Absolute Energy

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Summary: This paper presents a novel concept of Absolute Energy by defining it as the total kinetic energy required to bring a system to rest. In doing so, it introduces a frame-independent metric for energy analysis and addresses key gaps in classical mechanics.

Abstract

This paper presents a novel concept entitled as Absolute Energy, which defines energy as a quantified value of a system independent from the observer. Absolute Energy is defined as the total kinetic energy of all bodies within a system which would be required to bring the system to rest at a specific reference point and time, without transformation of kinetic energy into other forms or other forms into kinetic energy. This work extends the concepts of centrial motion and angular motion to present a unified understanding and quantification of energy in systems. This approach intends to resolve the concepts in classical and contemporary mechanics, addressing their shortcomings and limitations.

Classical and contemporary physics often define energy relative to an observer's frame of reference when analyzing isolated or multi-body systems, resulting in inconsistencies and ambiguities in establishing energy quantification. Absolute Energy is linked to the relative motions and momentums of the system’s components. In this paper, analysis of various scenarios, including two-body and multi-body systems using this new framework yields derivations for total energy quantifications. This, then, illustrates the credibility and robustness of the concept by detailing the interplay of linear and angular motions.

These findings demonstrate that Absolute Energy solves for derivations that are not often provided for in traditional classical definitions. Such findings can be seen to have new application to astrophysics, thermodynamics and classical mechanics, and are therefore challenging the foundations of classical and contemporary mechanics. As a result, this work is a beginning toward further research into energy properties of isolates systems and the universe as a whole.

Key words

Absolute Energy, The MM Theory, Linear Motion, Linear Momentum, Centrial Motion, Centrial Momentum, Angular Motion, Angular Momentum, Isolated Systems

1. Introduction

In physics, the concept of energy is a fundamental entity in our understanding that interrelates the concepts of bodies interactions and their motions within mechanical systems. This understanding of energy has evolved beginning with Newtonian mechanics, encompassing Einstein’s relativistic theories, and has increasingly been applied to address more complex systems. Nevertheless, remain significant gaps to accurately and universally quantify the energy of a system. As such, the focus of this paper is to present a new theoretical framework which postulates a new novel concept of Absolute Energy. Following on the author’s earlier work on centrial motion and centrial momentum [1], this new framework provides a consistent and universal quantification of energy and its analysis.

The initial work on classical mechanics that was originated by Newton and further formalized by Golstein, Poole, and Safko [2], focuses on a dependence of energy on an observer’s frame of reference, with properties defined across different coordinate systems. However, based on this dependence, classical mechanics fails to unify the definition of total energy for isolated system, particular for complex interactions. This limitation underscores the need for another framework, whereby the concept of Absolute Energy is introduced and allows the user to unify the process of energy analysis for a wide range of systems.

Marion and Thornton have similarly noted that traditional kinetic energy calculations often rely heavily on the observer's frame of reference, complicating attempts to establish universal energy definitions [3]. In contemporary times, studies by Taylor have explored energy transfer and momentum conservation in complex systems and the role of generalized coordinates and conserved quantities in systems with constraints [4]. However, Taylor's framework primarily focuses on systems with predefined reference points or symmetrical conditions, leaving a gap in clear understanding of cases where linear momentums converge to a central reference point, which is a critical condition for understanding centrial motion.

Arnold, in his seminal work on the mathematical methods in classical mechanics, provides a foundational basis for analyzing dynamic systems via different geometry and symplectic structures [5]. While this approach offers powerful tools in studying complex systems, his work does not address solutions for quantifying energy systems where the motion is centrial or dependant on the interplay of momentum interactions. This limitation demonstrates, once again, that there is a need for a unified framework like that based on Absolute Energy to bridge these gaps. Next, studying frameworks presented by Feynman provides an intuitive understanding of classical mechanics and dynamics, showcasing a focus on conservation and symmetry of physical systems [6]. However, although his discussions offer detailed examination of the interplay between energy and momentum in dynamic systems but fail to unify the metrics for quantifying energy states relative to a central reference point. This omission is significant when it becomes necessary to utilize consistent framework beyond observer-dependent measures for energy quantifications and analysis.

Studies done by Jose and Saletan also delve deeper into advanced classical mechanics and utilize Hamiltonian and Lagrangian techniques and methods to analyze complex systems [7]. Their work provides advanced tools for modeling energy and momentum conservation. Despite this, while their approach offers an efficient and effective method of analyzing these systems, it fails short of evaluating these systems relative to a central reference point. This indicates that a novel approach, such as one utilizing Absolute Energy, is required to address the challenges by more complicated systems and that it is needed to expand the applicability of classical mechanics to isolated systems involving central motion.

There is further study conducted by Symon that highlights the challenges in quantifying the total energetic states of isolated systems [8]. Although mathematical analysis has advanced the study of dynamic systems, the established frameworks are often found to be inadequate to provide a consistent quantitative measure for systems where linear and angular motions converge to a central point of reference. This gap underscores the need for a unified theoretical model capable of analyzing motion and energy under a single framework, most significantly in systems exhibiting centrial motion. As such, Absolute Energy offers a promising basis to bridge the gap in this theoretical framework, permitting a comprehensive understanding of these systems.

The purpose of this study is to address the challenges in energy quantification and analysis by introducing and formalizing the concept of Absolute Energy. Absolute Energy is defined as a constant, observer-independent quantity of a system's total kinetic energy that is calculated without the transformation of the system's kinetic energy into other forms or vice versa. This framework integrates linear motion (including centrial motion) and angular motion, providing a stable approach to energy analysis that extends beyond the traditional understanding of classical mechanics.

In the case of linear motion, Absolute Energy is determined by evaluating the linear momentums of all bodies within a system to a central reference point. For angular motion, Absolute Energy is evaluated for both pure and non-pure angular momentums by considering the angular momentums of all bodies to a conceptualized reference point. This demonstrates that Absolute Energy is inherently linked to the relative motions and momentums of the system's components rather than individual velocities or directions.

This paper involves analytical derivations and illustrative examples of two-body and multi-body systems, concentric centrial motions, pure and non-pure angular motions as well as systems with combined linear and angular motions, in order to demonstrate the definition and concept of Absolute Energy. The methodology focuses on determining Absolute Energy via the consideration of both linear and angular components and their relations to carefully defined reference points. These derivations and illustrative examples then demonstrate the practicality and theoretical significant of Absolute Energy that allows for the unification of energy metrics in isolate systems.

2. Theory

2.1. Definition of Absolute Energy (E_a)

The Absolute Energy of a system, (E_a), is defined as the total kinetic energy of all bodies within the system, expended internally by the bodies themselves to bring the system to rest at a specific reference point and time, without transformation of kinetic energy into other forms or transformation of other forms of energy into kinetic energy.

2.2. Linear Motion - Centrial Motion and Absolute Energy

Centrial motion and centrial momentum have been defined and their concepts have been extensively explained by the author in a paper titled "*The MM Theory: A Fundamental Revision of the Laws of Motion and Introducing Centrial Motion and Centrial Momentum* [1]." In this paper, based on a thorough examination of that article, it will be shown that the only system with linear motions that can satisfy the conditions for possessing Absolute Energy would be one that is in Centrial Motion.

To introduce Absolute Energy, first, as an example case, it is assumed that there are two bodies (1) and (2) with masses m_1 and m_2. These masses are moving in opposite directions, and their velocities are $\vec{v}_{1,O}$ and $\vec{v}_{2,O}$ with reference to point O respectively, as shown in [Fig. 1]. If these two bodies are considered to be the only bodies in the system or in the universe, the Absolute Energy of this entire set is the amount of energy that must be spent to bring both bodies to a complete rest. Here, since there are no other bodies in the system or in the universe, then body (1) must stop body (2), and vice versa body (2) must stop body (1). To stop each other, the magnitude of the linear momentum of body (1) must be equal to the magnitude of linear momentum of body (2) and also their directions must be in opposite to one another with respect to the point of reference along their axis.

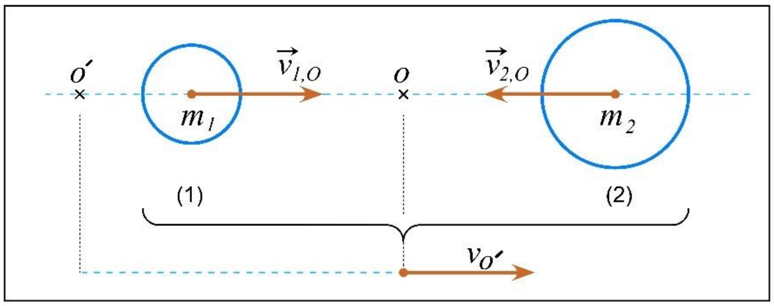

Fig. 1. Two bodies along an axis of motion and with reference to points O and O' for the purpose of analysis of finding absolute energy E_a.

If the magnitudes of momentum of bodies (1) and (2) are not equal, they cannot simultaneously bring each other to a complete stop relative to point O (considered as an initial reference point of their motions). However, if this condition is not met at O, it will still be possible to find another reference point where such a condition is satisfied. This new reference point should be determined such that the linear momentums of these two bodies are equal when considered with respect to that point. To satisfy this condition, the system of bodies as a whole must also be moving at a fixed velocity relative to this new reference point. This ensures that the reference frame is chosen such that the linear momentums of both bodies are equal and with opposite directions. Utilizing the momentum/velocity of these two bodies based on this new reference point, laying along the axis of their motions, enables us to determine the system's absolute energy. It is important to emphasize that this reference point can be any point along the axis of their motions. According to the definition and concept of centrial motion, this point is the same as the center of the centrial motion.

In effect, the reference point utilized to define Absolute Energy is distinctly independent of any observer. This marks a significant advancement over classical definitions that are dependant on external frames of reference. For instance, in the above case with the two bodies, the reference point is determined such that the total momentum of the system relative to this new point is zero.

This unique point is derived via the relative velocities of the two bodies and corresponds to the concept of centrial motion, where velocities are measured with respect to a centrial point, independent of any external observer. As a result, the Absolute Energy of a system is integrally associated to this centrial point, providing an observer-independent framework for energy quantification.

It is crucial to emphasize that per the case being studied here, the assumptions made are with the aim of finding the Absolute Energy. Here it does not matter if these two bodies collide or not. If both bodies are moving towards each other then they will finally collide. And if the two bodies are moving away from each other, it can be assumed for theoretically purposes that they have already collided with each other. Even if the two bodies are moving in the same direction [Fig. 2] but with different speeds, there is always a point of reference to which both motions can be viewed to be moving in opposite directions. In all cases we can assume that these two bodies must stop each other by any possible ways.

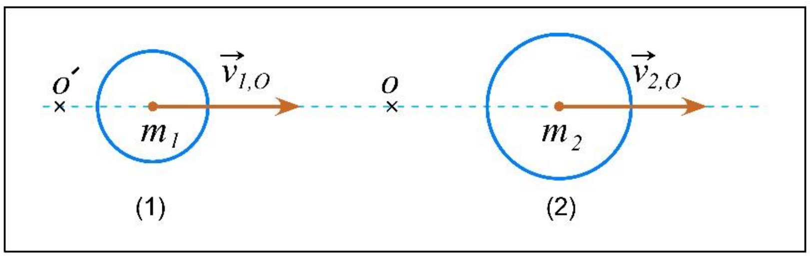

Fig. 2. Referencing of the motion of two bodies to the central point of centrial motion, O', in order to calculate their absolute energy E_a.

