

# The Proof of the Riemann conjecture

LIAOTENG

*Tianzheng International Mathematical Research Institute, Xiamen, China*

## **Abstract:**

In order to strictly prove the hypotheses and conjectures in Riemann's 1859 paper "The Number of prime Numbers not greater than x" from a purely mathematical point of view, and to strictly prove the generalized hypotheses and conjectures, this paper studies the relationship between symmetric and conjugate zeros of Riemann  $\zeta(s)$  function and Riemann  $\xi(t)$  function by using Euler's formula, it is found that the symmetry and conjugation of the nontrivial zeros of the Riemann  $\zeta(s)$  function are consistent, and the zeros of the Riemann  $\xi(t)$  function are symmetric and non-conjugated, it is proved that the zeros of the Riemann  $\xi(t)$  function must be all real numbers, the Riemann hypothesis and the Riemann conjecture are completely correct.

## **Key words:**

*Euler's formula, Riemann  $\zeta(s)$  function, Riemann function  $\xi(t)$ , Riemann hypothesis, Riemann conjecture, symmetric zeros, conjugate zeros, uniqueness.*

## **I. Introduction**

The Riemann hypothesis and the Riemann conjecture is an important and famous mathematical problem left by Riemann in his 1859 paper "On the Number of primes not greater than x", which is of great significance to the study of the distribution of prime numbers and is known as the greatest unsolved mystery in mathematics. After years of hard work, I solved this problem and rigorously proved that both the Riemann conjecture and the generalized Riemann conjecture are completely correct. The Polignac conjecture, the twin prime conjecture, and Goldbach's conjecture are also completely correct. It would be nice if you understood Riemann's conjecture thoroughly from the outset of his paper "On Prime Numbers not Greater than x" and were completely convinced of the logical reasoning behind it. You need to do this before you read my paper. The following is about the first half of Riemann's paper "On the Number of primes not Greater than x", which I have explained and derived, which is the premise and basis for your understanding of Riemann's conjecture.

In 1859, Riemann was admitted to the Berlin Academy of Sciences as a corresponding member, and in order to express his gratitude for the honor, he thought it would be best to use the permission he received immediately to inform the Berlin Academy of a study on the density of the distribution of prime numbers, a subject in which Gauss and Dirichlet had long been interested. It does not seem entirely unworthy of a report of this nature.

Riemann used Euler's discovery of the following equation as his starting point:

$$\prod_{p=1}^{\infty} \left( \frac{1}{1 - p^{-s}} \right) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

(1)

Where  $p$  on the left side of the equation takes all prime numbers,  $n$  on the right side takes all natural numbers, and the function of the complex variable  $s$  represented by the two series above (when they converge) is denoted by  $\zeta(s)$ . That is, to define a function of complex variables:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-s}} \right).$$

The two series above converge only if the real part of  $s$  is greater than 1, is also say when

$\text{Re}(s) > 1$ , then  $\sum_{n=1}^{\infty} \frac{1}{n^s}$  and  $\prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-s}} \right)$  converge only. if  $s=1$ , then  $\sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ , It's

called a harmonic series, and it diverges. If  $\text{Re}(s) < 1$ ,  $\sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \dots$ , it's more

divergent. Because if  $\text{Re}(s) < 1$ , then  $\frac{1}{1^s} = \frac{1}{1}, \frac{1}{2^s} > \frac{1}{2}, \frac{1}{3^s} > \frac{1}{3}, \frac{1}{4^s} > \frac{1}{4} \dots$ , . But if  $s$  is a negative

number, for example  $s = -1$ , then it does not satisfy the condition that  $\text{Re}(s) > 1$ . So you need to find

an expression for  $\zeta(s)$  function a that is always valid for any  $s$ . In modern mathematical language,

that is, to carry out an analytical extension of a complex function  $\zeta(s)$ , and the best way to analyze

the extension is to find a more extensive and effective representation of the function such as an

integral representation or an appropriate function representation. Therefore, we want to define a

new function, this new function also  $\zeta(s)$  to represent, this new function of the independent

variable  $s$  is not only full  $\text{Re}(s) > 1$ , but also satisfy  $\text{Re}(s) \leq 1 (s \neq 1)$ , and the function image is

smooth, every point on the function image can find its tangent slope, that is, the function

everywhere can find the derivative. However, it is no longer called the Euler zeta() function, but

the Riemann zeta() function. Riemann used the integral to express the function  $\zeta(s)$ . In this paper,

I have added another complex variable to express the Riemann function  $\zeta(s)$ .

Because  $\Pi(s) = \Gamma(s+1) = s\Gamma(s)$ , where  $\Pi(s)$  is the factorial function,  $\Gamma(s)$  is the Euler gamma

function,  $\Gamma(s) = \int_0^{\infty} x^{s-1} e^{-x} dx$ , Let the variable  $x \rightarrow nx (n \in \mathbb{Z}^+)$  in the integral symbol, then

$$\int_0^{\infty} (nx)^{s-1} e^{-nx} d(nx) = n \int_0^{\infty} e^{-nx} n^{s-1} x^{s-1} = n^s \int_0^{\infty} e^{-nx} x^{s-1} = \Gamma(s) = \Pi(s-1), \text{ so}$$

$$\int_0^{\infty} e^{-nx} x^{s-1} = \frac{\Pi(s-1)}{n^s}$$

That's exactly what Riemann says in his paper, he says he's going to use

$$\int_0^{\infty} e^{-nx} x^{s-1} = \frac{\Pi(s-1)}{n^s}$$

Since  $n$  is all positive integers, we need to assign  $\sum$  to  $e^{-nx}$  and  $\frac{1}{n^s}$  on both sides of the equation,

$$\text{so } \sum_{n=1}^{\infty} e^{-nx} = 1 + \sum_{n=1}^{\infty} e^{-nx} - 1 = (1 + e^{-x} + e^{-2x} + e^{-2x} + \dots) - 1 = \frac{1}{1-e^{-x}} - 1 = \frac{e^{-x}}{1-e^{-x}} = \frac{1}{e^x - 1},$$

The common ratio  $q$  satisfies  $0 < q = |e^{-x}| < 1 (0 < x \rightarrow +\infty)$ ,  $\frac{\Pi(s-1)}{n^s} = \frac{\Pi(s-1)}{1^s + 2^s + 3^s + 4^s + 5^s + \dots}$ ,

and  $\sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s + 2^s + 3^s + 4^s + 5^s + \dots} = \zeta(s)$ , so according

$$\int_0^{\infty} e^{-nx} x^{s-1} = \frac{\Gamma(s)}{n^s}$$

,can get  $\Gamma(s) \zeta(s) = \int_0^{\infty} \frac{x^{s-1} dx}{e^x - 1}$ , this is exactly what Riemann found in his paper.

Now consider the following integral

$$\int \frac{(-x)^{s-1} dx}{e^x - 1}$$

According to modern mathematical notation, the integral should be denoted as  $\int_C \frac{(-z)^{s-1} dz}{e^z - 1}$ , or considering that the complex number is generally represented by z, the integral should be denoted as  $\int_C \frac{(-Z)^{s-1} dZ}{e^Z - 1}$ , Its integral path proceeds from  $+\infty$  to  $+\infty$  on the forward boundary of a region containing the value 0 but not any other singularities of the integrable function, where the integral path C is shown in Figure 1 below.

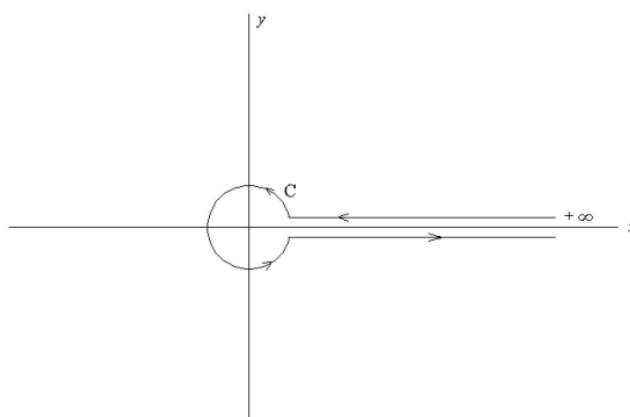


Figure 1

To obtain the value of this integral, we assume that there is a complex number of arbitrarily small moduli  $\delta$ , and that the moduli  $|\delta|$  of  $\delta, |\delta| \rightarrow 0$ , Because  $(-Z)^s = e^{s \ln(-Z)}$ , and  $\ln(-Z) =$

$\ln(Z) + \pi i$  or  $\ln(-Z) = \ln(Z) - \pi i$ , so

$$\int_C \frac{(-Z)^{s-1} dZ}{e^Z - 1} = \int_{+\infty}^{\delta} \frac{(-Z)^{s-1} dZ}{e^Z - 1} + \int_{\delta}^{\infty} \frac{(-Z)^{s-1} dZ}{e^Z - 1} + k \int_{|\delta| \rightarrow 0} \frac{(-Z)^{s-1} dZ}{e^Z - 1} = \int_{+\infty}^{\delta} \frac{(-Z)^s dZ}{(e^Z - 1)Z} + \int_{\delta}^{+\infty} \frac{(-Z)^s dZ}{(e^Z - 1)Z} + k \int_{|\delta| \rightarrow 0} \frac{(-Z)^s dZ}{(e^Z - 1)Z} = (e^{\pi s i} - e^{-\pi s i}) \int_{\delta}^{\infty} \frac{e^{s \ln(Z)} dZ}{(e^Z - 1)Z} + k \int_{|\delta| \rightarrow 0} \frac{(-Z)^s dZ}{(e^Z - 1)Z}, \quad k \text{ is a constant.}$$

The definition of trigonometric functions of complex variables is given by Euler's formula

$\sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$ , if  $z = \pi s$ , then  $\sin(\pi s) = \frac{e^{\pi si} - e^{-\pi si}}{2i}$ . so  $e^{\pi si} - e^{-\pi si} = 2i \sin(\pi s)$ ,  $i = \frac{e^{\pi si} - e^{-\pi si}}{2 \sin(\pi s)}$ . so

