<u>Evading the Challenges of the Higgs Sector through Complex Dynamics</u>

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Abstract

The Higgs sector of particle physics is confronted by several conceptual difficulties, which are still outstanding today. This brief note points out that the onset of complex dynamics in the ultraviolet regime (UV) of field theory evades three challenges of the Higgs sector, namely *fine-tuning, triviality* and the *tachyonic mass problems*. The approach taken here circumvents several attempts at solving the Higgs challenges based on supersymmetry, Technicolor models, composite Higgs or anthropically based arguments.

Key words: Higgs boson, fine-tuning, triviality, tachyonic mass problem, complex dynamics, Physics beyond the Standard Model.

1. Introduction

The physics of the Higgs boson involves several nagging questions that continue to be unanswered to this day. Because the in-depth details of how electroweak symmetry breaking occurs are still unclear, the true nature of the Higgs boson remains an active topic of investigation. In particular,

- The ***fine-tuning problem*** reflects the large discrepancy in theoretical expectation of the Higgs mass versus its observed value. The measured value of the Higgs mass ($m_H \approx 125\, GeV$) is far lower than what is predicted by quantum corrections to the Higgs propagator.
- The ***triviality problem*** arises in four-dimensional scalar field theory, which predicts that the Higgs coupling (λ) goes to zero with the boost in energy scale. It follows that the theory becomes "trivial" or non-interacting in the UV limit.
- The Standard Model postulates that the (mass)^2 term of the Higgs potential is *negative*. This choice is deemed unnatural, it leads to the ***tachyonic mass problem***, and it calls into question the stability of the

Higgs vacuum. Specifically, a tachyonic mass implies that the Higgs vacuum (v) is inherently unstable, as it tends to decay to a lower energy state ($v' < v$).

Elaborating from the likely onset of complex dynamics (CD) in the UV limit of field theory [1 - 2], we indicate here that - at least in principle - CD can evade the three challenges outlined above.

The paper is organized as follows: starting from [3 - 4], next section draws on the bifurcation mechanism of particle physics, which begins with the formation of a Higgs condensate and ends up with the formation of a top-antitop ($t\bar{t}$) condensate. As explained in the text, an added benefit of this mechanism is that the fine-tuning problem is no longer relevant and goes away by default. Section 3 delves into the emergence of the Wilson-Fisher (WF) fixed point in the Renormalization Group analysis of scalar field theory, whose properties naturally solve the triviality and tachyonic mass problems.

2. Evading the fine-tuning problem through bifurcations

Refs. [3 - 4] indicate that, taking complex-scalar field theory as baseline, recovers the composition of the Standard Model and Dark Matter structures from sequential bifurcations driven by the running observation scale. Fig. 1 shows the bifurcation diagram of the electroweak sector, beginning with the primary vertex containing a Higgs condensate and a secondary vertex filled with vector boson condensates (W^+W^-) and Z. The topological nature of condensates is a direct consequence of *spacetime fractality* conjectured to develop in the UV limit of field theory. Note that, in this context, bifurcations represent *stability changing events* organized in a hierarchical pattern.

According to this scenario,

a) Bifurcations generate classical states on account of *decoherence* induced by the far-of-equilibrium conditions of the CD regime. In this picture, all quantum corrections to the Higgs propagator vanish away by default.

b) the Higgs boson vertex is (by definition) unstable as it quickly splits into the gauge bosons vertex (WW) and Z.

It is apparent from this analysis that conditions 1) and 2) render the fine-tuning problem superfluous.