The above analysis is also true for any system, such as, for example: an isolated set of particles or objects. Again, in order to find the Absolute Energy of an isolated system, the sum of the energies required to bring all the particles and objects by themselves to a point of complete rest must be considered. The entire universe can also be considered as an isolated system, and therefore the Absolute Energy of the universe can be viewed and analyzed from this perspective.

2.2.1. Absolute Energy for a System with Two Bodies Moving Along a Single Axis

For the case in which two bodies are moving along a straight line [Fig. 1] is considered. First, to begin, we must determine at what point of reference the conditions for centrial motion are fulfilled. If the conditions of centrial motion are not satisfied with respect to the initial point of reference O, then we have to find the velocity of the system with respect to a new point of reference, for example O', where these conditions are satisfied.

Once this is determined, we assume that point O' is fixed, then, we can consider that the whole system is moving with velocity $\vec{v}_{O'}$ with respect to the point O'. For $\vec{v}_{O'}$, we can obtain,

$$\vec{v}_{O'} = -\frac{m_1\,\vec{v}_{1,O} + m_2\,\vec{v}_{2,O}}{m_1 + m_2} \tag{2.1}$$

And the Absolute Energy of the system is,

$$E_a = \frac{1}{2} m_1 v_{1,O'}^2 + \frac{1}{2} m_2 v_{2,O'}^2 \tag{2.2}$$

$$E_a = \frac{1}{2} m_1 \left|\vec{v}_{1,O} + \vec{v}_{O'}\right|^2 + \frac{1}{2} m_2 \left|\vec{v}_{2,O} + \vec{v}_{O'}\right|^2 \tag{2.3}$$

$$E_a = \frac{1}{2} m_1 \left|\vec{v}_{1,O} - \frac{m_1 \vec{v}_{1,O} + m_2 \vec{v}_{2,O}}{m_1 + m_2}\right|^2 + \frac{1}{2} m_2 \left|\vec{v}_{2,O} - \frac{m_1 \vec{v}_{1,O} + m_2 \vec{v}_{2,O}}{m_1 + m_2}\right|^2 \tag{2.4}$$

(2.5)

$$E_a = \frac{1}{2} m_1 \left|\frac{m_2(\vec{v}_{1,O} - \vec{v}_{2,O})}{m_1 + m_2}\right|^2 + \frac{1}{2} m_2 \left|\frac{m_1(\vec{v}_{2,O} - \vec{v}_{1,O})}{m_1 + m_2}\right|^2 \tag{2.6}$$

$$E_a = \frac{1}{2} \frac{m_1 m_2^2}{(m_1 + m_2)^2} \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right|^2 + \frac{1}{2} \frac{m_1^2 m_2}{(m_1 + m_2)^2} \left|\vec{v}_{2,O} - \vec{v}_{1,O}\right|^2 \tag{2.7}$$

Since, $\left|\vec{v}_{1,O} - \vec{v}_{2,O}\right| = \left|\vec{v}_{2,O} - \vec{v}_{1,O}\right|$, then,

$$E_a = \frac{1}{2}\left(\frac{m_1 m_2^2}{(m_1 + m_2)^2} + \frac{m_1^2 m_2}{(m_1 + m_2)^2}\right) \cdot \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right|^2 \tag{2.8}$$

$$E_a = \frac{1}{2}\left(\frac{m_1 m_2 (m_1 + m_2)}{(m_1 + m_2)^2}\right) \cdot \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right|^2 \tag{2.9}$$

$$E_a = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right) \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right|^2 \tag{2.10}$$

Now we can find the Absolute Energy of a set of two bodies based on the magnitude of its centrial momentums, Q [1],

$$E_a = \underbrace{\frac{1}{4}\left(\frac{2 m_1 m_2}{m_1 + m_2} \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right|\right)}_{Q} \cdot \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right| \tag{2.11}$$

$$E_a = \frac{1}{4} Q \cdot \left|\vec{v}_{1,O} - \vec{v}_{2,O}\right| \tag{2.12}$$

Eq. (2.12) demonstrates the relationship between Absolute Energy and the magnitude of the centrial momentum of the two bodies. Eq. (2.10) demonstrates that the Absolute Energy is not solely dependents on the speed of the bodies alone, but that it depends on the speed of the two bodies with respect to one another.

If we consider a scenario where $\vec{v}_{1,O}$ is equal to $\vec{v}_{2,O}$, then E_a is equal to zero. This means that if the two bodies have the same speed and are moving in the same direction, they are not able to stop one another at any point in time and therefore the Absolute Energy is equal to zero.

Now re-writing Eq. (2.11) as:

$$E_a = \frac{1}{4}\left(\underbrace{\frac{2m_1m_2}{m_1+m_2}\left|\vec{v}_{1,O}-\vec{v}_{2,O}\right|}_{Q}\cdot\left(\frac{m_1+m_2}{2m_1m_2}\right)\underbrace{\left(\frac{2m_1m_2}{m_1+m_2}\right)\cdot\left|\vec{v}_{1,O}-\vec{v}_{2,O}\right|}_{Q}\right) \quad (2.13)$$

$$E_a = \frac{1}{8}\left(\frac{m_1+m_2}{m_1m_2}\right)Q^2 \quad (2.14)$$

$$E_a = \frac{1}{8}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)Q^2 \quad (2.15)$$

Eq. (2.15) demonstrates the relationship between Absolute Energy and magnitude of the centrial momentum in a two-body system in centrial motion. As can be seen, the Absolute Energy is proportional to the square quantity of the magnitude of centrial momentum.

Below, the Absolute Energy of a more general case of a system in centrial motion, as illustrated in [Fig. 3], is calculated as an additional example:

2.2.2. Absolute Energy of an Example System in Centrial Motion

Now we turn our attention to an example as illustrated in [Fig. 3] to apply the theoretical framework that we introduced earlier. To begin our analysis, it must first be determined whether this system is in centrial motion. To find out this, we can note that the axes of linear momentum vectors and the axes of the summations of considered groups of linear momentum vectors all pass through the central reference point *O*, whereupon there is an initial energy blast. As can be seen, $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_5$ and $\vec{p}_6$, all pass through point *O*. Considering $\vec{p}_3$ and $\vec{p}_4$ together as $\vec{p}_{(3+4)}$ in a vector summation, and $\vec{p}_7$ and $\vec{p}_8$ as $\vec{p}_{(7+8)}$, then it is noted that both of these resultant summations pass through *O* as well. This, then, clearly satisfied the first condition for centrial motion. In addition, we assume that the initial energy blast at *O* occurs concurrently at the same point in time. We are also given the below for the purposes of this analysis as follows:

$$\vec{p}_1+\vec{p}_2=0, \qquad \vec{p}_{(3+4)}+\vec{p}_5=0, \qquad \vec{p}_6+\vec{p}_{(7+8)}=0$$

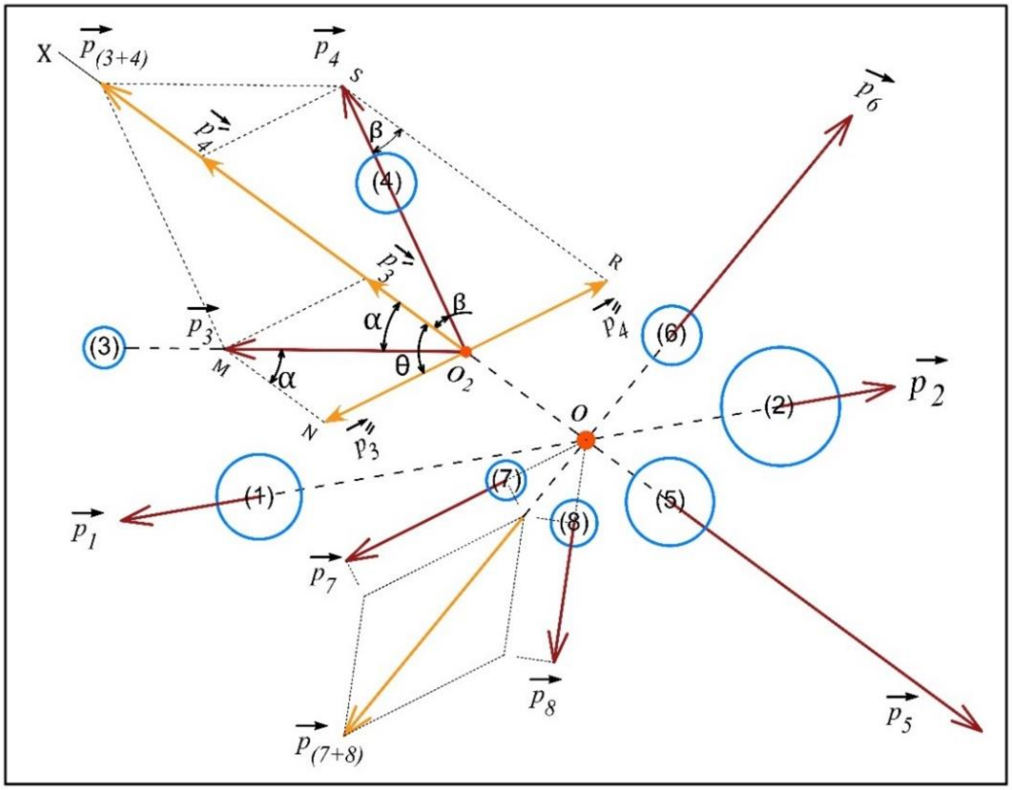

Fig. 3. Absolute energy calculations involving analysis of momentums subsequent to two energy blasts occurring at points O and O_2, noting the momentum vectors.

Per the above, all momentums cancel one another via reference point O and thus, $\sum \vec{p} = 0$. Therefore, it can be concluded that this system is indeed in centrial motion. This conclusion is based on that the fact that the two conditions for centrial motion are satisfied and noting that all the bodies can be shown to cancel each other's respective momentums via reference point O.

In this scenario, momentum vectors of $\vec{p}_3$ and $\vec{p}_4$ are intersecting at O_2 along the axis of that from vector $\vec{p}_5$ as shown in [Fig. 3]. As their summation vector, $\vec{p}_{(3+4)}$, is equal in magnitude but opposite in direction to that of $\vec{p}_5$, then it can be derived that there is a secondary energy blast that occurs at a later point in time at O_2. This results in a secondary centrial motion with momentums components $\overrightarrow{p''}_3$ and $\overrightarrow{p''}_4$. This occurs at an angle $\angle\theta$ as shown in [Fig. 3]. As a result, this system can be considered to encompass two centrial motions. For further analysis, the vector decomposition of the velocities of the bodies is also obtained and is presented in [Fig. 4].