$\int_C \frac{(-z)^{s-1} dz}{e^z - 1} = (e^{\pi si} - e^{-\pi si}) \int_{\delta}^{\infty} \frac{e^{s \ln(z)} dz}{e^z - 1} + k \int_{|\delta| \rightarrow 0} \frac{(-z)^s dz}{(e^z - 1)z}$ , if  $\delta$  is a real number and the

absolute value  $|\delta|$  of  $\delta$ ,  $|\delta| \rightarrow 0$ ,

then  $\int_{|\delta| \rightarrow 0} \frac{(-z)^s dz}{(e^z - 1)z} = 0$  then  $\int_C \frac{(-z)^{s-1} dz}{e^z - 1} = 2i \sin(\pi s) \int_0^{\infty} \frac{x^{s-1} dx}{e^x - 1}$  ( $x \in \mathbb{R}$ ). then

$\frac{1}{2i \sin(\pi s)} \int_C \frac{(-z)^{s-1} dz}{e^z - 1} = \int_0^{\infty} \frac{x^{s-1} dx}{e^x - 1}$  ( $x \in \mathbb{R}$ ). We got  $\Pi(s-1)\zeta(s) = \int_0^{\infty} \frac{x^{s-1} dx}{e^x - 1}$  ( $x \in \mathbb{R}$ ) before,

so  $2 \sin(\pi s) \Pi(s-1)\zeta(s) = i \int_0^{\infty} \frac{x^{s-1} dx}{e^x - 1}$ . Where we agree that in the many-valued function

$(-x)^{s-1}$ , the value of  $\ln(-x)$  is real for negative  $x$ , thus obtaining  $2 \sin(\pi s) \Pi(s-1)\zeta(s) =$

$i \int_{-\infty}^{\infty} \frac{(-x)^{s-1} dx}{e^x - 1}$  ( $x \in \mathbb{R}$ ). This equation now gives the value of the function  $\zeta(s)$  for any

complex variable  $s$ , and shows that it is single-valued analytic, and takes a finite value

for all finite  $s$  except 1, and zero when  $s$  is equal to a negative even number. The right side

of the above equation is an integral function, so the left side is also an integral function,  $\Pi(s-1) = \Gamma(s)$ , and the first-order poles of  $\Gamma(s)$  at  $s = 0, -1, -2, -3, \dots$  cancels out  $\sin(\pi s)$ 's zero. When the real part of  $s$  is negative, the above integral can be performed not along the region positively surrounding the given value, but along the region negatively containing all the remaining complex values. See Figure 2 below, where the radius of the great circle  $C'$  approaches infinity and thus contains all poles of the integrand, i.e., all zeros of the denominator  $e^x - 1$ ,  $n\pi i$  ( $n$  is an integer), and the following calculation applies Cauchy's residue theorem.

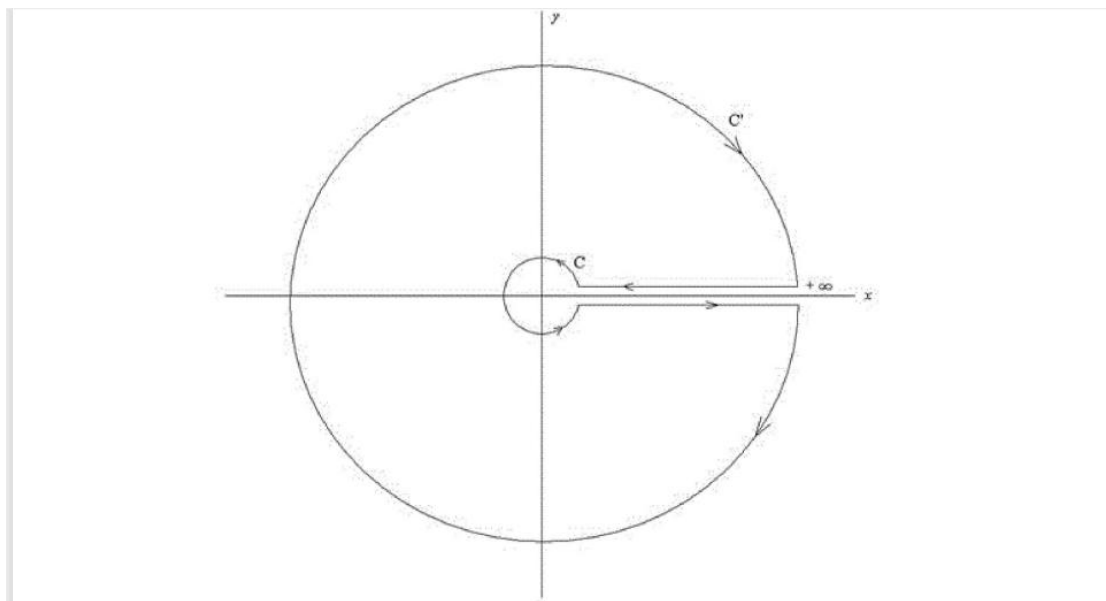


Figure 2

Since the value of the integral is infinitesimal for modular infinite complex numbers, and in this

Since the value of the integral is infinitesimal for modular infinite complex numbers, and in this region the integrand has a singularity only if  $x$  is equal to an integral multiple of  $2\pi i$ , the integral is equal to the sum of the integrals negatively around these values, but the integral around the value  $n2\pi i$  ( $n \in \mathbb{R}^+$ ) is equal to  $(-n2\pi i)^{s-1}(-2\pi i)$  ( $n \in \mathbb{R}^+$ ). The residue of the integrand at  $n2\pi i$  ( $n \neq 0$ ) is equal to

$$\left[ \frac{(-x)^{s-1}}{(e^x-1)^s} \right]_{x=n2\pi i} = \left[ \frac{(-x)^{s-1}}{e^x} \right]_{x=n2\pi i} = (n2\pi i)^{s-1} (n \neq 0).$$

So we get

$$2\sin(\pi s) \prod (s-1) \zeta(s) = (2\pi)^s \sum n^{s-1} ((-i)^{s-1} + i^{s-1}) \quad [1] \text{ (Formula 3)},$$

It reveals a relationship between  $\zeta(s)$  and  $\zeta(1-s)$ , using known properties of the function  $\Pi(s)$ , that is, using the coelements formula of the gamma function  $\Gamma(s)$  and Legendre's formula. It can also be expressed as:

$$\Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s) \text{ is invariant under the transformation } s \rightarrow 1-s.$$

based on euler's  $e^{ix} = \cos(x) + i \sin(x)$  ( $x \in \mathbb{R}$ ), can get

$$e^{i\left(-\frac{\pi}{2}\right)} = \cos\left(\frac{-\pi}{2}\right) + i \sin\left(\frac{-\pi}{2}\right) = 0 - i = -i,$$

$$e^{i\left(\frac{\pi}{2}\right)} = \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) = 0 + i = i,$$

then

$$(-i)^{s-1} + i^{s-1} = (-i)^{-1} (-i)^s + (i)^{-1} (i)^s = (-i)^{-1} e^{i\left(-\frac{\pi}{2}\right)s} + i^{(-1)} e^{i\left(\frac{\pi}{2}\right)s} =$$

$$i e^{i\left(-\frac{\pi}{2}\right)s} - i e^{i\left(\frac{\pi}{2}\right)s} = i \left( \cos\frac{-\pi s}{2} + i \sin\frac{-\pi s}{2} \right) - i \left( \cos\frac{\pi s}{2} + i \sin\frac{\pi s}{2} \right) = i \cos\left(\frac{\pi s}{2}\right) - i \cos\left(\frac{\pi s}{2}\right) + \sin\left(\frac{\pi s}{2}\right) + \sin\left(\frac{\pi s}{2}\right)$$

$$= 2\sin\left(\frac{\pi s}{2}\right) \text{ (Formula 4)}.$$

According to the property of  $\Pi(s-1) = \Gamma(s)$  of the gamma function, and

$$\sum_{n=1}^{\infty} n^{s-1} = \zeta(1-s) \quad (n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer, } s \in \mathbb{C}, \text{ and } s \neq 1),$$

Substitute the above (Formula 4) into the above (Formula 3), will get

$$2\sin(\pi s) \Gamma(s) \zeta(s) = (2\pi)^s \zeta(1-s) 2\sin\left(\frac{\pi s}{2}\right) \text{ (Formula 5)},$$

according to the double Angle formula  $\sin(\pi s) = 2\sin\left(\frac{\pi s}{2}\right)\cos\left(\frac{\pi s}{2}\right)$ , we Will get

$$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 6)},$$

Substituting  $s \rightarrow 1-s$ , that is taking  $s$  as  $1-s$  into Formula 6, we will get

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 7)},$$

This is the functional equation for  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). To rewrite it in a symmetric form, use the residual formula of the gamma function

$$\Gamma(z) \Gamma(1-z) = \frac{\pi}{\sin(\pi z)} \text{ (Formula 8)}$$

and Legendre's formula  $\Gamma(\frac{z}{2})\Gamma(\frac{z+1}{2})=2^{1-z}\pi^{\frac{1}{2}}\Gamma(z)$  (Formula 9) ,

Take  $z=\frac{s}{2}$  in (Formula 8) and substitute it to get

$$\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})}$$
 (Formula 10) ,

In (Formula 9), let  $z=1-s$  and substitute it in to get

$$\Gamma(1-s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1-\frac{s}{2})$$
 (Formula 11)

By substituting (Formula 10) and (Formula 11) into (Formula 7), can get

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)(s\in\mathbb{C} \text{ and } s\neq 1),$$

also

$$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s) \text{ is invariant under the transformation } s\rightarrow 1-s,$$

And that's exactly what Riemann said in his paper. That is to say:

$$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s) \text{ is invariant under the transformation } s\rightarrow 1-s,$$

also

$$\Pi(\frac{s}{2}-1)\pi^{-\frac{s}{2}}\zeta(s)=\Pi(\frac{1-s}{2}-1)\pi^{-\frac{1-s}{2}}\zeta(1-s)(s\in\mathbb{C} \text{ and } s\neq 1),$$

or

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)(s\in\mathbb{C} \text{ and } s\neq 1)$$
 (Formula 2),

Then  $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)(s\in\mathbb{C} \text{ and } s\neq 1)$  (Formula 7) .

