x - i\pi) - \pi^2}} = e^{\sqrt{z-\pi^2}} \tag{22}$$

so that

$$x^i = e^{i(\sqrt{\ln(-x)(\ln x - i\pi) - \pi^2})} = e^{i(\sqrt{z-\pi^2})} \tag{23}$$

Also

$$xe^{ix} = e^{ix(\sqrt{\ln(-x)(\ln x - i\pi) - \pi^2})} = e^{x(\sqrt{z-\pi^2})} \tag{24}$$

Example If $\ln x = 5$ work out $\ln(-x)$

solution By (20)

$$z = 5^2 + \pi^2 = 25 + \pi^2$$

therefore by (19)

$$\ln(-5) = \frac{25 + \pi^2}{5 - i\pi} = \frac{(25 + \pi^2)(5 + i\pi)}{25 + \pi^2} = 5 + i\pi$$

The problem can also be worked directly as

$$\ln(-5) = \ln 5 + \ln -1 = 5 + i\pi$$

Complex functions with infinite number of zeroes and turning points of interest. On the Distribution of primes

The function

$$y = x \sin x + \frac{x}{\ln x} \tag{25}$$

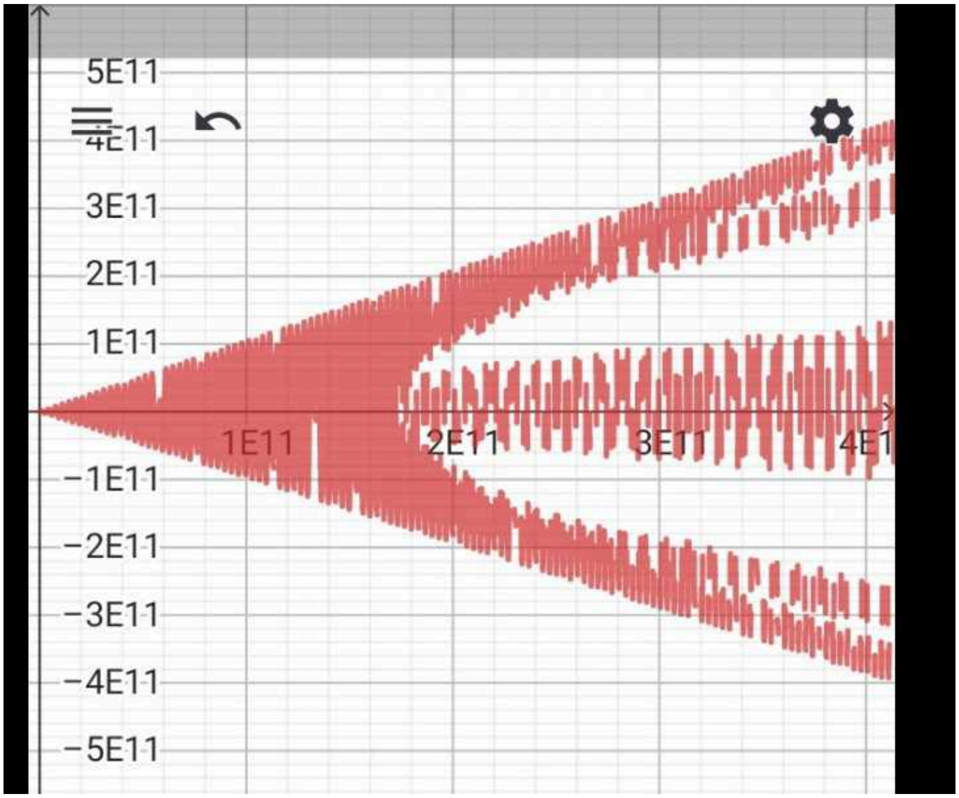
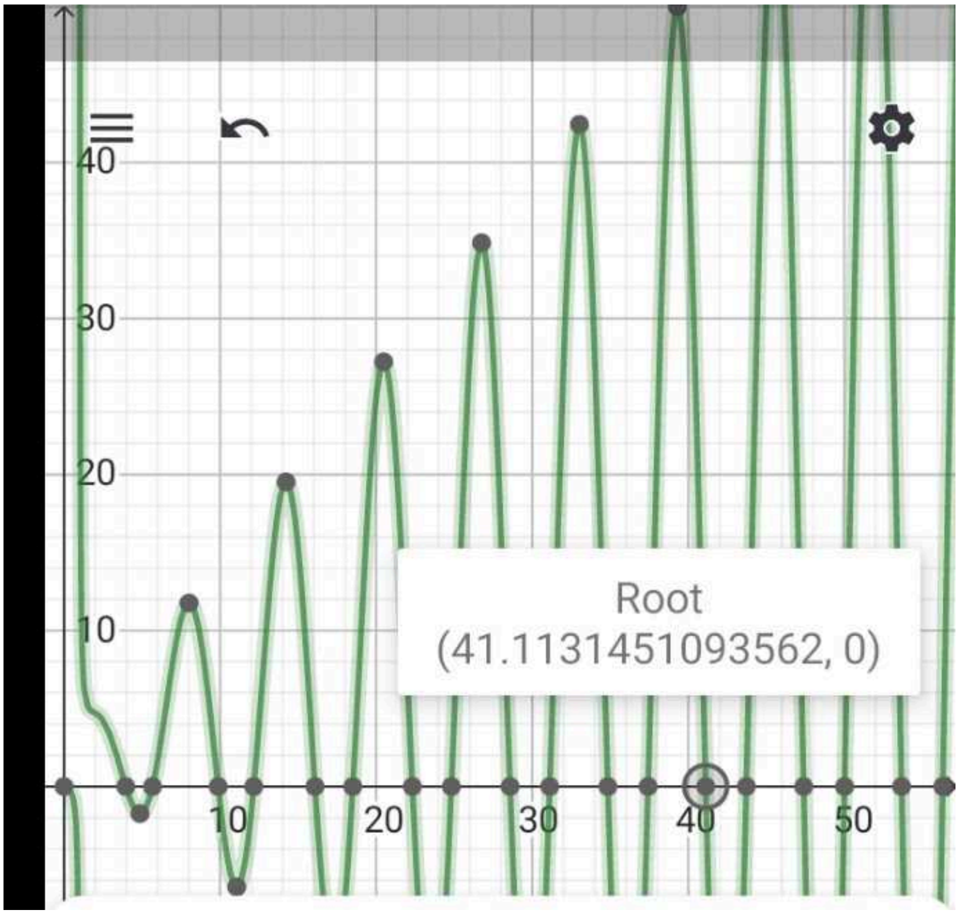
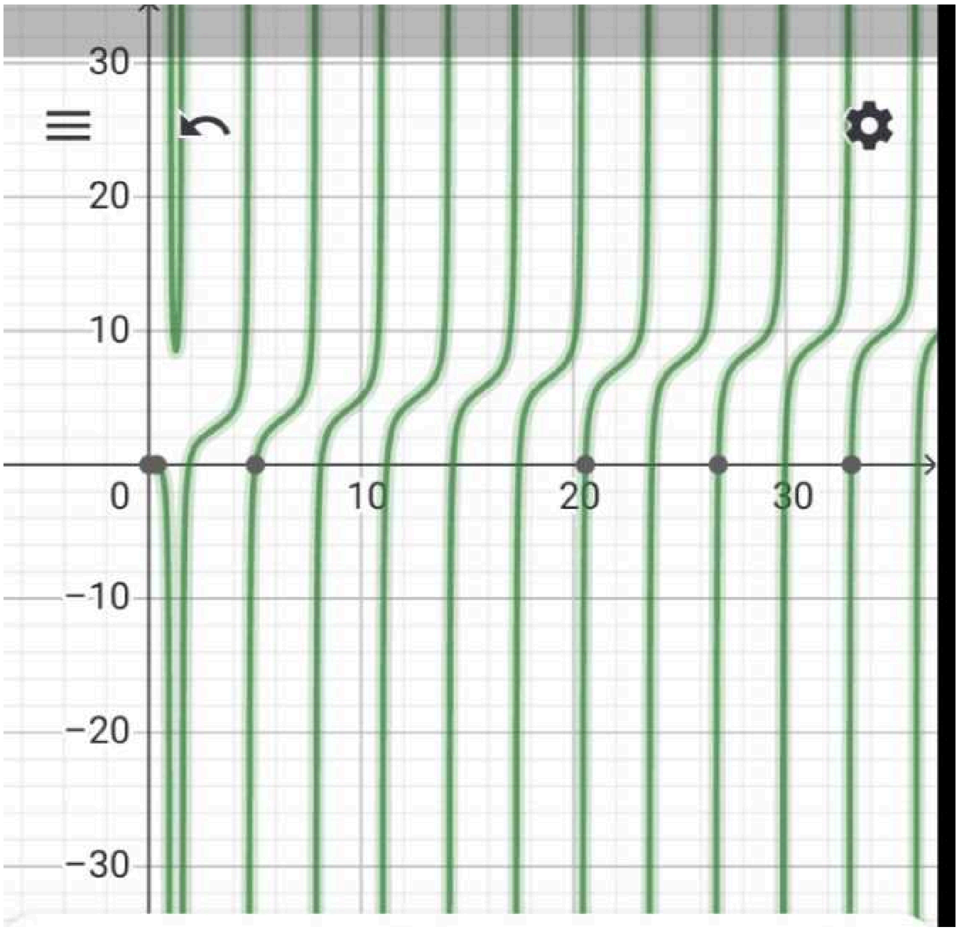