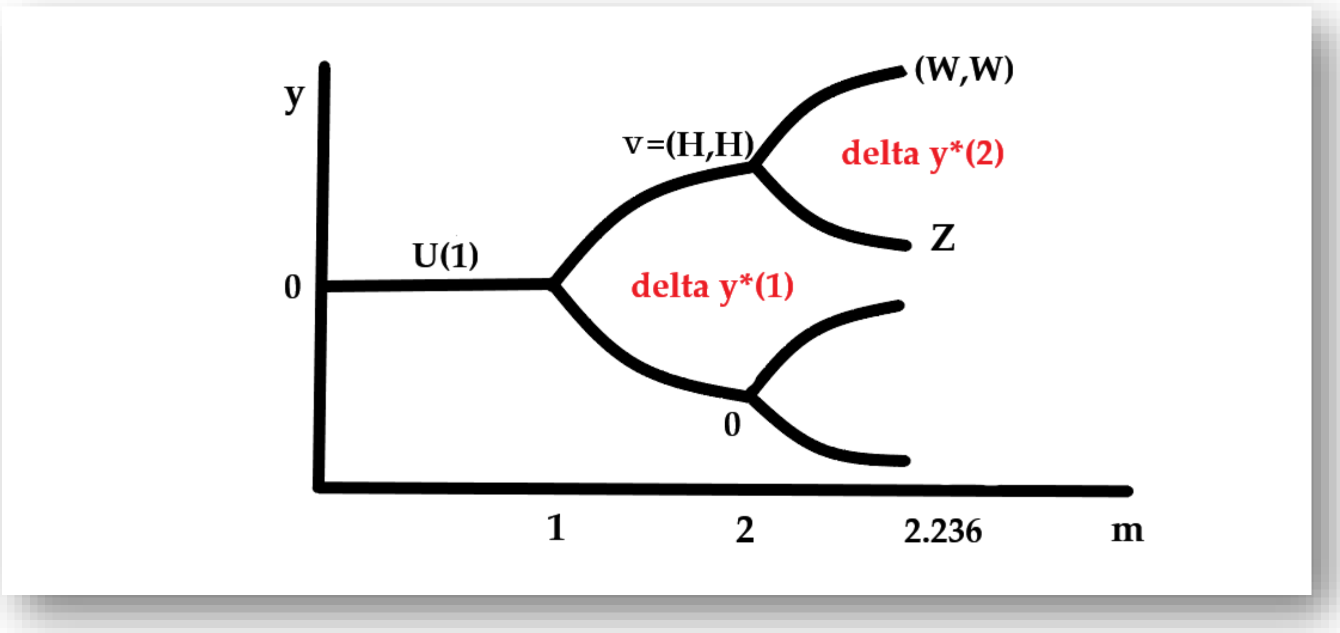

Fig. 1 Bifurcations in the electroweak sector of the Standard Model

The reader is directed to [3 - 4] for additional details and clarifications.

3. Evading triviality and tachyonic mass term problems

With reference to [5], consider the Higgs potential of field theory written as,

$$V(\varphi)=\lambda(|\varphi|^2-\frac{1}{2}\mathrm{v}^2)^2 \tag{1}$$

where v stands for the vacuum expectation value of the Higgs boson and φ is considered a real scalar field for simplicity. (1) can be cast in the form

$$V_H(\varphi)=V(\varphi)-\frac{1}{4}\lambda v^4=-\lambda v^2\varphi^2+\lambda\varphi^4 \tag{2}$$

(1) or (2) can be associated with the partition function of Statistical Physics based upon the functional integral

$$Z[j]=\int D\varphi\exp(-S[\varphi]) \tag{3}$$

where the action has the form,

$$S[\varphi]=\frac{1}{2}\int d\vec{x}\varphi(\vec{x})[r-\nabla^2]\varphi(\vec{x})+\frac{1}{4}u\int d\vec{x}\varphi^4(\vec{x})-\int d\vec{x}\,j(\vec{x})\varphi(\vec{x}) \tag{4}$$