This property of the function induces me to introduce  $\Pi(\frac{s}{2}-1)$  instead of  $\Pi(s-1)$  into the general term of the series  $\sum_{n=1}^{\infty}\frac{1}{n^s}$ , from which we obtain the function a very convenient expression for  $\zeta(s)$ , which we actually have

$$\frac{1}{n^s}\Pi\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}=\int_0^{\infty}e^{-n^2\pi x}x^{-\frac{s}{2}}dx.$$

To derive the above equation, let's look at  $\Pi(\frac{s}{2}-1)=\Gamma(\frac{s}{2})=\int_0^{\infty}x^{\frac{s}{2}-1}e^{-x}dx$  , in

$\Pi(\frac{s}{2}-1)=\Gamma(s)=\int_0^{\infty}x^{\frac{s}{2}-1}e^{-x}dx$ , replace  $x\rightarrow n^2\pi x$  as follows, then

$$\Pi(\frac{s}{2}-1)=\Gamma(s)=\int_0^{\infty}(n^2\pi x)^{\frac{s}{2}-1}e^{-n^2\pi x}dx=n^s\cdot n^{-2}\cdot\pi^{\frac{s}{2}}\cdot\pi^{-1}\int_0^{\infty}e^{-n^2\pi x}x^{-\frac{s}{2}}d(n^2\pi x)=$$

$$n^s\cdot n^{-2}\cdot\pi^{\frac{s}{2}}\cdot\pi^{-1}\cdot n^2\cdot\pi\int_0^{\infty}e^{-n^2\pi x}x^{-\frac{s}{2}}dx=n^s\cdot\pi^{\frac{s}{2}}\int_0^{\infty}e^{-n^2\pi x}x^{-\frac{s}{2}}dx, \text{ so}$$

$$\frac{1}{n^s}\Pi\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}=\int_0^{\infty}e^{-n^2\pi x}x^{-\frac{s}{2}}dx.$$

So, if we call  $\sum_{n=1}^{\infty} e^{-n^2\pi x} = \psi(x)$ , get immediately

$$\frac{1}{n^s} \Pi \left( \frac{s}{2} - 1 \right) \pi^{-\frac{s}{2}} = \int_0^{\infty} e^{-n^2\pi x} x^{-\frac{s}{2}} dx = \int_0^{\infty} (\sum_{n=1}^{\infty} e^{-n^2\pi x}) x^{-\frac{s}{2}} dx = \int_0^{\infty} \psi(x) x^{-\frac{s}{2}} dx.$$

According to the Jacobi theta function

$$\theta(x) = \sum_{n=-\infty}^{\infty} e^{-n^2\pi x} = e^{-0^2\pi x} + 2\sum_{n=1}^{\infty} e^{-n^2\pi x} = 1 + 2(e^{-\pi x} + e^{-4\pi x} + e^{-9\pi x} + e^{-16\pi x} + \dots),$$

Easy to see  $\psi(x) = \sum_{n=1}^{\infty} e^{-n^2\pi x} = \frac{\theta(x)-1}{2}$ .

The transformation formula of theta function is derived as follows:  $\theta\left(\frac{1}{x}\right) = \sqrt{x} \theta(x)$ .

Let the first class of complete elliptic integrals  $k, k'$  is called modulus and complement of Jacobi elliptic functions or elliptic integrals, respectively.

$$k = k(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{(1-k^2\sin^2\theta)}},$$

$$k' = k(k') = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{(1-k'^2\sin^2\theta)}},$$

let  $\tau = k'/k$ , then get

$$\sqrt{\frac{2k}{\pi}} = \theta(\tau) = 1 + 2(e^{-\pi\tau} + e^{-4\pi\tau} + e^{-9\pi\tau} + e^{-16\pi\tau} + \dots),$$

The modulo  $k$  and the complement  $k'$  are interchangeable

$$\sqrt{\frac{2k'}{\pi}} = \theta\left(\frac{1}{\tau}\right) = 1 + 2(e^{-\pi/\tau} + e^{-4\pi/\tau} + e^{-9\pi/\tau} + e^{-16\pi/\tau} + \dots),$$

Compare the two formulas to obtain  $\theta\left(\frac{1}{\tau}\right) = \sqrt{\tau} \theta(\tau)$ . It was first obtained by Cauchy using Fourier analysis, and later proved by Jacobi using elliptic functions.

We have again

$$\begin{aligned} \Pi \left( \frac{s}{2} - 1 \right) \pi^{-\frac{s}{2}} \zeta(s) &= \int_1^{\infty} \psi(x) x^{\frac{s}{2}-1} dx + \int_1^{\infty} \psi\left(\frac{1}{x}\right) x^{\frac{s-3}{2}} dx + \frac{1}{2} \int_0^1 (x^{\frac{s-3}{2}} - x^{\frac{s}{2}-1}) dx \\ &= \frac{1}{s(s-1)} + \int_1^{\infty} \psi(x) (x^{\frac{s}{2}-1} + x^{-\frac{1+s}{2}}) dx, \end{aligned}$$

Let's look at the last part of the equation, if  $s \rightarrow 1-s$ , then

$$\frac{1}{s(s-1)} = \frac{1}{(1-s)(1-s-1)} = \frac{1}{(1-s)(-s)} \frac{1}{(s-1)s},$$

$$x^{\frac{s}{2}-1} + x^{-\frac{1+s}{2}} = x^{\frac{1-s}{2}-1} + x^{-\frac{1+(1-s)}{2}} = x^{-\frac{1-s}{2}} + x^{-\frac{2-s}{2}} = x^{-\frac{1+s}{2}} + x^{\frac{s}{2}-1}, \text{ so}$$

$$\Pi \left( \frac{s}{2} - 1 \right) \pi^{-\frac{s}{2}} \zeta(s) \text{ is invariant under the transformation } s \rightarrow 1-s.$$

Riemann then derived the function equation for  $\zeta(s)$  again, which is simpler than the previous derivation using the circum-channel integral and residue theorems.

If we introduce auxiliary function  $\Phi(s) = \Pi \left( \frac{s}{2} - 1 \right) \pi^{-\frac{s}{2}} \zeta(s)$ .

This can be succinctly written as  $\Phi(s) = \Phi(1-s)$ , But it is more convenient to add the factor  $s(s-1)$  to  $\Phi(s)$ , which is what Riemann does next, i.e. (To keep with Riemann's notation, the number

factor  $\frac{1}{2}$  is introduced):  $\zeta(s) = \frac{1}{2}s(s-1) \prod \left(\frac{s}{2} - 1\right) \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s)$ .

Because factor  $(s-1)$  cancels out the pole of  $\zeta(s)$  at  $s=1$ , factor  $s$  cancels out the pole of  $\Gamma\left(\frac{s}{2}\right)$  at  $s=0$ , and  $\zeta(s)$ 's trivial zeros  $-2, -4, -6, \dots$ , and the rest of the poles of  $\Gamma\left(\frac{s}{2}\right)$  cancel out, so  $\zeta(s)$  is an integral function and is zero only at the nonnormal zero points of  $\zeta(s)$ . Note that since  $s(s-1)$  obviously does not change under  $s \rightarrow 1-s$ , there is a function equation  $\xi(s) = \xi(1-s)$ .

Now suppose  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ),  $\prod \left(\frac{s}{2}\right) (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \xi(t)$ , thus get

$$\xi(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \int_1^\infty \psi(x) x^{-\frac{3}{4}} \cos\left(\frac{1}{2} t \ln x\right) dx$$

Or

$$\xi(t) = 4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \psi'(x))}{dx} x^{-\frac{1}{4}} \cos\left(\frac{1}{2} t \ln x\right) dx .$$

The function  $\prod \left(\frac{s}{2}\right) (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \xi(t)$  defined by Riemann is essentially the same as the function

$\xi(s) = \frac{1}{2}s(s-1) \prod \left(\frac{s}{2} - 1\right) \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s)$  commonly used today. Because

$$\prod \left(\frac{s}{2}\right) = \Gamma\left(\frac{s}{2} + 1\right) = \frac{s}{2} \Gamma\left(\frac{s}{2}\right), \text{ so } \prod \left(\frac{s}{2}\right) (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \frac{s}{2} \Gamma\left(\frac{s}{2}\right) (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \frac{1}{2}s(s-1) \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \zeta(s) = \xi(s).$$

The only difference is that Riemann takes  $t$  as the independent variable, while  $\xi(s)$ , which is now commonly used, still takes  $s$  as the independent variable, and  $s$  and  $t$  differ by a linear transformation:  $s = \frac{1}{2} + ti$ , that's a 90 degree rotation plus a translation of  $\frac{1}{2}$ . In this way, the line

$\text{Re}(s) = \frac{1}{2}$  in the complex plane of  $s$  corresponds to the real axis in the  $t$  plane, and the zero of the

zeta function on the critical line  $\text{Re}(s) = \frac{1}{2}$  corresponds to the real root of the function  $\xi(t)$ . Note

that in Riemann's notation, the functional equation  $\xi(s) = \xi(1-s)$  becomes  $\xi(t) = \xi(-t)$ , that is,  $\xi(t)$  is an even function, so its power series expansion is only an even power, and the zeros are symmetrically distributed with respect to  $t = 0$ .

In addition, it is also clear from the above two integral representations that  $\xi(t)$  is an even function, since  $\cos\left(\frac{1}{2} t \ln x\right)$  is an even function of  $t$ .