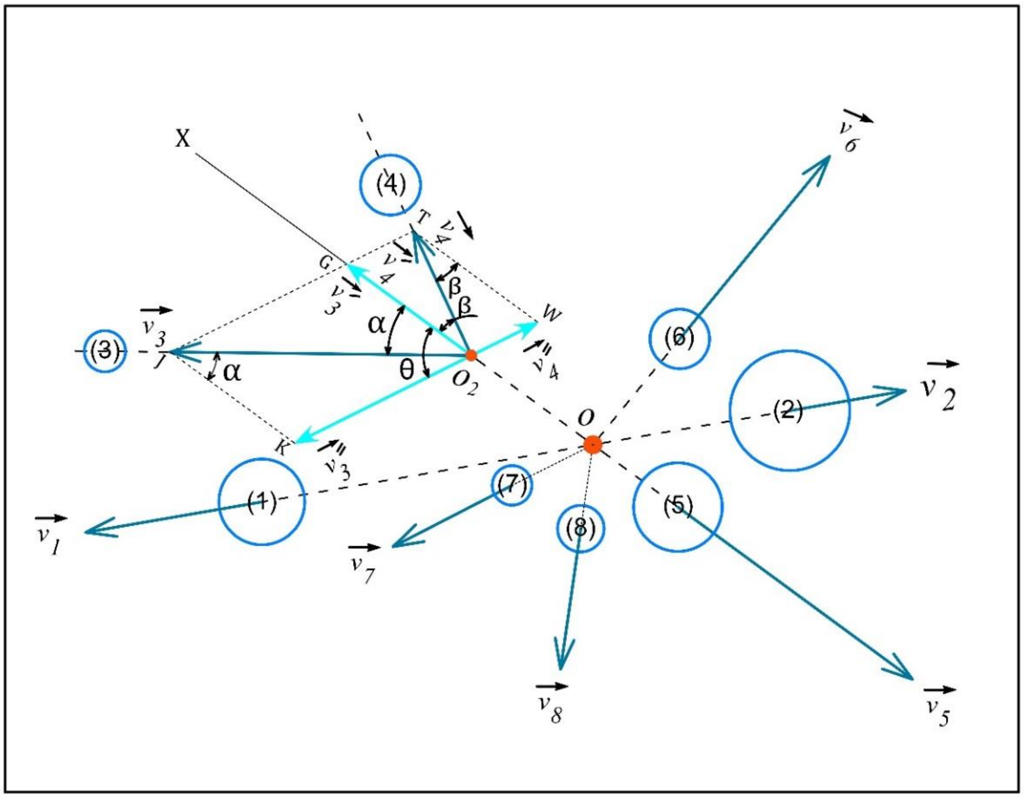

Fig 4. Absolute energy calculations involving analysis of velocities subsequent to two energy blasts occurring at points O and O_2 noting the velocity vectors.

Now, to obtain the total of two Absolute Energies of the entire system (i.e., the total Absolute Energy of these two centrial motions), we consider each momentum and its corresponding velocity and therefore the total Absolute Energy of the two energy blasts is calculated as follows:

$$E_{a,total} = \frac{1}{2}(p_1 v_1 + p_2 v_2 + p_3 v_3 + p_4 v_4 + p_5 v_5 + p_6 v_6 + p_7 v_7 + p_8 v_8) \qquad (2.16)$$

To separately calculate the magnitude of each energy blast, we first determine the value of $\angle\theta$. By given the angles of motion of bodies (3) and (4) relative to their initial axes of motion, denoted as $\angle\alpha$ and $\angle\beta$ respectively, and the magnitudes of $\vec{v}_3$ and $\vec{v}_4$, we can derive $\angle\theta$. Based on the principles outlined in the paper on centrial motion and centrial momentum, the initial velocities of bodies (3) and (4), $\vec{v'}_3$ and $\vec{v'}_4$, must be equal.

As it shown in [Fig. 4], the velocities $\vec{v''}_3$ and $\vec{v''}_4$ gained by bodies (3) and (4) from the second blast, must be in opposite directions but along the same axis. Consequently, as shown in the figure and based on this analysis, $\vec{v}_3$, the final velocity of body (3), is the vector sum of $\vec{v'}_3$ and $\vec{v''}_3$, Similarly, $\vec{v}_4$, the final velocity of body (4), is the vector sum of $\vec{v'}_4$ and $\vec{v''}_4$. In considering triangles $\triangle\, O_2JG$ and $\triangle\, O_2TG$, we obtain the below geometric relationships:

$$\frac{v'_3}{sin(\theta-\alpha)} = \frac{v_3}{sin(\pi-\theta)} \Rightarrow v'_3 = v_3 \frac{sin(\theta-\alpha)}{sin\,\theta} \quad (2.17)$$

$$\frac{v'_4}{sin(\pi-(\theta+\beta))} = \frac{v_4}{sin\,\theta} \Rightarrow v'_4 = v_4 \frac{sin(\theta+\beta)}{sin\,\theta} \quad (2.18)$$

Since, v'_3 is equal to v'_4,

$$v_3\,sin(\theta-\alpha) = v_4\,sin(\theta+\beta) \quad (2.19)$$

In Eq. (2.19), all of parameters are known except ∠θ, therefore we can obtain ∠θ.

Now that we know the value for $\angle\theta$, we are able to separately calculate each of the energy blast's magnitude. For calculating the primary energy blast at O, we may utilize one of two methods. In the first method, energy from momentums ($\overrightarrow{p'}_3$ and $\overrightarrow{p'}_4$) of body (3) and (4) that are directed in the direction of X-axis can be calculated by utilizing velocities ($\overrightarrow{v'}_3$ and $\overrightarrow{v'}_4$). This can then be added to the energy from the other bodies (1), (2), (5), (6), (7), and (8) to calculate the total Absolute Energy of the primary energy blast. In the second method, the energy from the secondary energy blast can be calculated and subsequently, this can be deducted from the total energy of the system $E_{a,total}$. In the second method, we must find $\overrightarrow{p''}_3$ and $\overrightarrow{p''}_4$ and their corresponding velocities $\overrightarrow{v''}_3$ and $\overrightarrow{v''}_4$. In this example, we are going to utilize the second method and to solve for its components as follows:

$$E_{a,2nd} = \frac{1}{2}\left(p''_3 v''_3 + p''_4 v''_4\right) \quad (2.20)$$

In considering triangle $\triangle\, O_2$MN in [Fig. 3], the below geometric relationship is utilized to solve for $\overrightarrow{p''}_3$:

$$\frac{p''_3}{sin\,\alpha} = \frac{p_3}{sin(\pi-\theta)} \Rightarrow p''_3 = \frac{sin\,\alpha}{sin\,\theta} p_3 \quad (2.21)$$

And similarly, in [Fig. 4], the same geometric relationship in $\triangle\, O_2JK$ can be utilized to solve for $\overrightarrow{v''}_3$ as per the below:

$$\frac{v''_3}{sin\,\alpha} = \frac{v_3}{sin(\pi-\theta)} \Rightarrow v''_3 = \frac{sin\,\alpha}{sin\,\theta} v_3 \quad (2.22)$$

Now, using the triangle relationship in $\triangle\, O_2SR$, in [Fig. 3] and $\triangle\, O_2TW$ in [Fig. 4], we can also calculate for $\overrightarrow{p''}_4$ and $\overrightarrow{v''}_4$,

$$\frac{p''_4}{sin\,\beta} = \frac{p_4}{sin\,\theta} \Rightarrow p''_4 = \frac{sin\,\beta}{sin\,\theta} p_4 \quad (2.23)$$

$$\frac{v''_4}{\sin\beta} = \frac{v_4}{\sin\theta} \Rightarrow v''_4 = \frac{\sin\beta}{\sin\theta} v_4 \tag{2.24}$$

Now substituting $\overrightarrow{p''}_3$, $\overrightarrow{v''}_3$, $\overrightarrow{p''}_4$, and $\overrightarrow{v''}_4$ in Eq. (2.20),

$$E_{a,2nd} = \frac{1}{2}\left(\frac{\sin\alpha}{\sin\theta} p_3 \cdot \frac{\sin\alpha}{\sin\theta} v_3 + \frac{\sin\beta}{\sin\theta} p_4 \cdot \frac{\sin\beta}{\sin\theta} v_4\right) \tag{2.25}$$

$$E_{a,2nd} = \frac{1}{2}\left(\frac{\sin^2\alpha}{\sin^2\theta} p_3 v_3 + \frac{\sin^2\beta}{\sin^2\theta} p_4 v_4\right) \tag{2.26}$$

$$E_{a,2nd} = \frac{1}{2\sin^2\theta}(\sin^2\alpha \cdot p_3 v_3 + \sin^2\beta \cdot p_4 v_4) \tag{2.27}$$

$E_{a,2nd}$ is the Absolute Energy of the secondary energy blast derived in terms of the known values for p_3, v_3, p_4, v_4, α, β, and θ. It can be deducted from the total energy to obtain the value for the primary energy blast.

2.2.3. Absolute Energy for a Concentric Bodies Centrial Motion

As defined by the author in the paper entitled "*The MM Theory: A Fundamental Revision of the Laws of Motion and Introducing Centrial Motion and Centrial Momentum*" the "Concentric Bodies Centrial Motion" is a motion where the start or end of the motions of all considered bodies in a centrial motion system is a single point.

Here, it is assumed that, in a centrial motion, *n* bodies with masses m_1, m_2…and m_n are moving so that their axes of motions pass through the center point and their velocities with respect to the center are denoted as v_1, v_2… and v_n respectively and all bodies moving outwardly or inwardly. Then,

$$E_a = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots \ldots + \frac{1}{2} m_n v_n^2 \tag{2.28}$$

$$E_a = \frac{1}{2}\left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \cdots \ldots + \frac{p_n^2}{m_n}\right) \tag{2.29}$$

$$E_a = \frac{1}{2}(p_1 v_1 + p_2 v_2 + \cdots \ldots + p_n v_n) = \frac{1}{2}\sum_{i=1}^{n} p_i v_i \tag{2.30}$$

2.2.4. Absolute Energy for a Concentric Bodies Centrial Motion with Equal Magnitudes of Linear Momentums

Here the magnitude of the linear momentum of all bodies of a concentric centrial motion are equal to one another. The Absolute Energy for this centrial motion is derived as follows:

$$E_a = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \cdots \ldots + \frac{1}{2} m_n v_n^2 \quad (2.31)$$

$$E_a = \frac{1}{2}\left(\frac{p_1^2}{m_1} + \frac{p_2^2}{m_2} + \cdots \ldots + \frac{p_n^2}{m_n}\right) \quad (2.32)$$

Since, $p_1 = p_2 = \cdots \ldots = p_n = p$

$$E_a = \frac{1}{2} p^2 \left(\frac{1}{m_1} + \frac{1}{m_2} + \cdots \ldots + \frac{1}{m_n}\right) \quad (2.33)$$

$$Q = p_1 + p_2 + \cdots \ldots + p_n = np \quad \Rightarrow \quad p = \frac{Q}{n} \quad (2.34)$$

$$E_a = \frac{1}{2n^2}\left(\frac{1}{m_1} + \frac{1}{m_2} + \cdots \ldots + \frac{1}{m_n}\right) Q^2 \quad (2.35)$$