Here, $j(\vec{x})$ plays the role of an external current and the coefficient r has the dimensions of [mass]^2. Casting (4) in field theoretic language leads to the side-by-side identification

$$r = m^2 \Leftrightarrow 2\lambda v^2 \tag{5}$$

$$\lambda \Leftrightarrow -\frac{1}{4}u \tag{6}$$

The Renormalization Group (RG) analysis of (4) starts by splitting the field into its long and short wavelengths components according to

$$\varphi(\vec{k}) = \varphi_{long}(\vec{k}) + \varphi_{short}(\vec{k}) \tag{7}$$

$$\varphi_{long}(\vec{k}) = \begin{cases} \varphi(\vec{k}), & 0 < k < \Lambda/b \\ 0, & \Lambda/b < k < \Lambda \end{cases} \tag{8}$$

$$\varphi_{short}(\vec{k}) = \begin{cases} 0\ , & 0 < k < \Lambda/b \\ \varphi(\vec{k}), & \Lambda/b < k < \Lambda \end{cases} \tag{9}$$

where b is a scaling factor and $\Lambda < \Lambda_{UV}$ denotes the upper energy scale of RG calculations. The RG flow of parameters (r, λ, j) is described by the β-

functions of the theory. To compute the β-functions, one considers an infinitesimal momentum shell integration defined by

$$b = \exp(\delta l) \approx 1 + \delta l \tag{10}$$

and the RG flow equations in near 4-dimensional spacetime read,

$$\beta_r = \frac{dr}{dl} = 2r + 3K_4\Lambda^4 \frac{u}{r+\Lambda^2} + O(u^2) \tag{11}$$

$$\beta_u = \frac{du}{dl} = \varepsilon u - 9K_4\Lambda^4 \frac{u^2}{(r+\Lambda^2)^2} + O(u^2) \tag{12}$$

in which $K_4 = (8\pi^2)^{-1}$. Equations (11) and (12) have a trivial (Gaussian) fixed point solution defined as

$$\beta_r = \beta_u = 0 \Rightarrow r^* = u^* = 0 \tag{13}$$

To analyze the behavior of RG flows near (13), one proceeds by linearizing (11) and (12) and solving the corresponding eigenvalue equation. The pair of solutions satisfying the eigenvalue equation is given by,

$$\lambda_1^a = 2 > 0 \tag{14}$$

$$\lambda_2^a = \varepsilon = 4 - d \tag{15}$$

It is seen that (15) is positive (relevant) for $d < 4$ but negative (irrelevant) for $d > 4$. Since the Gaussian fixed point (13) corresponds to a vanishing coupling (6), it follows from this analysis that Higgs sector is *unstable* in less than $d = 4$ dimensions but turns *stable* in $d > 4$ dimensions. A non-trivial fixed-point (called the Wilson-Fisher or WF point) of RG equations (11) – (12) emerges if one considers the small dimensional deviation $\varepsilon = 4 - d << 1$ as a *tunable parameter.* Expanding the RG equations to quadratic order yields

$$\frac{dr}{dl} \approx 2r + au - bur \tag{16}$$

$$\frac{du}{dl} \approx \varepsilon u - 3bu^2 \tag{17}$$

with $a = 3K_4\Lambda^2$ and $b = 3K_4$. The WF point derived from (16) - (17) is located at,

$$\boxed{|u^*| = 4\lambda^* = \frac{1}{3b}\varepsilon} \tag{18}$$

$$\boxed{r^* = (m^*)^2 = -\frac{a}{6b}\varepsilon} \tag{19}$$

The pair of eigenvalues associated with the WF point are found to be,

$$\lambda_1^{WF} = 2 - \frac{\varepsilon}{3} > 0 \tag{20}$$

$$\lambda_2^{WF} = -\varepsilon < 0 \tag{21}$$

which makes the WF point *stable* in less than four dimensions ($d < 4$). The flows corresponding to the Gaussian and WF fixed points are displayed in Figs. 2-3 below.

The key point of this analysis is that, according to (18) – (19), both mass^2 term and coupling parameter of the Higgs field *arise from the continuous and nonvanishing dimensional deviation* ε.