For all finite  $t$ , function  $\xi(t) = \frac{1}{2} - \left(t^2 + \frac{1}{4}\right) \int_1^\infty \psi(x) x^{-\frac{3}{4}} \cos\left(\frac{1}{2} t \ln x\right) dx$  or function  $\xi(t) =$

$$4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \psi'(x))}{dx} x^{-\frac{1}{4}} \cos\left(\frac{1}{2} t \ln x\right) dx \text{ is finite in value,}$$

And can be expanded to a power of  $t^2$  as a rapidly convergent series, because for an  $s$  value with a real part greater than 1, the value of  $\ln \zeta(s) = -\sum \ln(1 - p^{-s})$  is also finite. It is same true for the logarithm of the other factors of  $\xi(t)$ , so the function  $\xi(t)$  can take zero only if the imaginary part of  $t$  lies between  $\frac{1}{2}$  and  $-\frac{1}{2}$ . That is,  $A$  can take a zero value only if the real part of

$s$  lies between 0 and 1. The number of roots of the real part of the equation  $\xi(t)$  between 0 and  $T$



is approximately equal to  $N(T) = \frac{T}{2\pi} \ln \frac{T}{2\pi} - \frac{T}{2\pi} + O(\ln T)$ , approximately to  $(\frac{T}{2\pi} \ln \frac{T}{2\pi} - \frac{T}{2\pi})$  (this result of Riemann's estimate of the number of zeros was not strictly proved until 1859 by Mangoldt). This is because the value of the integral  $\int d \ln \xi(t)$  (after omitting small quantities of order  $\frac{1}{T}$ ) approximately equal to  $(T \ln \frac{T}{2\pi} - T)i$ . The value of this integral is equal to the number of roots of the equation in this region multiplied by  $2\pi i$  (this is the application of the amplitude Angle principle). In fact, Riemann found that the number of real roots in this region is approximately equal to this number, and it is highly likely that all the roots are real. Riemann naturally hoped for a rigorous proof of this, but after some hasty and unsuccessful initial attempts, Riemann temporarily set aside the search for proof because it was not necessary for the purposes of Riemann's subsequent studies. What Riemann wrote down is the famous Riemann conjecture, the most famous conjecture in mathematics!

According to Riemann's hypothesis in the paper :  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), then the Riemann conjecture is equivalent to that for  $\zeta(s) = 0$ , its complex roots  $s$  (except for negative even numbers) must all be complex numbers satisfying only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), and they all lie on the critical boundary of the vertical real number axis satisfying  $\text{Re}(s) = \frac{1}{2}$ . These complex roots  $s$  (except negative even numbers) are called nontrivial zeros of Riemann  $\zeta(s)$  ( $n \in \mathbb{R}^+$  and  $s \neq 1$  and  $s \neq -2n$  functions).

Let's call the prime counting function  $\pi(x)$  ( $x \in \mathbb{R}^+$ ), the name of this function has nothing to do with  $\pi$ . According to the prime number theorem,  $\pi(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{R}^+$ ). The number of primes less than or equal to 1 is 1, the number of primes other than 1 is 0, so  $\pi(1) = 0$ . The primes less than or equal to 2 are 1 and 2, the number of primes other than 1 is 1, so  $\pi(2) = 1$ . The primes less than or equal to 3 are 1, 2, 3, and the number of primes other than 1 is 2, so  $\pi(3) = 2$ . The primes less than or equal to 4 are 1, 2, 3, and the number of primes other than 1 is 2, so  $\pi(4) = 2$ . The primes less than or equal to 5 are 1, 2, 3, 5, and the number of primes other than 1 is 3, so  $\pi(5) = 3$ . So  $\pi(6) = 3$ ,  $\pi(7) = 4$ ,  $\pi(11) = 5$ ,  $\pi(13) = 6$ , ... , and so on. If we get a simple expression to calculate the prime number counting function, it will lead to amazing results, which will have great significance for the theory and application of mathematical distribution and the development of the mathematical discipline.

Riemann improved the prime counting function, and the prime counting function Riemann obtained was called  $J(x)$  ( $x \in \mathbb{R}^+$ ). The relationship between  $J(x)$  ( $x \in \mathbb{Z}^+$ ) and  $\pi(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{Z}^+$ ) is as follows:

$$\pi(x) = \sum_{n=1}^{\infty} \frac{\mu(n)}{n} J\left(\frac{x}{n}\right) = J(x) - \frac{1}{2} J\left(\frac{x}{2}\right) - \frac{1}{3} J\left(\frac{x}{3}\right) - \frac{1}{5} J\left(\frac{x}{5}\right) + \frac{1}{6} J\left(\frac{x}{6}\right) - \dots \quad (x \in \mathbb{Z}^+, n \in \mathbb{Z}^+),$$

The relationship between  $J(x)$  ( $x \in \mathbb{R}^+$ ) and  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) is as follows:

$$\frac{1}{s} \ln \zeta(s) = \int_0^{\infty} J(x) x^{-s-1} dx ,$$

$\mu(n)$  is called the Mobius function.

The Mobius function  $\mu(n)$  has only three values, which are 0 and plus or minus 1, if  $n$  is not divisible by the square of any prime number, that is, an exponent of one or more prime factors other than 1 in the prime factorization of  $n$ . If the power is raised to the second or higher power, then  $\mu(n)=0$ . If  $n$  is not divisible by the square of any prime number, that is to say, the exponent of any prime factor other than 1 in the prime factorization of  $n$  has the degree 1, then let's count the number of prime factors. If there are an even number of prime factors, then  $\mu(n)= 1$ . If the number of prime factors is odd, then  $\mu(n)=-1$ . This also includes the case of  $n=1$ , since 1 has no prime factors other than 1, then the number of prime factors of 1 other than 1 is 0, and 0 counts as an even number, so  $\mu(1)=1$ . In the above expansion, as  $n(n \in \mathbb{R}^+)$  increases,  $\frac{1}{n}(n \in \mathbb{Z}^+)$  becomes smaller and smaller,  $\frac{1}{x^n}(n \in \mathbb{Z}^+)$  also gets smaller and smaller, The  $n(n \in \mathbb{Z}^+)$  and  $n \rightarrow +\infty$ th term is going to get smaller and smaller. It shows that the largest contribution to the value of  $\pi(x)$  is the first term  $J(x)$ .

Now let's look at the following formula from Riemann:

$$J(x) = \text{Li}(x) - \sum_p \text{Li}(x^p) + \int_x^{+\infty} \frac{dt}{t^2(t^2-1)\ln t} - \ln 2 \quad (x \in \mathbb{Z}^+),$$

among ,  $\text{Li}(x) = \int_0^x \frac{dt}{\ln t} \quad (x \in \mathbb{Z}^+),$

$J(x)$  is called a step function, it equals zero where  $x$  equals zero, that is,  $J(0)=0$ , and then as the value of  $x$  increases, every time it passes through a prime number (such as 2,3,5,...). The value of  $J(x)$  increases by 1. Every time it square a prime number (4,9,25), the value of  $J(x)$  increases by  $\frac{1}{2}$ . Every time it pass through the third square of a prime

number (such as 8,9,25,...) The value of  $J(x)$  increases by  $1/3$ . Every time it pass 4 squares of a prime number (say, 16,81,256,625,...) , the value of  $J(x)$  increases by  $1/4$ . And so on, every time it passes a prime number to  $x^n$  ( $n \in \mathbb{Z}^+$  ,  $n \rightarrow +\infty$ ,  $x$  is a prime number), the value of  $J(x)$  increases  $\frac{1}{n}$  ( $n \in \mathbb{Z}^+$  and  $n \rightarrow +\infty$ ) .You can think of it as that every time it passes a prime number to  $x^n$  ( $n \in \mathbb{R}^+$  ,  $n \rightarrow +\infty$ ,  $x$  is a prime number) ,  $J(x)$  increases  $\frac{1}{n}$  ( $n \in \mathbb{Z}^+$  and  $n \rightarrow +\infty$ ). Obviously, this function is closely related to the distribution of prime numbers. If you look at the right-hand side of the equation, the first term is called the logarithmic integral function  $\text{Li}(x) = \int_0^x \frac{dt}{\ln t}$  ( $x \in \mathbb{Z}^+$ ) , When  $x$  is sufficiently large,  $\text{Li}(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{Z}^+$ ) ,  $\pi(x) \approx \text{Li}(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{Z}^+$ ,  $x$  is sufficiently large). Let's look at the second item  $\text{Li}(x^\rho)$  ( $x \in \mathbb{Z}^+$  ,  $\rho \in \mathbb{C}$ ) ,  $\rho$  is a complex number other than a negative even number,  $\rho$  is called the nontrivial zero of the  $\zeta(s)$  ( $n \in \mathbb{Z}^+$  and  $s \neq 1$  and  $s \neq -2n$ ) function by Riemann.  $\rho$  is denoted as:  $\rho = \sigma + it$  ( $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$ ). On the real number line, the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ) function has no zeros except for negative even numbers, So  $\rho$  is definitely not a real number other than a negative even number, so  $x^\rho$  ( $\rho \in \mathbb{C}$ ,  $x \in \mathbb{Z}^+$ , and  $\rho \neq 1$  and  $\rho \neq -2n$ ,  $n \in \mathbb{Z}^+$ ) is definitely not a real number other than a negative even number as also. So how do we compute  $\text{Li}(x^\rho)$  ( $x \in \mathbb{R}^+$  ,  $\rho \in \mathbb{C}$ , and  $\rho \neq 1$  and  $\rho \neq -2n$ ,  $n \in \mathbb{Z}^+$ )? Just extend the domain resolution of  $\text{Li}(x) = \int_0^x \frac{dt}{\ln t}$  ( $x \in \mathbb{R}^+$ ) to all complex numbers except divided by 1. Riemann proved that the non-trivial zero  $\rho$  of the Riemann  $\zeta(\rho)$  ( $\rho \in \mathbb{C}$  and  $s \neq 1$  and  $\rho \neq -2n$ ,  $n \in \mathbb{Z}^+$ ) function must satisfy  $0 \leq \text{Re}(\rho) \leq 1$ . The vertical strip of width 1 on the complex plane is called the critical strip. and the line perpendicular to the real number axis satisfying  $\text{Re}(s) = \frac{1}{2}$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$  ,  $n \in \mathbb{Z}^+$ ) is called the critical boundary, that is, the center

line of the critical band. Riemann guessed that the non-trivial zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function all lie on the critical boundary, which is a very surprising conclusion.