We can also rewrite Eq. (2.31), as follows:

$$E_a = \frac{1}{2}\big(m_1 v_1 (v_1) + m_2 v_2 (v_2) + \cdots \ldots + m_n v_n (v_n)\big) \quad (2.36)$$

$$E_a = \frac{1}{2} p (v_1 + v_2 + \cdots \ldots + v_n) \quad (2.37)$$

$$E_a = \frac{1}{2n} Q (v_1 + v_2 + \cdots \ldots + v_n) = \frac{1}{2n} Q \sum_{i=1}^{n} v_i \quad (2.38)$$

2.2.5. *Absolute Energy for a System with Symmetrical Centrial Motion*

As defined in the above-mentioned paper, the "symmetrical centrial motion is a motion where there are a number of bodies with equal and uniform mass that are equidistant from one another, with each body having the same distance to the central point of reference, and having the same speeds with respect to that central point. Additionally, the directions of all their motions are either inward or outward." The Absolute Energy of this system is derived as follow where: *m* is the mass of each body, *n* is the number of bodies, and *v* is the velocity of each body with respect to the center point:

$$E_a = n\left(\frac{1}{2} m v^2\right) = \frac{1}{2}(n m v^2) \quad (2.39)$$

Since the mass of all bodies sum together would be $M = nm$, then,

$$\boxed{E_a = \frac{1}{2} M v^2} \quad (2.40)$$

also,

$$E_a = n\left(\frac{1}{2}mv^2\right) = \frac{1}{2n}\left(\frac{n^2m^2v^2}{m}\right) = \frac{1}{2nm}Q^2 \tag{2.41}$$

$$\boxed{E_a = \frac{1}{2M}Q^2} \tag{2.42}$$

To obtain the Absolute Energy for a symmetrical centrial motion base on the speed of momentum,

$$E_a = n\left(\frac{1}{2}mv^2\right) = \frac{1}{2}(np)v \tag{2.43}$$

$$\boxed{E_a = \frac{1}{2}Qv} \tag{2.44}$$

Therefore, in the case presented above, the relationship in Eq. (2.42) shows that Absolute Energy is proportional to Q^2. In turn, Eq. (2.44) shows a linear relationship with respect to v.

2.3. Angular Motion and Absolute Energy

In the study of rotational bodies, angular motion plays a critical role in understanding the behavior of rigid bodies and systems in motion. The concept of Absolute Energy, introduced in this paper, provides a framework for analyzing systems with such motion as well as linear motion. As detailed, the concept of Absolute Energy has already been established for linear motion in the context of centrial motion. Now to derive its application to angular motion, a careful examination of angular momentum is required. By identifying the relationship between angular motion and Absolute Energy, the definition of Absolute Energy is expanded to encompass systems of rotational bodies.

To begin our discussion, it is important to firstly define Angular Momentum. Angular momentum $(\vec{L})$ is defined by a vector quantity that describes the rotational motion of a rigid body or a system of rigid bodies. For a rigid body rotating about an axis, angular momentum is expressed as $\vec{L} = I\vec{\omega}$, where I is the moment of inertia of the body about the axis of rotation, and $\vec{\omega}$ is the angular velocity vector that defines the magnitude and direction of rotation.

For systems of multiple bodies, when the total angular momentum relative to a given point is zero $\left(\vec{L}_{Total} = 0\right)$, this means that the angular momentums of all bodies have cancelled one another. This condition is the basis for defining the Absolute Energy of rotational system of bodies. However, it is not a sole sufficient condition for establishing the definition of Absolute Energy that we will find later on in this paper.

2.3.1. Definition of Pure Angular Momentum

Angular momentum is defined as pure when it remains at a constant quantity and direction irrespective of the choice of the reference point in space. It is crucial to emphasize that this invariance is not limited to just symmetric systems; it can also arise in systems where the distribution of mass and motion are with a consistent total angular momentum relative to any point.

To illustrate pure angular momentum, an evenly mass-distributed ring rotating about its center *O* with angular velocity $\vec{\omega}$ can be considered as shown in [Fig. 5]. This ring is noted to lie in what is denoted as Plane (1), parallel to the XY-plane, with its mass distributed at a radius *r* from *O*. It can be shown, in this example, that the angular momentum of this ring remains invariant when calculated with respect to an arbitrary reference point O' in 3D space.

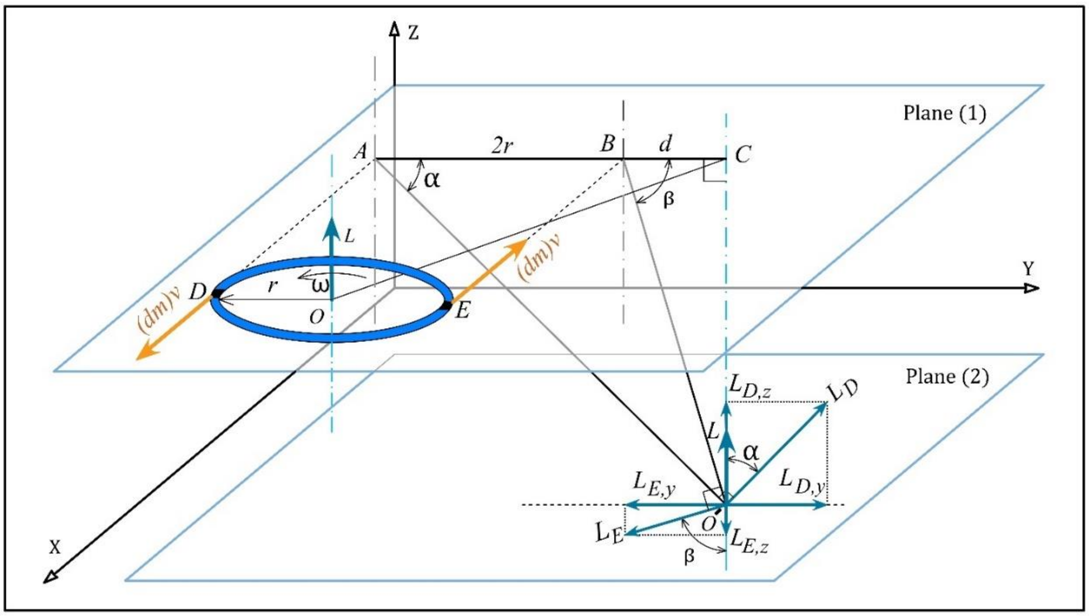

Fig. 5. Illustration of a ring demonstrating that its angular momentum remains invariant regardless of the reference point chosen, helping to establish a new framework for rotational motion.

The analysis to confirm the invariance of angular momentum in this example is as follows:

The ring's mass can be divided into infinitesimal elements *dm*, each with linear momentum $\overrightarrow{dp} = dm \cdot \vec{v}$, where $\vec{v} = \vec{\omega} \times \vec{r}$. Consider two such elements located at points *D* and *E* on opposite sides of the ring at which their axes of motions are perpendicular to the ZY-plane. To calculate the angular momentum of these elements relative to O', the perpendicular distances from O' to the axes of motions of elements at *D* and *E* are determined. Line $O'A$ is drawn perpendicular to *DA*, the axis of motion of *dm* at *D*, and line $O'B$ is drawn perpendicular to *EB*, the axis of motion of *dm* at *E*.

The angular momentum contributions from each of the two elements can now be calculated with respect to O' and are denoted as L_D and L_E as seen in [Fig 5]. L_D denotes the angular momentum of *dm* at *D* relative to O', directed perpendicular to the plane formed by O' and the axis *DA*, while

L_E represents the angular momentum of *dm* at *E* relative to O', directed perpendicular to the plane formed by O' and the axis *EB*. These angular momentum contributions can be decomposed into components along the Z-axis and Y-axis: $L_{D,z}$ and $L_{E,z}$ are the projections of L_D and L_E along the Z-axis, respectively, while $L_{D,y}$ and $L_{E,y}$ are the projections of L_D and L_E along the Y-axis. From the geometric relationships illustrated in the figure, α is the angle between line $O'A$ and Plane (1), while β is the angle between line $O'B$ and Plane (1). Additionally, point *C* is where the perpendicular line taken from O' to Plane (1) intersects at Plane (1). From this, the distance between points *B* and *C* is denoted as *d*.

Based on the above discussion, first, we begin by calculating the angular momentum of these elements with respect to *O*:

$$L_O = I\omega = 2(dmr^2)\omega = 2(dmr^2)\frac{v}{r} \tag{2.45}$$

$$L_O = 2rdmv \tag{2.46}$$

Next, the angular momentum of each element (at *D* and *E*) is individually calculated with respect to O':

$$L_D = (O'A)dmv, \qquad L_E = (O'B)dmv \tag{2.47}$$

Now, projections of L_D and L_E on the Z-axis and combining their quantities produces the below result for the total angular momentum, L_Z:

$$L_Z = L_{D,z} - L_{E,z} = L_D \cos\alpha - L_E \cos\beta \tag{2.48}$$

$$L_Z = (O'A)dmv\frac{2r+d}{O'A} - (O'B)dmv\frac{d}{O'B} \tag{2.49}$$

$$L_Z = 2rdmv \tag{2.50}$$

Similarly, projections of L_D and L_E along the Y-axis yields the below:

$$L_Y = L_{D,y} - L_{E,y} = L_D \sin\alpha - L_E \sin\beta \tag{2.51}$$

$$L_Y = (O'A)dmv\frac{O'C}{O'A} - (O'B)dmv\frac{O'C}{O'B} \tag{2.52}$$

$$L_Y = 0 \tag{2.53}$$

As a result, the Z-axis components of angular momentums $(\vec{L}_{D,z} + \vec{L}_{E,z})$ summate to the same angular momentum $(\vec{L}_O)$ as that taken from the elements about *O*. This is while the Y-axis components $(\vec{L}_{D,y} + \vec{L}_{E,y})$ cancel one another due to their opposing directions.

Thus, the total angular momentum of the ring relative to O' is identical to the angular momentum relative to *O*. The above analysis demonstrates that the angular momentum of the ring remains invariant with respect to the choice of reference point in space, establishing the presence of pure angular momentum.

Now having established that a rotating ring possesses pure angular momentum when it rotates about its center, this conclusion can be extended to other geometrical bodies via inference. For example, a disk, being composed of an infinite number of concentric rings, can also logically be shown to exhibit pure angular momentum when it rotates about its center. Similarly, a sphere, which can be viewed as consisting of an infinite number of rotating rings or an infinite number of concentric disks, likewise then possesses pure angular momentum when it rotates about its central axis. As another example, a cylinder can be viewed as an aggregation of infinite coaxial rings or disks. As each individual ring or disk contributes pure angular momentum when rotating about their shared axis, the entire cylinder, by extension, also exhibits pure angular momentum about its central axis of rotation.

Based on the above, the concept of pure angular momentum can be generalized for systems with symmetric mass distributions. That is, the contributions of individual mass elements in symmetry ensures that the total angular momentum remains invariant with respect to the choice of reference point, which leads to being classified as pure. This result highlights a direct relationship between rotational symmetry and the invariance of angular momentum. As such, this forms the basis for understanding further complex rotational dynamics in both theoretical and practical contexts.