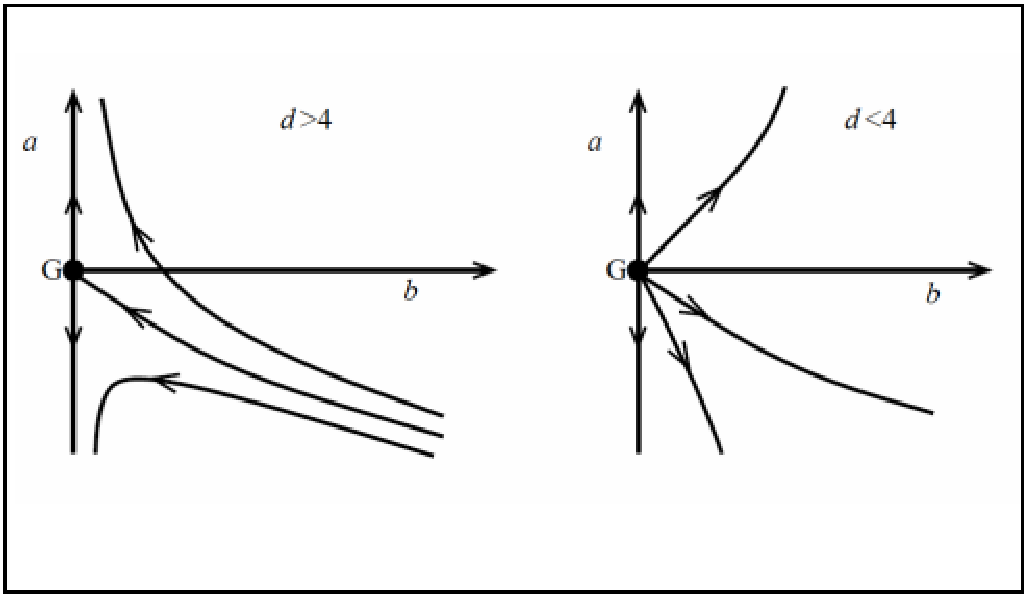

Fig. 2: The Gaussian fixed point of scalar field theory

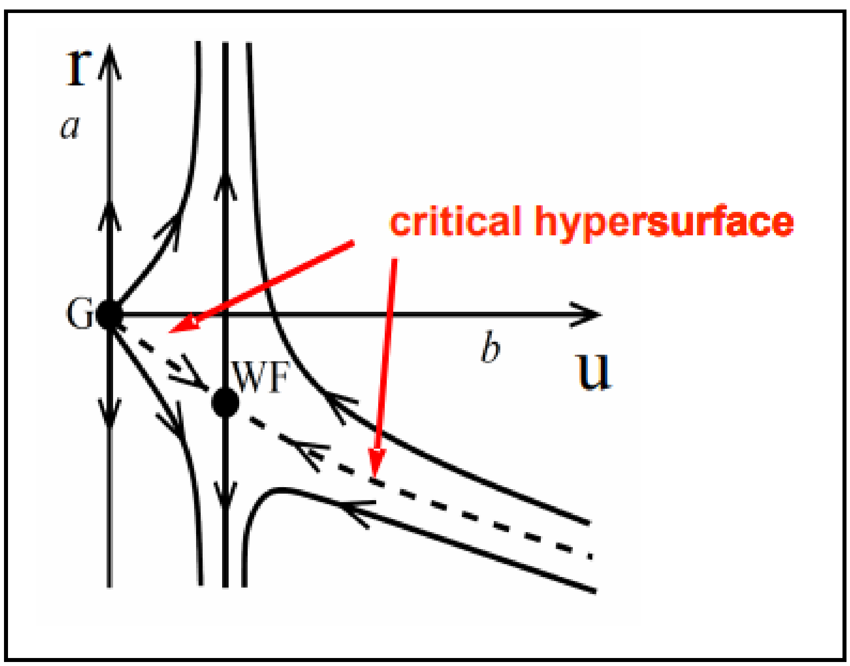

Fig. 3: The Wilson-Fisher (WF) fixed point of scalar field theory

Following these findings,

a) *Triviality* occurs in classical four spacetime continuum, where $\varepsilon = 0$ and the only fixed point is the Gaussian one ($\lambda^* = m^* = 0$). By default, the triviality problem goes away when $\varepsilon \neq 0$.

b) Relations (18) – (19) show that the parameters of the WF fixed point carry opposite signs, with a negative sign attached to the mass parameter. This conclusion brings the *tachyonic mass problem* to a close.

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