If the real part of the nontrivial zero of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function takes random values between 0 and 1, then the probability that it reaches exactly  $\frac{1}{2}$  should equal 0, which Riemann thought was 100%. If the Riemann conjecture is strictly true, then the occurrence of prime numbers or the distribution of prime numbers is not random at all, but occurs in a definite way, and there must be a deep reason behind this. The proof of the prime number theorem is an intermediate product in the process of studying Riemann conjecture. In 1896, Hadamar and De la Vabsan proved that the nontrivial zero  $\rho$  of the Riemann  $\zeta(\rho)$  ( $\rho \in \mathbb{C}$  and  $s \neq 1$  and  $\rho \neq -2n, n \in \mathbb{Z}^+$ ) function has no zero when  $\text{Re}(\rho)=0$  and  $\text{Re}(\rho)=1$ , thus easily proving the prime number theorem  $\pi(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{Z}^+$ ).

The prime number theorem  $\pi(x) \approx \frac{x}{\ln x}$  ( $x \in \mathbb{R}^+$ ) holds, showing that for the prime counting function  $\pi(x)$ , the largest part of its value comes from the logarithmic integral function  $\text{Li}(x) = \int_0^x \frac{dt}{\ln t}$  ( $x \in \mathbb{R}^+$ ) while the minor part of its value comes from  $\text{Li}(x^\rho)$  ( $x \in \mathbb{Z}^+, \rho \in \mathbb{C}$  and  $s \neq 1$  and  $\rho \neq -2n, n \in \mathbb{Z}^+$ ), since the calculation of  $x \ln x, x \in \mathbb{Z}^+$  is simple, but for the accurate calculation of the prime counting function  $\pi(x)$ , the calculation of the non-trivial zero  $\rho$  of the Riemann  $\zeta(\rho)$  ( $\rho \in \mathbb{C}$  and  $s \neq 1$  and  $\rho \neq -2n, n \in \mathbb{Z}^+$ ) function is very important, and the strict proof of the Riemann conjecture is very important. In 1921, the British mathematician Hardy proved that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function has infinitely many nontrivial zeros on the critical

boundary. But this conclusion is actually quite different from the Riemann conjecture, because the fact that there are infinitely many nontrivial zeros on the critical boundary does not mean that all zeros are on the critical boundary. Just as a line segment has an infinite number of points, but a line segment has an infinite number of lines, the percentage of Hardy's proof is almost zero compared to the number of all nontrivial zeros. It wasn't until 1942 that mathematicians pushed this percentage significantly higher than zero. That year, the Norwegian mathematician Selberg proved that the percentage was greater than zero, but did not give a specific value. In 1974, the American mathematician Liesen proved that at least 34% of nontrivial zeros lie on the critical boundary. In 1980, Chinese mathematicians Lou Shituo and Yao Qi proved that 35% of nontrivial zeros lie on the critical boundary. In 1989, the American mathematician Conrey proved that 40% of nontrivial zeros are located on the critical boundary. The calculation of the nontrivial zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function is more complicated. Graham calculated the first 15 nontrivial zeros of the Riemann  $\zeta(s)$  function, and after 25 years, another 138 nontrivial zeros were calculated. Since then, the calculation of the nontrivial zeros of the Riemann  $\zeta(s)$  function has stalled because of the clumsy methods and the lack of computers to assist it. After the calculation was halted for seven years, the deadlock was broken, and German mathematician Siegel found in Riemann's manuscript that Riemann was far ahead of the time 70 years of clever algorithm, so that the calculation of non-trivial zero points was suddenly bright. In honor of Siegel, this algorithm formula is also known as the Riemann-Siegel formula, and Siegel himself won the Fields Medal for it.

A mathematician's manuscript is worth far more than an antique. Since then, the non-trivial zeros of the Riemann  $\zeta(s)$  function have been computed much faster. Hardy's students pushed the calculation of the non-trivial zeros of the Riemann  $\zeta(s)$  function to 1041, the father of artificial intelligence Alan Turing pushed the calculation of the non-trivial zeros of the Riemann  $\zeta(s)$  function to 11,041, and later with the application of computers, the calculation of the non-trivial zeros of the Riemann- $\zeta(s)$  function from 3.5 million to 300 million, 1.5 billion, 850 billion, and now 10 trillion, of these nontrivial zeros lie on Riemann's critical boundary. The Riemann conjecture stands firm. It has been inferred that the nontrivial zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function are symmetric with respect to the real number axis. Although this guess is correct, it needs to be rigorously proved, otherwise such a guess has no meaning. In the following paper, I give a strict proof of this conjecture, and give a strict proof of Riemann conjecture, which is indeed true. Riemann guess is equivalent to  $\zeta(s)=\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\zeta(1-s)=\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) are both established.

$\zeta(1-s)=\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) can be deduced by  $\zeta(s)=2^s \pi^{s-1} \text{Sin}\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

when  $\zeta(s)=0$ , and  $\zeta(s)=\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) can be deduced by  $\zeta(s)=\overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

when  $\zeta(s)=0$ .  $\zeta(s)=\overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) must use euler's formula

$e^{ix}=\cos(x)+isin(x)$  ( $x \in \mathbb{R}$ ) and  $e^{iz}=\cos(Z)+isin(Z)$  ( $Z \in \mathbb{C}$ ) and amplitude Angle principle (after index

was extended to general real Numbers), and strict proof. If we want to solve the

Riemann conjecture, the proof of it must follow such principles and methods, otherwise

it may not be correct.

First of all, there are:

$$\prod\left(\frac{1}{1-p^{-s}}\right)=\sum\frac{1}{n^s},$$

this is a formula of Euler, in which  $n$  is a natural number and  $p$  is a prime number. Euler has already proved it, and I will repeat it below. If you are familiar with Euler's formulas and know exactly that they are correct, you can omit them.

Turn this Euler formula around and get:

$$\sum\frac{1}{n^s}=\prod\left(\frac{1}{1-p^{-s}}\right)$$

When Euler first proposed this formula,  $s$  only represented a positive integer more than

1. Obviously, both sides of this formula are series. Euler found that there is such a series:

$$\sum\frac{1}{n^s}=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\dots(\text{equation 1}).$$

The above equation is multiplied by  $\frac{1}{2^s}$  on both sides,  $\frac{1}{2^s}$  on the left and  $\frac{1}{2^s}$  on the right.

we can get:

$$\frac{1}{2^s}\sum\frac{1}{n^s}=\frac{1}{2^s}+\frac{1}{4^s}+\frac{1}{6^s}+\frac{1}{8^s}+\frac{1}{10^s}+\frac{1}{12^s}+\dots(\text{equation 2})$$

By subtracting the left and right sides of the two equations (equation 1) and (equation 2), the following results can be obtained:

$$\left(1-\frac{1}{2^s}\right)\sum\frac{1}{n^s}=1+\frac{1}{3^s}+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{9^s}+\frac{1}{11^s}+\frac{1}{13^s}+\frac{1}{15^s}+\dots(\text{equation 3})$$

It can be observed that the product term on the left side increases by  $\left(1-\frac{1}{2^s}\right)$  as the left term of equation 3 relative to equation 1. When the items on the right side of equation 1 are multiplied by  $\frac{1}{2^s}$ , the items whose denominator is even are eliminated, and the remaining items are regarded as the items on the right side of equation 3.

By multiplying the left and right sides of equation 3 by  $\frac{1}{3^s}$ , we can get:

$$\frac{1}{3^s}\left(1-\frac{1}{2^s}\right)\sum\frac{1}{n^s}=\frac{1}{3^s}+\frac{1}{9^s}+\frac{1}{15^s}+\frac{1}{21^s}+\frac{1}{27^s}+\frac{1}{33^s}+\frac{1}{39^s}+\frac{1}{45^s}+\dots(\text{equation 4})$$

By subtracting the left and right sides of the two equations (equation 3) and (equation 4), we can get:

$$(1 - \frac{1}{3^s})(1 - \frac{1}{2^s})\sum \frac{1}{n^s} = 1 + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{17^s} + \frac{1}{19^s} + \frac{1}{23^s} + \frac{1}{25^s} + \frac{1}{29^s} + \frac{1}{31^s} + \dots$$

... (equation 5)

Similarly, multiply the left and right sides of equation 5 by  $\frac{1}{5^s}$  we can get:

$$(\frac{1}{5^s})(1 - \frac{1}{3^s})(1 - \frac{1}{2^s})\sum \frac{1}{n^s} = \frac{1}{5^s} + \frac{1}{25^s} + \frac{1}{35^s} + \frac{1}{55^s} + \frac{1}{65^s} + \frac{1}{85^s} + \frac{1}{95^s} + \frac{1}{115^s} + \frac{1}{145^s} + \dots$$

(equation 6)

By subtracting the left and right sides of the two equations (equation 5) and (equation 6), the following results can be:

$$(1 - \frac{1}{5^s})(1 - \frac{1}{3^s})(1 - \frac{1}{2^s})\sum \frac{1}{n^s} = 1 + \frac{1}{7^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{17^s} + \frac{1}{19^s} + \frac{1}{23^s} + \frac{1}{29^s} + \frac{1}{31^s} + \frac{1}{37^s} + \dots$$

(equation 7)

Referring to this method, in equation (2k-1) (k is a positive integer), we multiply the items on the left by  $\frac{1}{p_1^s}$  and the items on the right by  $\frac{1}{p_i^s}$  (i is a positive integer).