2.3.2. Another example for Pure Angular Momentum

Another case example consists of two bodies m_1 and m_2 rotating about point *O* with both having angular velocity $\vec{\omega}$. These bodies are positioned on opposite sides of *O* with respective radii r_1 and r_2 [Fig 6].

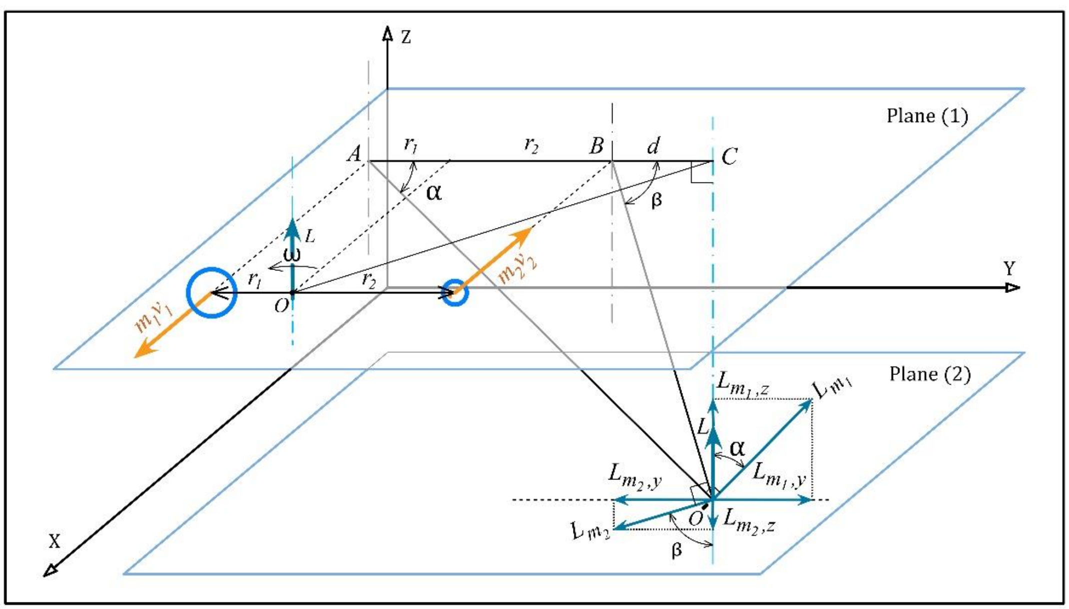

Fig. 6. An example for the studding pure angular momentum. This scenario consists of two bodies m_1 and m_2 rotating about point *O*, both with the same angular velocity $\vec{\omega}$

In this scenario, the centrifugal forces of both bodies relative to O are equal:

$$F_1 = F_2 \Rightarrow m_1 \omega_1^2 r_1 = m_2 \omega_2^2 r_2 \Rightarrow m_1 v_1 \omega_1 = m_2 v_2 \omega_2 \quad (2.54)$$

Since the angular velocity for both are the same, it follows that:

$$m_1 v_1 = m_2 v_2 \quad (2.55)$$

As we did for the previous scenario, we begin by determining the angular momentum of the system with respect to O.

$$L_O = r_1 m_1 v_1 + r_2 m_2 v_2 \qquad \text{since} \quad m_1 v_1 = m_2 v_2, \quad (2.56)$$

$$L_O = (r_1 + r_2) m_1 v_1 \quad (2.57)$$

Next, we determine the angular momentum of the system with respect to O':

$$L_{m_1} = (O'A) m_1 v_1, \qquad L_{m_2} = (O'B) m_2 v_2 \quad (2.58)$$

Now, projections of L_{m_1} and L_{m_2} on the Z-axis and combining their quantities produces the below result:

$$L_Z = L_{m_1,z} - L_{m_2,z} = L_{m_1} \cos\alpha - L_{m_2} \cos\beta \quad (2.59)$$

$$L_Z = (O'A) m_1 v_1 \frac{(r_1 + r_2 + d)}{O'A} - (O'B) m_2 v_2 \frac{d}{O'B} \quad (2.60)$$

Since, $m_1 v_1 = m_2 v_2$,

$$L_Z = (r_1 + r_2) m_1 v_1 + m_1 v_1 d - m_1 v_1 d = (r_1 + r_2) m_1 v_1 \quad (2.61)$$

Therefore, the total projections of angular momentums on Z-axis remains constant. Similarly, projections of L_{m_1} and L_{m_2} along the Y-axis yields the below:

$$L_Y = L_{m_1,y} - L_{m_2,y} = L_{m_1} \sin\alpha - L_{m_2} \sin\beta \quad (2.62)$$

$$L_Y = (O'A) m_1 v_1 \frac{O'C}{O'A} - (O'B) m_2 v_2 \frac{O'C}{O'B} \quad (2.63)$$

$$L_Y = 0 \quad (2.64)$$

Therefore, this system exhibits pure angular momentum. As we noticed, the linear momentums of the two bodies were equal in magnitude. As a result, we can conclude that if two bodies rotate about a common point with equal linear momentums, the system inherently possesses pure angular momentum.

Another key insight that we can conclude from this scenario is that the reference point O remains fixed. In other words, the forces exerted on point O by the two bodies due to their angular motions cancel each other out. This prevents any displacement of the center point. This concept is crucial to canalize such these systems.

2.3.3. Invariance of Pure Angular Momentum in Systems with Zero Linear Momentum Projections

Now, we will demonstrate that any system in which all motions lie within a single plane and the total projections of linear momentum along both axes are zero possesses pure angular momentum.

Consider a system of n bodies moving in a plane, each with linear momentum $\vec{p}_i\,(p_{i,x}, p_{i,y})$, where $i = 1, 2, \ldots, n$. Assume that the projections of all linear momentums along the X-axis and Y-axis sum to zero, such that $\sum_{i=1}^{n} p_{i,x} = 0$ and $\sum_{i=1}^{n} p_{i,y} = 0$. We want to show that this condition leads to the conclusion that the system possesses pure angular momentum, independent of the choice of reference point.

To find the total angular momentum of the system relative to an arbitrary reference point on the plane for example *O*, first, the angular momentum of each body is determined. The angular momentum of *i*-th body relative to *O* is expressed as:

$$\vec{L}_{i,O} = \vec{r}_{i,O} \times \vec{p}_i \qquad (2.65)$$

were, $\vec{r}_{i,O} = (x_i - x_O,\ y_i - y_O)$ is the position vector of *i*-th body with respect to *O* and $\vec{p}_i = (p_{i,x},\ p_{i,y})$ is its linear momentum. In two dimensions, the cross product simplifies to a scalar quantity as follow:

$$L_{i,O} = (x_i - x_O)p_{i,y} - (y_i - y_O)p_{i,x} \qquad (2.66)$$

Then, the total angular momentum of the system is derived by summing the contributions of all bodies:

$$L_{total} = \sum_{i=1}^{n} \left[(x_i - x_O)p_{i,y} - (y_i - y_O)p_{i,x}\right] \qquad (2.67)$$

Expanding this expression gives:

$$L_{total} = \sum_{i=1}^{n} (x_i p_{i,y}) - x_O \sum_{i=1}^{n} p_{i,y} - \sum_{i=1}^{n} (y_i p_{i,x}) + y_O \sum_{i=1}^{n} p_{i,x} \qquad (2.68)$$

Since $\sum_{i=1}^{n} p_{i,x} = 0$ and $\sum_{i=1}^{n} p_{i,y} = 0$, then the terms involving x_O and y_O vanish. Thus, the total angular momentum simplifies as:

$$L_{total} = L_O = \sum_{i=1}^{n} \left[x_i p_{i,y} - y_i p_{i,x}\right] \qquad (2.69)$$

This result shows that if the projections of all linear momentums along the X- and Y-axes are zero, then the total angular momentum of the system is independent of the choice of the reference point *O*.

Our next objective is to demonstrate that the angular momentum of that system where the linear momentum projections along the X- and Y-axes of the plane of motion sum to zero, remains also pure with respect to any arbitrary reference point O' in three-dimensional space.

Let's consider the same system consists of *n* bodies moving within a plane, where their linear momentums satisfy:

$$\sum_{i=1}^{n} p_{i,x} = 0 \qquad and \qquad \sum_{i=1}^{n} p_{i,y} = 0 \tag{2.70}$$

Consider an arbitrary reference point O' in space, outside the plane of motion. We draw a line from O' perpendicular to the plane of motion. This line intersects the plane at a point O. We have already proved that the angular momentum of the system relative to O, denoted $\vec{L}_O$, is pure. This means that the angular momentum of the system is invariant with respect to any reference point within the plane, which is:

$$\vec{L}_O = \sum_{i=1}^{n} \vec{r}_{i,O} \times \vec{p}_i \tag{2.71}$$

where $\vec{r}_{i,O}$ is the position vector of the *i*-th body relative to O, and $\vec{p}_i$ is its linear momentum.

Now, we determine the angular momentum of the system relative to O'. Let $\vec{r}_{i,O'} = \vec{r}_{i,O} + \vec{r}_{O'O}$, where $\vec{r}_{O'O}$ is the vector from O' to O. The angular momentum of the *i*-th body relative to O' is:

$$\vec{L}_{i,O'} = \vec{r}_{i,O'} \times \vec{p}_i = (\vec{r}_{i,O} + \vec{r}_{O'O}) \times \vec{p}_i \tag{2.72}$$

Expanding this cross product gives us:

$$\vec{L}_{i,O'} = (\vec{r}_{i,O} \times \vec{p}_i) + (\vec{r}_{O'O} \times \vec{p}_i) \tag{2.73}$$

Then, the total angular momentum relative to O' is the sum over all *n* bodies,

$$\vec{L}_{O'} = \sum_{i=1}^{n} \vec{L}_{i,O'} = \sum_{i=1}^{n} (\vec{r}_{i,O} \times \vec{p}_i) + \sum_{i=1}^{n} (\vec{r}_{O'O} \times \vec{p}_i) \tag{2.74}$$

Since the projections of all linear momentums along X-and Y-axes is zero ($\sum_{i=1}^{n} \vec{p}_i = 0$), then the second term will be vanished, because:

$$\sum_{i=1}^{n} (\vec{r}_{O'O} \times \vec{p}_i) = \vec{r}_{O'O} \times \sum_{i=1}^{n} \vec{p}_i = \vec{r}_{O'O} \times 0 = 0 \tag{2.75}$$

Thus, the total angular momentum relative to O'simplifies to:

$$\vec{L}_{O'} = \sum_{i=1}^{n} (\vec{r}_{i,O} \times \vec{p}_i) = \vec{L}_O \quad (2.76)$$

Since, the angular momentum of the system relative to O is pure, and $\vec{L}_{O'} = \vec{L}_O$, it is concluded that the angular momentum relative to O' is also pure.

2.3.4. Invariance of Pure Angular Momentum Under a Uniformly Moving Reference Frame

Now, we want to prove that a system exhibits pure angular momentum in an initial reference frame, maintains pure angular momentum with respect to a reference frame that moves with uniform velocity.