Figure 1: graph of the equaton $y = x \sin x + \frac{x}{\ln x}$



The figure 1 is a compact image of the function (25). In the figure (2) above the number of dots (roots) on the x axis represent nearly the number of primes to a given interval. Thus the number of roots up to 50 are 16. the number of primes up to 50 are 15. However the number of turning points up to 50 are 15. Each of the root points in the above graph represents a prime number. To get a clearer picture we will use the graph of the function:

$$y = \tan x + \frac{x}{\ln x} \tag{26}$$



The above graph has 10 roots representing primes between 1 and 30. The actual number of primes is 10.

Summary and Conclusion

There are many possible ways of constructing the complex variable for the Riemann zeta function. Some of these complex variables when used in the Riemann zeta function, will generate non-trivial zeroes outside of the category envisioned in the Riemann hypothesis. These results falsify the Riemann hypothesis.
The Riemann hypothesis is therefore false. A number functions showing distribution of primes do exist.

References

[1] Samuel Bonaya Buya and John Bezaleel Nchima (2024). A Necessary and Sufficient Condition for Proof of the Binary Goldbach Conjecture. Proofs of Binary Goldbach, Andrica and Legendre Conjectures. Notes on the Riemann Hypothesis. International Journal of Pure and Applied Mathematics Research, 4(1), 12-27. doi: 10.51483/IJPAMR.4.1.2024.12-27