$p_i$  is the nearest prime number of the prime number  $p_{i-1}$  in the first item  $(1 - \frac{1}{p_{i-1}^s})$  on the left side of equation (2k-1). The "nearest prime" here refers to the one closest to  $p_{i-1}$ . There is no other prime between them, and  $p_i > p_{i-1}$ , equation (2k-1) add  $\frac{1}{p_1^s}$  to the left. equation (2k-1) the right side becomes: item 1 is  $\frac{1}{p_1^s}$ , item 2 is  $\frac{1}{p_1^s} \times \frac{1}{p_i^s}$ , item 3 is  $\frac{1}{p_1^s} \times \frac{1}{p_{i+1}^s}$ , item 4 is  $\frac{1}{p_1^s} \times \frac{1}{p_{i+2}^s}$ , item 5 is  $\frac{1}{p_1^s} \times \frac{1}{p_{i+3}^s}$ , ...,  $\frac{1}{p_1^s} \times \frac{1}{p_{(i+k)}^s}$ , ..., k is a positive integer. So go on and add them up, where  $p_1, p_2, p_3, \dots, p_{i+1}, p_{i+2}, p_{i+3}, p_{i+4}, \dots, p_{i+k}$  ... It is an infinite sequence of prime numbers arranged in the order of numerical size from small to large, and  $p_3 = 5, p_2 = 3, p_1 = 2$ . In this way, we get the expression on the right side of equation (2k-1) and mark the whole equation as equation (2k). By The coefficient of  $\sum \frac{1}{n^s} (n \in \mathbb{Z}^+)$ . on its left side is a continuous product of some forms such as  $(1 - \frac{1}{p_i^s})$ . n is a natural number and p takes all prime numbers. In order to write conveniently, the symbol is introduced and the left side is written as:



referring to this method and doing it over and over again, we will eventually get such an equation:

On the right is 1, plus a score:  $\frac{1}{p_i^s \times p_{i+k}}$ . The values of  $p_i^s$  and  $p_{i+k}$  are two infinite prime numbers, so the value of is zero, which can be omitted. So, the right side is 1. So you can get it:

$$\sum \frac{1}{n^s} = \frac{1}{\prod(1-\frac{1}{p^s})} = \prod \frac{1}{(1-\frac{1}{p^s})} = \prod \frac{1}{1-p^{-s}}$$

Riemann extends Euler's definition of positive integer  $s$  analytic to complex number, that is, the variable  $s$  is defined as complex number. And we use a function  $\zeta(s)$  constructed by Euler himself to record the two series on both sides of the above equation:

$$\zeta(s) = \sum \frac{1}{n^s} = \prod \frac{1}{1-p^{-s}} .$$

Secondly, there is another Euler formula:  $e^{ix} = \cos(x) + i\sin(x)$ ,  $x$  is a real number, representing the radian of an angle. This formula has been proved by Euler and can be used directly. Let me prove it again in my own way:

If we have a function  $f_1(x) = e^x$ , we derive  $f_1'(x) = e^x (x \in \mathbb{R})$ , " ' " means derivative, then  $(e^x)' = e^x$ , the derivative of  $e^x$  is itself. So if we make the independent variable  $cx$  ( $c$  is constant) of function  $f_1(x) = e^x$ , we will get function  $f_1(cx) = e^{cx}$ , and derivative of function  $[f_1(x)]' = (e^{cx})' = ce^{cx}$ , then  $[f_1(x)]' = (e^{cx})' = ce^{cx}$ . If the function  $f_1(cx) = e^{cx}$ ,  $c = i$  ( $i$  is also constant), then  $f_1(ix) = e^{ix}$ , then  $[f_1(ix)]' = [e^{ix}]' = ie^{ix}$ . Suppose that  $f_2(x) = \cos(x) + i\sin(x) = s$ , then  $s$  is a complex number. Now the derivative of function  $f_2(x)$  is obtained:  $[f_2(x)]' = [\cos(x) + i\sin(x)]' = [\cos(x)]' + [i\sin(x)]' = -\sin x + i\cos x$  (equation 1). If  $f_1(ix) = e^{ix} = \cos x + i\sin x$  is correct, then suppose that  $e^{ix} = \cos(x) + i\sin(x)$  is correct based on the above  $[f_1(x)]' = [e^{ix}]' = ie^{ix}$ ,  $[f_1(ix)]' = [e^{ix}]' = ie^{ix}$  (equation 2), replacing  $e^{ix}$  with  $\cos x + i\sin x$ , then:  $[f_1(ix)]' = [e^{ix}]' = ie^{ix} = i(\cos x + i\sin x) = -\sin x + i\cos x$  (equation 2). By comparing (equation 1) and (equation 2), it can be found that the derivatives of  $f_1(ix)$  and  $f_2(x)$  are equal, and since both  $f_1(ix)$  and  $f_2(x)$  have no constant terms, the expressions of  $f_1(ix)$  and  $f_2(x)$  should be consistent. We found  $f_1(ix) = e^{ix} = \cos x + i\sin x = f_2(x)$ . The expressions of  $f_1(ix)$  and  $f_2(x)$  are exactly the same, which shows that the Euler's formula  $e^{ix} = \cos(x) + i\sin(x) (x \in \mathbb{R})$  is

correct. To prove  $e^{ix} = \cos(x) + i\sin(x) (x \in \mathbb{R})$ , a better method is the following, but more

complex. Everyone First of all, look at the function  $y = e^x$ . If we find the derivative of this function, we will get  $y' = (e^x)' = e^x$ . That is to say, the derivative of  $y = e^x$  is itself. This is a very special exponential function. Let  $y' = \frac{dy}{dx}$ , when  $\frac{dy}{dx} = 0$ , then  $y = e^x$ , when  $\frac{dy}{dx} = 1$ , then  $y = e^x = 1 + x$ , when

$$\frac{dy}{dx} = 1 + x, y = e^x = 1 + x + \frac{1}{2}x^2, \text{ when } \frac{dy}{dx} = 1 + x + \frac{1}{2}x^2, \text{ then } y = e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3, \text{ when}$$

$$\frac{dy}{dx} = e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3, \text{ then } y = e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4, \text{ when } \frac{dy}{dx} = e^x = 1 +$$

$$x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4, \text{ then } y = e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5, \text{ by analogy, this is a}$$

preliminary proof :  $y=e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5, + \dots$ , But what about the series

$y=x^n (n \in \mathbb{Z}^+)$ . in general? What about the series of  $y = e^x$  When  $x$  is treated as  $e$  and  $n$  as  $x$ ,  $y = e^x$  is obtained, which requires the introduction of the concept of power series.

This is the introduction of the concept of power series:  $1+x+x^2+x^3+x^4+x^5+\dots (x \in \mathbb{R})$ , Every item is a power in the form of  $x^n (n \in \mathbb{Z}^+)$ . Let function  $f(x) = 1+x+x^2+x^3+x^4+x^5+\dots (x \in \mathbb{R})$ , Equivalent to the sum of the items, if some numbers are used as the coefficients of the items, if these numbers are  $a_0, a_1, a_2, a_3, a_4, a_5, \dots, a_{i-1}, a_i, \dots$ , They are derivatives of order 0  $f^{(0)}(x)$  of the function  $f(x) = x^n (n \in \mathbb{Z}^+)$ , the derivatives of order 1  $f^{(1)}(0)$  of the function  $f(x) = x^n (n \in \mathbb{Z}^+)$ , the derivatives of order 2  $f^{(2)}(0)$  of the function  $f(x) = x^n (n \in \mathbb{Z}^+)$ , the derivatives of order 3  $f^{(3)}(0)$  of the function  $f(x) = x^n (n \in \mathbb{Z}^+)$ , ... ,the derivatives of order  $n$   $f^{(n)}(0)$  of the function  $f(x) = x^n (n \in \mathbb{Z}^+)$ . They are:  $a_0=f^{(0)}(0), a_1=f^{(1)}(0), a_2=f^{(2)}(0), a_3=f^{(3)}(0), \dots, a_{i-1}=f^{(i-1)}(0)$ ,

$a_i = f^{(i)}(0), \dots$ , If  $f(x) = x^n (n \in \mathbb{Z}^+)$  is taken as  $n$  times derivative, we will get:  $f^{(n)}(0) = n(n-1)(n-2)(n-3)\dots 2 \times 1 \times 0^0$ , so that  $f^{(n)}(0) = n!$ , For a particular function  $f = e^x$ , the values of all these derivatives at  $x = 0$ :  $f^{(0)}(0), f^{(1)}(0), f^{(2)}(0), f^{(3)}(0), \dots, f^{(n-1)}(0), f^{(n)}(0), \dots$ , they must be 1, because the derivative of any order of  $e^x$  is itself. But the value of derivatives of order  $x^n$  at  $x = 0$  are:  $f^{(n)}(0) = n(n-1)(n-2)(n-3)\dots 2 \times 1 \times 0^0 = n!$ , therefore

$a_0, a_1, a_2, a_3, a_4, a_5, \dots, a_{i-1}, a_i, \dots$ , have to divide one by  $n!$ , can make:  $f^{(0)}(0) = 1, f^{(1)}(0) = 1, f^{(2)}(0) = 1, f^{(3)}(0) = 1, \dots, f^{(n-1)}(0) = 1, f^{(n)}(0) = 1$ , In order to satisfy the coefficients of the series expression of function  $f(x) = e^x$

correctly:  $a_0, a_1, a_2, a_3, a_4, a_5, \dots, a_{i-1}, a_i, \dots$ , Namely:  $a_0 = \frac{1}{0!} = 1, a_1 = \frac{1}{1!}, a_2 = \frac{1}{2!}, a_3 = \frac{1}{3!}, a_4 = \frac{1}{4!}, \dots, a_{n-1} = \frac{1}{(n-1)!}, a_n = \frac{1}{n!}, \dots$ ,

For a particular function  $f(x) = e^x$ , the method here is to multiply the  $n$  power of  $x$  by the values of the derivative functions of the function  $x^n (n \in \mathbb{Z}^+)$  at the independent variable  $x = 0$ , and then divide by the factorial of  $n$ .