We consider a system of n bodies that move within a plane as previous one, where the total projections of their linear momentums along the X- and Y-axes are zero:

$$\sum_{i=1}^{n} p_{i,x} = 0 \qquad and \qquad \sum_{i=1}^{n} p_{i,y} = 0 \quad (2.77)$$

Previously, we showed that such a system possesses pure angular momentum with respect to any stationary reference point O in space. Now, we extend this proof to a moving reference frame, denoted O'', that has a motion along the axis of $r_{O'',O_{cm}}$ with a constant linear velocity V relative to the original reference frame, where $r_{O'',O_{cm}}$ is the position vector of O'' relative to O_{cm} and O_{cm} is the center of mass of the system in the initial reference frame.

To analyze this, we need to choose an appropriate reference point in the original frame. We select the center of mass O_{cm} as our reference point.

The center of mass O_{cm} of the system is given by:

$$r_{O_{cm},O} = \frac{1}{M} \sum_{i=1}^{n} m_i \, r_{i,O}, \quad (2.78)$$

where M is the total mass of the system; $M = \sum_{i=1}^{n} m_i$

By taking the time derivative, the velocity of the center of mass calculated as follow:

$$v_{O_{cm},O} = \frac{1}{M} \sum_{i=1}^{n} m_i \, v_{i,O} = \frac{1}{M} \sum_{i=1}^{n} p_{i,O} \quad (2.79)$$

As we assumed, the total linear momentum of the system is zero; $\sum_{i=1}^{n} p_{i,O} = 0$, we obtain:

$$v_{O_{cm},O} = 0 \quad (2.80)$$

Since the center of mass remains fixed in the original frame, this point, O_{cm}, is an appropriate reference point for analyzing.

We have chosen O'' as a reference point in the moving frame that moves with a uniform velocity V. The position vectors in the original and moving frames are related as follows:

$$r_{i,O''} = r_{i,O_{cm}} - r_{O'',O_{cm}} \tag{2.81}$$

Since O'' moves with velocity V, along the axis of $r_{O'',O_{cm}}$, then, its position relative to O_{cm} at time t is:

$$r_{O'',O_{cm}} = Vt \tag{2.82}$$

Thus,

$$r_{i,O''} = r_{i,O_{cm}} - Vt \tag{2.83}$$

The velocities in the moving frame follow from the Galilean transformation:

$$v_{i,O''} = v_{i,O_{cm}} - V \tag{2.84}$$

Since linear momentum is given by $p_i = m_i v_i$, the transformed linear momentum in the moving frame is:

$$p_{i,O''} = p_{i,O_{cm}} - m_i V \tag{2.85}$$

Summing over all bodies, we get:

$$\sum_{i=1}^{n} p_{i,O''} = \sum_{i=1}^{n} p_{i,O_{cm}} - \sum_{i=1}^{n} m_i V \tag{2.86}$$

Since we established that $\sum_{i=1}^{n} p_{i,O_{cm}} = 0$, we can conclude that:

$$\sum_{i=1}^{n} p_{i,O''} = -MV \tag{2.87}$$

Therefore, the entire system acquires a uniform linear momentum $-MV$, which does not affect the calculations of angular momentum. However, for analyzing and evaluating of Absolute Energy, this must be considered for such systems.

Next, transforming angular momentum; The angular momentum relative to O'' is:

$$L_{O''} = \sum_{i=1}^{n} r_{i,O''} \times p_{i,O''} \tag{2.88}$$

Substituting $r_{i,O''} = r_{i,O_{cm}} - Vt$ and $p_{i,O''} = p_{i,O_{cm}} - m_i V$, then, we obtain:

$$L_{O''} = \sum_{i=1}^{n} (r_{i,O_{cm}} - Vt) \times (p_{i,O_{cm}} - m_i V) \quad (2.89)$$

Expanding the terms we get,

$$L_{O''} = \sum_{i=1}^{n} r_{i,O_{cm}} \times p_{i,O_{cm}} - \sum_{i=1}^{n} (r_{i,O_{cm}} \times m_i V) - \sum_{i=1}^{n} (Vt \times p_{i,O_{cm}}) + \sum_{i=1}^{n} (Vt \times m_i V) \quad (2.90)$$

The first term:

$$\sum_{i=1}^{n} r_{i,O_{cm}} \times p_{i,O_{cm}} = L_{O_{cm}} \quad (2.91)$$

This term is the total angular momentum relative to the center of mass, that we already proved to be pure. Because this point is located at the original frame.

For the second term, using the center mass definition; $\sum_{i=1}^{n} m_i \; r_{i,O_{cm}} = 0$, we conclude:

$$\sum_{i=1}^{n} (r_{i,O_{cm}} \times m_i V) = V \times \sum_{i=1}^{n} m_i \, r_{i,O_{cm}} = 0 \quad (2.92)$$

For the third term, since *Vt* is constant for all bodies, we can factor it out,

$$\sum_{i=1}^{n} (Vt \times p_{i,O_{cm}}) = Vt \times \sum_{i=1}^{n} p_{i,O_{cm}} \quad (2.93)$$

From our given assumption; $\sum_{i=1}^{n} p_{i,O_{cm}} = 0$, we get:

$$Vt \times \sum_{i=1}^{n} p_{i,O_{cm}} = Vt \times 0 = 0 \quad (2.94)$$

For the fourth term, we assumed that the direction of motion of the moving frame is along the same axis as $r_{O'',O_{cm}}$. That means V is parallel to $r_{O'',O_{cm}}$. This leads to:

$$\sum_{i=1}^{n} (Vt \times m_i V) = r_{O'',O_{cm}} \times MV = 0 \quad (2.95)$$

Thus, the total angular momentum in the moving frame simplifies to:

$$L_{O''} = L_{O_{cm}} \quad (2.96)$$

Since we already proved that $L_{O_{cm}}$ is pure, it can be concluded that the system exhibits pure angular momentum in the moving frame as well. This proof confirms that pure angular momentum remains invariant under transformations to reference frames moving with constant velocity along the axis of $r_{O'',O_{cm}}$.

Next, we study a case where this time using a geometric solution. In comparison to examples of symmetrical systems, that introduced earlier, this case study features a system that is not in symmetry. Despite this, it will be shown that the angular momentum of this system is also pure.

The system consists of n bodies, labeled (1), (2), ..., (n), each possessing linear momentums $p_1, p_2, \dots, p_n$. All bodies are confined to a single plane, and their motions also occur within this plane [Fig. 7]. In tracing the trajectories of the linear momentums of each body, a closed geometric shape is generated. For instance, if the points of intersection between consecutive trajectories are denoted as *A, B, C, …, N*, then the resulting shape would be a closed polygon with vertices *A*, *B*, *C, …, N*.

The length of the segment between each two adjacent points is related to the magnitude of each corresponding linear momentum with constant *K*. As such,

$$AB = K \cdot p_1, \quad BC = K \cdot p_2, \quad CD = K \cdot p_3, \quad \text{and so on, up to } n. \tag{2.97}$$

The direction of each segment is treated as a vector, with each aligning to the direction of its corresponding linear momentum. For example, segment *AB* corresponds to p_1, and similarly for others. As these segments form a closed loop, it is deduced that the vector sum of all these segments is zero:

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \cdots + \overrightarrow{NA} = 0 \tag{2.98}$$

Here, an example which has five linear motions is considered [Fig. 7]. The initial reference point is *O* and the arbitrary reference point in space is O' and we have,

$$AB = K \cdot p_1, \quad BC = K \cdot p_2, \quad CD = K \cdot p_3, \quad DE = K \cdot p_4, \quad EA = K \cdot p_5 \tag{2.99}$$

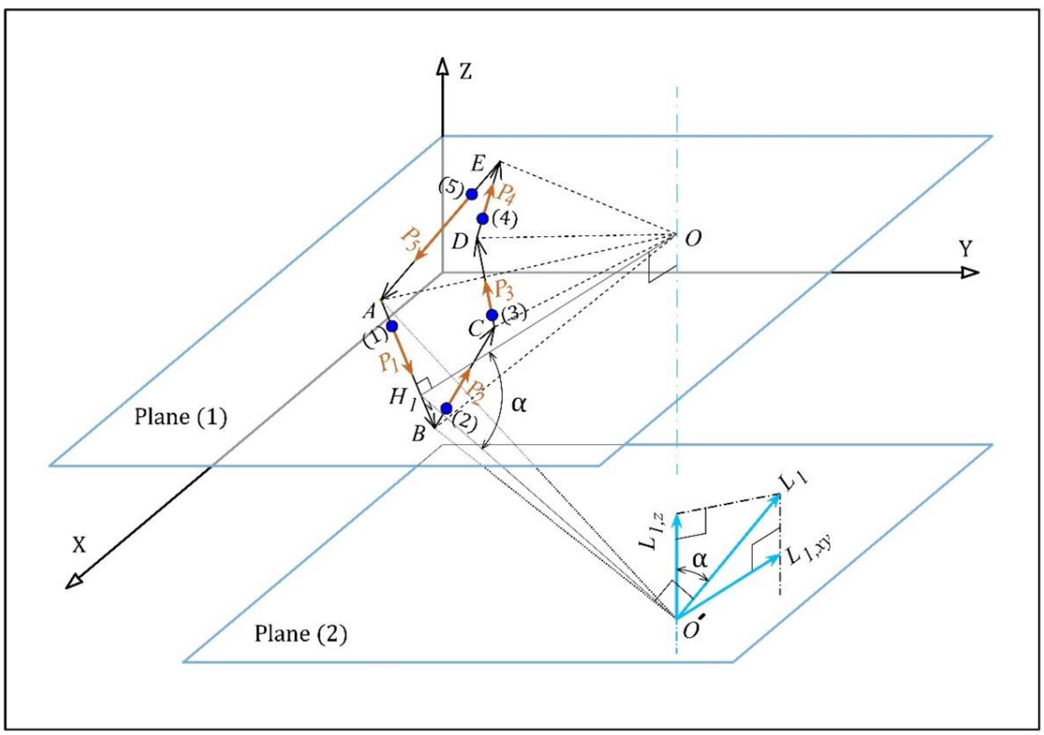

Fig. 7. an example case with five linear motions with respect to the initial reference point *O* and arbitrary reference point in space O'.

It follows then that these relationships can be re-written as per the below:

$$p_1 = \frac{AB}{K} \qquad p_2 = \frac{BC}{K} \qquad p_3 = \frac{CD}{K} \qquad p_4 = \frac{DE}{K} \qquad p_5 = \frac{EA}{K} \tag{2.100}$$

Next, the angular momentum derived from the linear momentum p_1 with respect to point O' is calculated as the product of $O'H_1$ (the perpendicular distance from point O' to the line of action of p_1) and the magnitude of p_1. It similarly follows that the angular momentums of the linear momentums p_2, p_3, p_4, and p_5 can be derived by multiplying their respective perpendicular distances from point O' to their corresponding lines of action by the magnitudes of their respective linear momentums.