So for a particular function  $f(x) = e^x$ ,  $a_0 = \frac{1}{0!} = 1, a_1 = \frac{1}{1!}, a_2 = \frac{1}{2!}, a_3 = \frac{1}{3!}, a_4 = \frac{1}{4!}, \dots$ ,

$a_{n-1} = \frac{1}{(n-1)!}, a_n = \frac{1}{n!}, \dots$ , So you can write the series of the function  $f(x) = e^x$  again:  $e^x =$

$$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \dots + \frac{1}{(n-1)!}x^{n-1} + \frac{1}{n!}x^n + \dots,$$

Let's assume  $f(x) = \cos(x)$  to find the power series of  $\cos(x)$ . The 0-th derivative of function  $f(x) = \cos(x)$  is  $f^{(0)}(x) = \cos(x)$  (the 0-th of a function is itself). The 1-th derivative of function  $f(x) = \cos(x)$  is  $f^{(1)}(x) = -\sin(x)$ , the 2-th derivative of function  $f(x) = \cos(x)$  is  $f^{(2)}(x) = -\cos(x)$ , the 3-th derivative of function  $f(x) = \cos(x)$  is  $f^{(3)}(x) = \sin(x)$ , the 4 - th derivative of function  $f(x) = \cos(x)$  is  $f^{(4)}(x) = -\sin(x)$ , the  $n$ -th derivative of function  $f(x) = \cos(x)$  is  $f^{(n)}(x) = \dots$ , If  $x = 0$  is substituted, the value of the derivative function of each order at 0 will be obtained. Because the series is derived by dividing the value of the derivative function at the independent variable  $x = 0$  by the factorial of  $N$  and multiplying by the expansion of  $x^n (n \in \mathbb{Z}^+)$ . Therefore, at  $x = 0$ , it is easy to get the value of each derivative function at  $x = 0$  by assigning the independent variable of each derivative function to zero:  $f^{(0)}(0) = \cos(0) = 1, f^{(1)}(0) = -\sin(0) = 0, f^{(2)}(0) = -\cos(0) = -1, f^{(3)}(0) = \sin(0) = 0, f^{(4)}(0) = \cos(0) = 1, f^{(5)}(0) = -\sin(0) = 0, f^{(6)}(0) = -\cos(0) = -1, f^{(7)}(0) = \sin(0) = 0, \dots$ , according to

1, 0, - 1, 0, 1, 0, - 1, 0,... In the form of 1,0, -1,0, the cycle section goes on indefinitely. The function value of the derivative function of order  $f = \cos(x)$  at 0 of its independent variable can be used to construct the coefficients needed for the power series of  $\cos(x)$ . They are divided by the factorial of  $n$ , which is the coefficients of the powers of  $x$ . Now we can construct the power series of  $\cos(x)$  by referring to the power series of  $e^x$  above,  $n$  is the order of the derivative function of order  $f = \cos(x)$ , and is also the  $n$ -th power of  $x$ . So the power series of  $\cos(x)$  expansion is: It starts with  $\frac{f^{(0)}(0)}{0!} x^0 = \frac{\cos(0)}{0!} x^0 = \frac{0}{0!} \times 0 = 1$  as the zero term, the constant term.

Next is:  $\frac{f^{(1)}(0)}{1!} x^1 = \frac{-\sin(0)}{1!} x^1 = \frac{0}{1!} \times x = 0$ , The result is zero, which means that there is no 1-th term, or that there is no first order term of  $x$ .

Next is:  $\frac{f^{(2)}(0)}{2!} x^2 = \frac{-\cos(0)}{2!} x^2 = \frac{-1}{2!} \times x^2 = -\frac{1}{2} x^2$ , which means that there is no 2-th term.

Next is:  $\frac{f^{(3)}(0)}{3!} x^3 = \frac{\sin(0)}{3!} x^3 = \frac{0}{3!} \times x^3 = 0$ , The result is zero, which means that there is no 3-th term, or that there is no 3-th power term of  $x$ .

Next is:  $\frac{f^{(4)}(0)}{4!} x^4 = \frac{\cos(0)}{4!} x^4 = \frac{1}{4!} x^4$ , which means that there is no 4-th term.

..., If we go on doing this, we will find that  $n$ -order derivative of  $f(x) = \cos(x)$ ,  $n$  is a nonnegative positive number. Starting from zero, if  $n$  is an even number, then the value of  $f^{(n)}(0)$  is either  $+1$  or  $-1$ , according to  $1, -1, 1, -1, 1, -1, \dots$ . The regular arrangement of, So for the power series expansion of  $\cos(x)$ , the sign of the value of the coefficients in front of the even power term of  $X$  is as follows:  $+, -, +, -, +, -, -, \dots$  regularly arranged. The coefficients are:  $\frac{f^{(n)}(0)}{n!} = \frac{1}{n!}$  or  $\frac{f^{(n)}(0)}{n!} = -\frac{1}{n!}$ , If

$n$  is an odd number, the value of its coefficient is:  $\frac{f^{(n)}(0)}{n!} = 0$ , So for the expansion of power series of  $\cos(x)$ , there is no odd term of  $X$ . So the power series of the function  $f(x) = \cos(x)$  is:

$$\cos(x) = \frac{1}{0!} x^0 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \frac{1}{8!} x^8 - \frac{1}{10!} x^{10} + \dots = 1 - \frac{1}{2} x^2 + \frac{1}{4!} x^4 - \frac{1}{6!} x^6 + \frac{1}{8!} x^8 - \frac{1}{10!} x^{10} + \dots$$

Let's assume  $f(x) = \sin(x)$  to find the power series of  $\sin(x)$ . The 0-th derivative of function  $f(x) = \sin(x)$  is  $f^{(0)}(x) = \sin(x)$  (the 0-th derivative of a function is itself), The 1-th derivative of function  $f(x) = \sin(x)$  is  $f^{(1)}(x) = \cos(x)$ , The 2-th derivative of function  $f(x) = \sin(x)$  is  $f^{(2)}(x) = -\sin(x)$ , The 3-th derivative of function  $f(x) = \sin(x)$  is  $f^{(3)}(x) = -\cos(x)$ , The 4-th derivative of function  $f(x) = \sin(x)$  is  $f^{(4)}(x) = \sin(x)$ , The  $n$ -th derivative of function  $f(x) = \cos(x)$  is  $f^{(n)}(x) = \dots$ . If  $x = 0$  is substituted, the value of the derivative function of each order at 0 will be obtained. Because the series is derived by dividing the value of the derivative function at the independent variable  $x = 0$  by the factorial of  $N$  and multiplying by the expansion of  $x^n (n \in \mathbb{Z}^+)$ . Therefore, at  $x = 0$ , it is easy to get the value of each derivative function at  $x = 0$  by assigning the independent variable of each derivative function to zero:  $f^{(0)}(0) = \sin(0) = 0$ ,  $f^{(1)}(0) = \cos(0) = 1$ ,  $f^{(2)}(0) = -\sin(0) = 0$ ,  $f^{(3)}(0) = -\cos(0) = -1$ ,  $f^{(4)}(0) = \sin(0) = 0$ ,  $f^{(5)}(0) = \cos(0) = 1$ ,  $f^{(6)}(0) = -\sin(0) = 0$ ,  $f^{(7)}(0) = \cos(0) = -1, \dots$  According to  $0, 1, -0, -1, 0, 1, 0, -1, \dots$  In the form of  $0, 1, 0, -1$ , the cycle section goes on indefinitely. The function value of the derivative function of order  $f = \sin(x)$  at 0 of its independent variable can be used to construct the coefficients needed for the power series of  $\sin(x)$ . They are divided by the factorial of  $n$ , which is the coefficients of the powers of  $x$ . Now

we can construct the power series of  $\sin(x)$  by referring to the power series of  $e^x$  above,  $n$  is the order of the derivative function of order  $f=\sin(x)$ , and is also the  $n$ -th power of  $x$ . So the power series of  $\sin(x)$  expansion is:

It starts with  $\frac{f^{(0)}(0)}{0!}x^0 = \frac{\sin(0)}{0!}x^0 = \frac{0}{0!} \times 1 = 0$  as the zero term, the constant term,

Next is:  $\frac{f^{(1)}(0)}{1!}x^1 = \frac{\cos(0)}{1!}x^1 = \frac{1}{1!} \times x = x$ , as 1-th term,

Next is:  $\frac{f^{(2)}(0)}{2!}x^2 = \frac{-\sin(0)}{2!}x^2 = \frac{0}{2!} \times x^2 = 0$ , which means that there is no 2-th term,

Next is:  $\frac{f^{(3)}(0)}{3!}x^3 = \frac{-\cos(0)}{3!}x^3 = \frac{-1}{3!} \times x^3 = -\frac{1}{3!}x^3$ , as 3-th term,

Next is:  $\frac{f^{(4)}(0)}{4!}x^4 = \frac{\sin(0)}{4!}x^4 = \frac{0}{4!} \times x^4 = 0$ , which means that there is no 4-th term.