For clarity, the Figure does not explicitly show all quantities related to $L_2, L_3, L_4,$ and L_5. The following relationships are derived from this analysis:

$$L_1 = O'H_1 \cdot p_1, \qquad L_2 = O'H_2 \cdot p_2, \qquad L_3 = O'H_3 \cdot p_3, \tag{2.101}$$

$$L_4 = O'H_4 \cdot p_4, \qquad L_5 = O'H_5 \cdot p_5$$

Projections of the angular momentum of L_1 is then calculated using geometric relationships,

$$L_{1,z} = L_1 \cos\alpha = O'H_1 \cdot p_1 \frac{OH_1}{O'H_1} = OH_1 \cdot p_1 \tag{2.102}$$

Similarly, the linear momentums for $L_{2,z}$, $L_{3,z}$ and $L_{4,z}$ can be calculated, albeit their directions are in the negative Z-axis in contrast to L_1. As such, we have the below:

$$L_{2,z} = -OH_2 \cdot p_2, \quad L_{3,z} = -OH_3 \cdot p_3, \quad L_{4,z} = -OH_4 \cdot p_4, \tag{2.103}$$

L_5 is in the same direction as that of the positive Z-axis,

$$L_{5,z} = OH_5 \cdot p_5$$

$$L_z = \sum L_{i,z} = L_{1,z} + L_{2,z} + L_{3,z} + L_{4,z} + L_{5,z} \tag{2.104}$$

$$L_z = OH_1 \cdot p_1 - OH_2 \cdot p_2 - OH_3 \cdot p_3 - OH_4 \cdot p_4 + OH_5 \cdot p_5 \tag{2.105}$$

$$L_z = \frac{OH_1 \cdot AB}{K} - \frac{OH_2 \cdot BC}{K} - \frac{OH_3 \cdot CD}{K} - \frac{OH_4 \cdot DE}{K} + \frac{OH_5 \cdot EA}{K} \tag{2.106}$$

$$L_z = \frac{2}{K}\left(\frac{OH_1 \cdot AB}{2} - \frac{OH_2 \cdot BC}{2} - \frac{OH_3 \cdot CD}{2} - \frac{OH_4 \cdot DE}{2} + \frac{OH_5 \cdot EA}{2}\right) \tag{2.107}$$

If we define $K' = \frac{2}{K}$ and consider each of the terms inside the bracket in Eq. (2.107) as the corresponding area of triangles, it follows that:

$$L_z = K'(A_{\triangle OAB} - A_{\triangle OBC} - A_{\triangle OCD} - A_{\triangle ODE} + A_{\triangle OEA}) \tag{2.108}$$

Refer to the [Fig. 7], we will find that:

$$L_z = K' \cdot A_{ABCDE} \quad (2.109)$$

Therefore L_z is constant because the area of polygon *ABCDE* is the same for any arbitrary reference point.

Now, we will find the sum of the projections of all angular momentum vectors on XY-plane:
From [Fig. 7]:

$$L_{1,xy} = L_1 \sin\alpha \quad = O'H_1 \cdot p_1 \frac{OO'}{O'H_1} = OO' \cdot p_1 \quad (2.110)$$

Since in this configuration OO'remains the same for the calculation of all angular momentum projections onto the XY-plane, each angular momentum projection is directly proportional to its respective linear momentum.

$$L_{2,xy} = OO' \cdot p_2, \qquad L_{3,xy} = OO' \cdot p_3, \quad (2.111)$$

$$L_{4,xy} = OO' \cdot p_4, \qquad L_{5,xy} = OO' \cdot p_5,$$

To determine the sum of all projections of the angular momentums onto the XY-plane, we show them as vectors on Plane (1) [Fig. 8(a)]. Because the axis of $L_{1,xy}$ is parallel to OH_1, and OH_1 is preopercular to segment *AB*, we can draw $L_{1,xy}$ perpendicular to segment *AB* on Plane (1). Similarly, all other projections are drawn perpendicular to their corresponding segments on Plane (1).

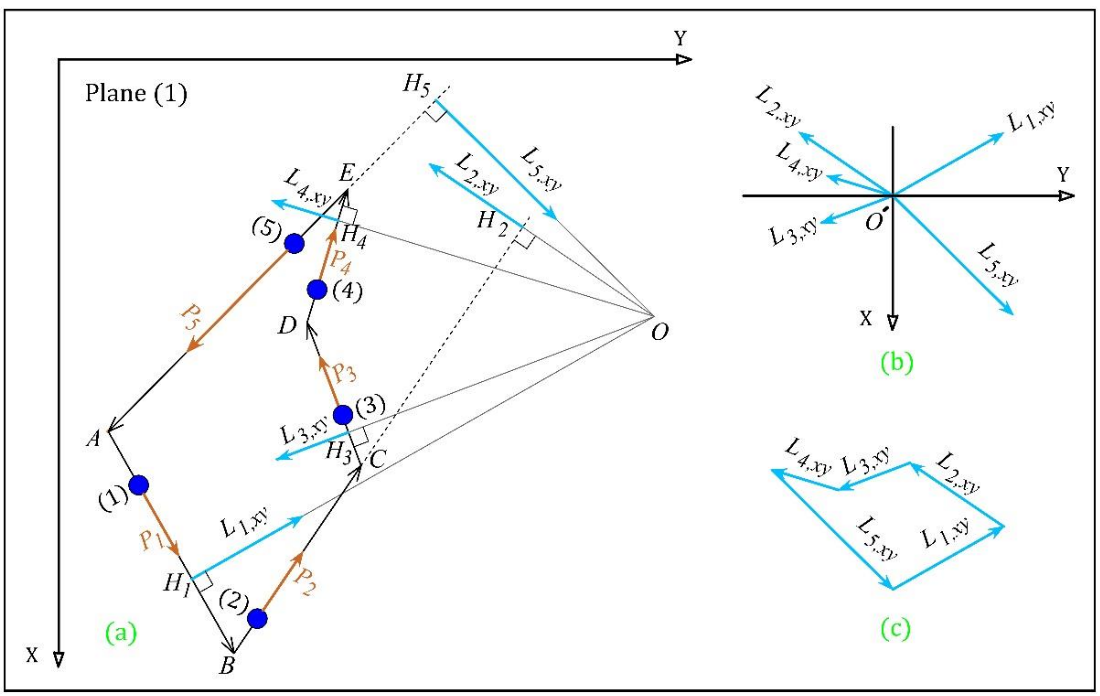

Fig. 8. projections of the angular momentums onto XY-plane: **a)** projections are drawn perpendicular to their corresponding segments, **b)** projections are drawn from point O', **c)** closed-loop shows the angular momentums projections to sum to zero.

To show a clearer visualization and determine the sum of the angular momentum projections, [Fig. 8(b)] depicts the vectors of the angular momentum projections onto Plane (2) and [Fig. 8(c)] illustrates the vectors of projections of all angular momentums as a closed-loop. Here in this configuration, each angular momentum vector is drawn perpendicular to the axis of its corresponding linear momentum and it is proportional to the magnitude of that linear momentum. For example, the direction of $L_{1,xy}$ is perpendicular to the axis of p_1, and similar relationships apply to the other vectors.

The closed-loop of these angular momentum projections vectors can be found directly from the configuration of the system. As it has been already mentioned, the trajectories of the linear momentums themselves form a closed geometric shape where each side of the shape is proportional to its corresponding linear momentum. Therefore, the projections of angular momentum vectors, that are perpendicular and proportional to the linear momentums, form a closed geometric shape as well. Because each segment of this closed shape is a vector, the sum of the vectors is zero resulting the sum of the projections of angular momentum vectors to zero.

The example in [Fig. 7] depicts a pentagon-shaped trajectory for simplicity. This scenario can be also applied to n-body systems (where $n > 1$), even for the cases where the trajectories of linear momentums cross each other and form shapes composed of multiple polygons. As long as the final trajectory forms a single closed loop and each segment follows the previous one, and sum of the vectors/segments (considering their directions) is zero, the system possesses pure angular momentum.

Here, the proportionality constant K is a key factor, to ensure that the magnitude of each linear momentum corresponds to the length of its respective segment.

2.3.5. Absolute Energy for Systems Composed of Pure Angular Momentums

For systems composed of multiple pure angular momentums in one reference frame, angular momentum for each body or subsystem can initially be defined relative to separate reference points. However, because the property of pure angular momentum is invariant, the angular momentum of each body or subsystem can be redefined with respect to a single point in space. By this, a unified reference point can be chosen for the entire system to simplify the analysis and evaluating the total angular momentum.

The total angular momentum of such a system can be expressed as:

$$\vec{L}_{total} = \vec{L}_1 + \vec{L}_2 + \cdots \vec{L}_n \quad (2.112)$$

If the system composed of multiple pure angular momentums and if $\vec{L}_{total} = 0$, the condition for defining Absolute Energy is met. Then, Absolute Energy for angular motion is defined as the total kinetic energy of the system required to bring it to rest relative to this conceptualized reference point, without transformation of kinetic energy into other forms or vis-versa.

For systems consists of bodies like ring, disk, sphere, cylindrical shape, with evenly mass-distributed and all subsystems with symmetric mass distributed, Absolute Energy can be expressed as:

$$E_a = \frac{1}{2} I_1 \ \omega_1^2 + \frac{1}{2} I_2 \ \omega_2^2 + \cdots + \frac{1}{2} I_n \ \omega_n^2 \, , \qquad (2.113)$$

where $I_{1,} \, I_{2,} \ldots I_n$ are the moments of inertia of the respective bodies or subsystems about their axes of rotation, and $\omega_{1,} \, \omega_{2,} \ldots \omega_n$ are their angular velocities.

To calculate the Absolute Energy for cases consists of systems like the one shown in [Fig. 7], we have two options: we can either consider the angular motion of each body relative to any point of reference or consider only their linear motions.

2.3.6. Definition of Non-Pure Angular Momentum

Non-pure angular momentum is the angular momentum of a body or system that is tied to a specific reference point. Unlike pure angular momentum, that is invariant regardless of the choice of reference point, non-pure angular momentum depends on the specific point of reference. In systems with one or more non-pure angular momentums, since each angular momentum may be defined with respect to a different reference point, the direct summation or analysis of the total angular momentum of such systems are more complex.

*2.3.7. Absolute Energy for Systems Composed of Non-Pure Angular Momentum*s

To calculate the total angular momentum and determine the Absolute Energy for systems with non-pure angular momentums, if there is a point that the total angular momentum relative to this unified reference point is zero, then the angular momentums cancel out algebraically. However, cancellation generally results in linear motions within the system. If the newly generated linear momentums are not fully canceled by other linear momentums generated at the same time within the system or by linear momentums of interacting systems, then the system will not be at true rest. As such, Absolute Energy cannot be defined for such systems unless all linear momentums are taken into account.

To understand this limitation, we again turn our attention to linear motions as we explored them for Absolute Energy earlier. For systems involving linear motions, for example, a scenario with two bodies with equal and opposite linear momentums along the same axis, they can collide, stop completely and change direction without changing linear momentum. This condition allows the system to be conceptually brought to rest, thus satisfying the criteria for defining Absolute Energy. Similarly, for systems with pure angular momentums for which the total angular momentum is zero, this condition is sufficient for defining Absolute Energy. However, to determine whether the components of systems with non-pure angular momentums can stop each other simultaneously, require further analysis.

To illustrate this limitation, consider a system consists of one pure angular momentum and one non-pure angular momentum. The system has three bodies: body (1) and body (2) each with mass *m* are located at a distance *r* from a central point *O* and are situated directly opposite to each other. These bodies are moving at velocities *v* tangentially and both rotating around *O* in the same direction forming a pure angular momentum [Fig. 9]. Body (3) is also of mass *m* and is located at *2r* from O and it also rotates in the same plane as bodies (1) and (2). This last body has a tangential velocity *v* and rotates in the opposite direction to bodies (1) and (2) and thus creates a non-pure angular momentum.

The angular momentum of bodies (1) and (2), which form a pure angular momentum, is calculated as $L_{(1,2)} = -2(r)mv$. For Body (3), which forms a non-pure angular momentum, the angular momentum is calculated as $L_{(3)} = (2r)mv$. By adding these, the total angular momentum of the system is:

$$L_{total} = L_{(1,2)} + L_{(3)} = -2(r)mv + (2r)mv = 0 \qquad (2.114)$$

Next, we calculate the linear momentum of the system along the Y-axis. Before any interaction occurs, the total linear momentum is:

$$\sum p_{initial} = p_1 - p_2 + p_3 = mv - mv + mv = mv \qquad (2.115)$$

If the system were to stop completely, the directions of motion for each body after the interaction would need to reverse, as a result:

$$\sum p_{final} = -p_1 + p_2 - p_3 = -mv + mv - mv = -mv \qquad (2.116)$$

Since the total linear momentum before and after the interaction in the Y-direction is not equal, the conservation of linear momentum is violated. This shows that the system cannot stop completely. Consequently, even though the system has zero total angular momentum, the conditions for defining Absolute Energy are not satisfied.

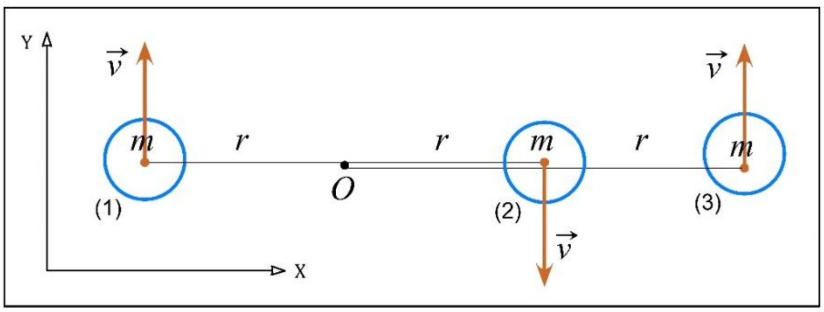

Fig. 9. a system consists of one pure angular momentum and one non-pure angular momentum.

This analysis shows that the condition $\sum L = 0$ is not sufficient to define Absolute Energy for the systems with non-pure angular momentums. The system must also have zero total linear momentum in all directions of X, Y, and Z. Only when these additional conditions are met, the

Absolute Energy for the system can be determined. This refinement highlights the importance of angular and linear momentum and their interrelationship in determining the Absolute Energy in complex systems.

2.3.8. Evaluation of Absolute Energy in Systems with Centrial Momentum and Pure Angular Momentums

For evaluating Absolute Energy, a single reference frame is needed for systems with both centrial momentum and pure angular momentum. Since pure angular momentum is reference point independent, the center of the centrial motion can be used as a single reference point to analyze and evaluate the Absolute Energy of the entire system. However, if the reference frames of the subsystems have motions with respect to one another, the linear momentums resulting from these relative motions must also be taken into account.

2.3.9. Evaluation of Absolute Energy in Systems with Pure and Non-Pure Angular Momentums

To calculate the Absolute Energy for systems composed of pure and non-pure angular momentums, a reference point that satisfy the conditions necessary for defining Absolute Energy for non-pure angular as described before, can be chosen. Since the reference point for pure angular momentums can be any point, the refence point of non-pure angular momentums can be used for pure angular momentums as well. However, if the reference frames of the subsystems have motions with respect to one another, the linear momentums resulting from these relative motions must also be taken into account."

Below is the calculation of Absolute Energy for systems with pure and non-pure angular momentum in a single frame of reference. Absolute Energy is calculated based on angular momentums of pure angular momentums and linear momentums of individual bodies in non-pure angular momentums.

$$E_a = \frac{1}{2} I_1\ \omega_1^2 + \frac{1}{2} I_2\ \omega_2^2 + \cdots + \frac{1}{2} I_n\ \omega_n^2 + \frac{1}{2} m_1\ v_1^2 + \frac{1}{2} m_2\ v_2^2 + \cdots + \frac{1}{2} m_n\ v_n^2 \tag{2.117}$$

2.3.10. Evaluation of Absolute Energy in Systems with Centrial Momentum and Pure and Non-Pure Angular Momentums

Consider a system where all motions; centrial, pure angular, and non-pure angular momentums exist within a single frame of reference. Since for pure angular momentums any point can be chosen as a reference point, any point may serve as a valid reference. If the center of centrial motion satisfies the necessary conditions for defining Absolute Energy for non-pure angular momentum, it can be used for all motions for analyzing and evaluating Absolute Energy for the entire system.

With these discussions, the concept of Absolute Energy can be easily extended from linear motion (centrial motion) to angular motion. In this way, a deeper understanding of energy dynamics in systems involving both translational and rotational motion can be found, and the framework can be used more broadly in theoretical and applied physics.

Now, return to the previous discussion and based on the above results, Table [1] presents a comparison of the similarity of linear motion, symmetrical centrial motion, and angular motion equations.

Table. 1. Comparison of the similarities for the cases of linear momentum and symmetrical centrial momentum.

Linear Motion Equations	Symmetrical Centrial Motion Equations	Angular Motion Equations
$E = \frac{1}{2}mv^2 = \frac{1}{2m}p^2 = \frac{1}{2}pv$	$E_a = \frac{1}{2}Mv^2 = \frac{1}{2M}Q^2 = \frac{1}{2}Qv$	$E = \frac{1}{2}I\omega^2 = \frac{1}{2I}L^2 = \frac{1}{2}L\omega$
$p = \frac{dE}{dv} = mv$	$Q = \frac{dE_a}{dv} = Mv$	$L = \frac{dE}{d\omega} = I\omega$
$m = \frac{dp}{dv}$	$M = \frac{dQ}{dv}$	$I = \frac{dL}{d\omega}$

2.4. Law of Conservation of Absolute Energy

Absolute Energy is a conserved quantity, offering a universal framework for energy estate which is not limited by traditional definitions of energy. Unlike the conventional definition and evaluation of energy, which relies on the observer's frame of reference, Absolute Energy is defined as independent of any specific frame of reference. This independence ensures that Absolute Energy remains conserved and constant across all contexts, regardless of the observer and frame of reference.

In contemporary physics, energy conservation is tied to reference-dependant quantities such as velocity and momentum. Such dependencies present ambiguities anywhere from evaluating complex systems to calculating the total energy of the universe. For instance, in classical and relativistic frameworks, total energy of a system depends primarily on establishing an initial point of reference, leading to complications when describing energy in a universal sense. The definition of Absolute Energy resolves this problem by relying solely on the internal motions and dynamics of the system itself.

Absolute Energy, as presented in this paper, is a unique property that is defined relative to a conceptualized system-wide reference point. This enables us to analyze motions, including linear and angular, to be accounted for without an observer's bias or any introduction of arbitrariness. In this sense, the total Absolute Energy of a closed system remains conserved and is solely dependant on a system's intrinsic properties and motions.

3. Discussion

3.1. Applications of Absolute Energy

Absolute Energy presents unique opportunities and applications across various fields of physics. In **astrophysics,** the total energy of celestial systems can be calculated by conceptualizing them as isolated systems with defined reference points. This approach corrects and enhances existing models developed for galactic dynamics, energy distributions within clusters, and cosmic interactions, offering a reliable method for quantifying the intrinsic energy in such systems.

In **thermodynamics,** Absolute Energy provides a consistent, reference-independent metric for evaluating energy transformations in isolated systems. This is found to be especially significant in cases where entropy plays a key role, allowing detailed calculations of system dynamics by excluding the ambiguity of observer dependence.

In **particle physics,** the concept of Absolute Energy redefines the fundamentals of high-energy collision analysis. Its observer-independent property ensures consistent results in determining outcomes, bridging the gap between classical mechanics and quantum theory in complex, high-energy scenarios.

Finally, there is a profound application to the **entire universe** when discussing Absolute Energy, as presented in this paper. By modelling the universe as a system that exhibits both **centrial motion** and **angular motion** the Absolute Energy framework provides a consistent methodology for quantifying the total energy of the universe. This framework has the potential to deepen our understanding of the fundamental structure, evolution, and dynamic properties of the universe and to open new avenues for cosmological studies.

3.2. Future Directions

The framework in this paper establishes a foundation for expanding on the theoretical concepts presented here and to unlock other practical applications. Future research on this work may explore more complicated systems involving linear, angular, and centrial motion, as well as by applying the concept of Absolute Energy to diverse physical cases. This may include the study of this framework when applied to high-energy particles and astrophysical phenomena. Additionally, as the universe is seen to exhibit centrial and angular motion itself, this new framework presents a novel approach to evaluating the total energy of the universe.

4. Conclusion

This paper has presented a novel approach to energy quantification by the introduction and definition of Absolute Energy. It has been defined as the total kinetic energy required to bring all bodies in a system to reset at a specific reference point. In defining Absolute Energy as such, this paper has resolved significant limitations in classical mechanic and its associated approaches.

The study highlighted the theoretical significance and applications of Absolute Energy, as newly defined in this paper. Two-body systems, concentric centrial motion, symmetric centrial motion, as well as systems involving pure and non-pure angular momentums were utilized to illustrate in examples the analytical derivations that were laid to present the foundations of the theoretical framework. In doing so, these derivations proved that this new framework presents a conserved and reference-independent methodology for analyzing linear and angular motions. As such, it provides for more precise and reliable energy quantification across a wide spectrum of systems in physics.

These findings have significant applications across multiple disciplines in physics. In astrophysics, Absolute Energy offers a novel method for studying galactic bodies and systems and in studying celestial dynamics. In thermodynamics, if offers a new method of analyzing energy transformation in isolated systems. In particle physics, it presents a consistent quantification for high-energy collisions and physical interactions. In cosmology, it offers the capability to calculate total energy of the universe, all the while showcasing that it can be done in a frame-independent manner, which poses interesting fundamental questions about the universe's total energic state.

In challenging conventional concepts and ideas, and in introducing a fresh and universal methodology for the understanding of energy dynamics, this research lays the foundation for future theoretical work, advancements, and physical experiments. The new novel concept of Absolute Energy is a significant step forward in unifying our understanding of energy in physics. With this framework, there is a substantial potential for it to refine our approach to evaluating energy dynamics in both isolated and universal systems.

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6. References

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