..., If we go on doing this, we will find that  $n$ -order derivative of  $f(x)=\sin(x)$ ,  $n$  is not a nonnegative positive number. Starting from zero, If  $n$  is an odd number, then the value of  $f^{(n)}(0)$  is either  $+1$  or  $-1$ , according to  $1, 0, 1, -1, 1, -1, -1, -1, \dots$ . Regular arrangement, if  $n$  is an even number, then the value of  $f^{(n)}(0)$  is either  $+1$  or  $-1$ , according to  $0, 1, 0, -1, 0, 1, 0, -1, \dots$ , the regular arrangement of, so for the power series expansion of  $\sin(x)$ , the sign of the value of the coefficients in front of the odd power term of  $x$  is as follows:  $+, -, +, -, +, -, -, -, \dots$  regularly arranged. The coefficients are:  $\frac{f^{(n)}(0)}{n!} = \frac{1}{n!}$  or  $\frac{f^{(n)}(0)}{n!} = -\frac{1}{n!}$ . If  $n$  is an even number, the value of

its coefficient is:  $\frac{f^{(n)}(0)}{n!} = 0$ , So for the expansion of power series of  $\sin(x)$ , there is no even term of  $x$ . So the power series of the function  $f(x) = \sin(x)$  is:

$$\sin(x) = \frac{1}{1!}x^1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \dots = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \dots,$$

Previously obtained

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \dots + \frac{1}{n!}x^n = 1 + x + \frac{1}{3}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \dots + \frac{1}{n!}x^n (x \in \mathbb{R})$$

If we change  $x$  to  $ix$ , We can get:

$$e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \frac{1}{5!}(ix)^5 + \dots + \frac{1}{n!}(ix)^n = (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + \dots) + (x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \dots) (x \in \mathbb{R})$$

because  $\cos(x) = (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + \dots)$ ,  $\sin(x) = (x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \dots)$ , therefore  $e^{ix} = \cos(x) + i\sin(x) (x \in \mathbb{R})$ , So this is another Eulerian formula.

In the formula above, if  $x$  equals  $\pi$ , we will get:  $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1 + 0 = -1$ , therefore  $e^{i\pi} + 1 = 0$ , It's also called Euler's formula. It puts all the most important things in mathematics,  $0, 1, e, i$  and  $\pi$ , into one formula. It is a special case of Euler formula  $e^{ix} = \cos(x) + i\sin(x) (x \in \mathbb{R})$ . when  $Z \in \mathbb{C}$ , then  $e^{iZ} = \cos(Z) + i\sin(Z) (Z \in \mathbb{C})$ .

## II .Conclusion Reasoning

**Femma 1:**

$\sum_{n=1}^{\infty} n^{-s} = \prod_p (1 - p^{-s})^{-1}$  ( $s \in \mathbb{Z}^+$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers,  $p \in \mathbb{Z}^+$  and  $p$  takes all the prime numbers), this formula was proposed and proved by the Swiss mathematician Leonhard Euler in 1737 in a paper entitled "Some Observations on Infinite Series", Euler's product formula connects a summation expression for natural numbers with a continuative product expression for prime numbers, and contains important information about the distribution of prime numbers. This information was finally deciphered by Riemann after a long gap of 122 years, which led to Riemann's famous paper "On the number of primes less than a Given Value"<sup>[1]</sup>. In honor of Riemann, the left end of the Euler product formula was named after Riemann, and the notation  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) used by Riemann was adopted as the Riemann zeta function .

Because  $e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \sum_{n=0}^{\infty} \frac{1}{n!} \approx 2.7182818284\dots$  ,  $e$  is a natural constant, I use

" $\times$ " for Multiplication, then based on euler's  $e^{ix} = \cos x + i \sin(x)$  ( $x \in \mathbb{R}$ ) and the principle of amplitude Angle, get  $(e^{3i})^2 = (\cos(3) + i \sin(3))^2 = \cos(2 \times 3) + i \sin(2 \times 3) = \cos(6) + i \sin(6)$ , because  $e^{6i} = \cos(6) + i \sin(6)$ ,

so

$$(e^{3i})^2 = e^{6i} ,$$

In general,  $(e^{bi})^c = e^{b \times ci}$  ( $b \in \mathbb{R}$  ,  $c \in \mathbb{R}$ ) is established, the angle principle is extended to the case where the exponent is a real number.

so when  $x > 0$  ( $x \in \mathbb{R}$ ), suppose  $e^y = x$  ( $e = 2.7182818284\dots$  ,  $x$  is a natural constant,  $x \in \mathbb{R}$  and  $x > 0$ ,

$y \in \mathbb{R}$ ), then  $y = \ln(x)$  ( $x > 0$ ), based on euler's  $e^{ix} = \cos(x) + i \sin(x)$  ( $x \in \mathbb{R}$ ), will get

$$e^{yi} = e^{\ln(x)i} = \cos(\ln x) + i \sin(\ln x) \quad (x \in \mathbb{R} \text{ and } x > 0).$$

Suppose  $t \in \mathbb{R}$  and  $t \neq 0$ , now let's figure out expression for  $x^{ti}$  ( $x \in \mathbb{R}$  and  $x > 0$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ) is  $x^{ti} = (e^y)^{ti} = (e^{yi})^t = (\cos(\ln x) + i \sin(\ln x))^t$  ( $x > 0$ ).

Suppose  $s$  is any complex number, and Suppose  $s = \sigma + ti$  ( $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$ ), then let's find the expression of  $x^s$  ( $x \in \mathbb{R}$  and  $x > 0$ ,  $s \in \mathbb{C}$ ) ,

You can put  $s = \sigma + ti$  ( $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$ ) and  $x^{ti} = (e^y)^{ti} = (e^{yi})^t = (\cos(\ln x) + i \sin(\ln x))^t$  ( $x > 0$ ) into  $x^s$  ( $x > 0$ ) and you will get

$x^s = x^{(\sigma + ti)} = x^\sigma x^{ti} = x^\sigma (\cos(\ln x) + i \sin(\ln x))^t = x^\sigma (\cos(t \ln x) + i \sin(t \ln x))$  ( $x > 0$ ) , if You put  $s = \sigma - ti$  ( $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ) and  $x^{ti} = (e^y)^{ti} = (e^{yi})^t = (\cos(\ln x) + i \sin(\ln x))^t$  ( $x > 0$ ) into  $x^s$ , you will get

$$x^{\bar{s}} = x^{(\sigma - ti)} = x^\sigma (x^{ti})^{-1} = x^\sigma (\cos(\ln x) + i \sin(\ln x))^{-t} = x^\sigma (\cos(t \ln x) - i \sin(t \ln x)) \quad (x > 0) .$$

Then

$$\begin{aligned} \zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} \frac{1}{n^{\sigma + ti}} = \sum_{n=1}^{\infty} \frac{1}{n^{\sigma + ti}} = \sum_{n=1}^{\infty} \left( \frac{1}{n^\sigma} \times \frac{1}{n^{ti}} \right) = \sum_{n=1}^{\infty} (n^{-\sigma}) \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^t} \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n)))^{-t}) \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(t \ln(n)) - i \sin(t \ln(n)))) \end{aligned}$$

)

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers), or

$\zeta(s) =$

$$\prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-s}} \right) = \prod_{p=1}^{\infty} (1-p^{-s})^{-1} = \prod_{p=1}^{\infty} (1-p^{-\sigma-ti})^{-1} = \prod_{p=1}^{\infty} \left( 1 - \frac{1}{p^{\sigma+ti}} \right)^{-1} = \prod_{p=1}^{\infty} \left[ 1 - (p^{-\sigma}) \frac{1}{(\cos(\ln p) + i \sin(\ln p))^t} \right]^{-1}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers).

And

$$\begin{aligned} \zeta(\bar{s}) &= \sum_{n=1}^{\infty} \frac{1}{n^{\bar{s}}} = \sum_{n=1}^{\infty} \frac{1}{n^{\sigma-ti}} = \sum_{n=1}^{\infty} \frac{1}{n^{\sigma-ti}} = \sum_{n=1}^{\infty} \left( \frac{1}{n^{\sigma}} \times \frac{1}{n^{-ti}} \right) = \sum_{n=1}^{\infty} (n^{-\sigma}) \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^{-t}} \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n)))^t) \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n)))) \end{aligned}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers),

or

$$\begin{aligned} \zeta(\bar{s}) &= \prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-\bar{s}}} \right) = \prod_{p=1}^{\infty} (1-p^{-\bar{s}})^{-1} = \prod_{p=1}^{\infty} (1-p^{-\sigma+ti})^{-1} = \prod_{p=1}^{\infty} \left( 1 - \frac{1}{p^{\sigma-ti}} \right)^{-1} = \\ &= \prod_{p=1}^{\infty} \left[ 1 - (p^{-\sigma}) \frac{1}{(\cos(\ln p) - i \sin(\ln p))^t} \right]^{-1} = \prod_{p=1}^{\infty} \left[ 1 - (p^{-\sigma}) (\cos(\ln p) + i \sin(\ln p)) \right]^{-1} \end{aligned}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers).

And

$$\zeta(1-s) = \sum_{n=1}^{\infty} \frac{1}{n^{1-s}} = \sum_{n=1}^{\infty} \frac{1}{n^{1-\sigma-ti}} = \sum_{n=1}^{\infty} (n^{\sigma-1}) \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^{-t}} =$$

$$\sum_{n=1}^{\infty} (n^{\sigma-1}) (\cos(\ln(n)) + i \sin(\ln(n)))^t = \sum_{n=1}^{\infty} (n^{\sigma-1}) (\cos(\ln(n)) + i \sin(\ln(n)))$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers),

Or

If  $k \in \mathbb{R}$ , then

$$\zeta(k-s) = \sum_{n=1}^{\infty} \frac{1}{n^{k-s}} = \sum_{n=1}^{\infty} \frac{1}{n^{k-\sigma-ti}} = \sum_{n=1}^{\infty} (n^{\sigma-k}) \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^{-t}} =$$

$$\sum_{n=1}^{\infty} (n^{\sigma-k}) (\cos(\ln(n)) + i \sin(\ln(n)))^t = \sum_{n=1}^{\infty} (n^{\sigma-k}) (\cos(\ln(n)) + i \sin(\ln(n)))$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $k \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers),

and

$$\begin{aligned} \zeta(k-s) &= \prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-k+s}} \right) = \prod_{p=1}^{\infty} (1-p^{-k+s})^{-1} = \prod_{p=1}^{\infty} (1-p^{-\sigma-k+ti})^{-1} = \prod_{p=1}^{\infty} \left[ 1 - (p^{\sigma-k}) (\cos(\ln p) + i \sin(\ln p)) \right]^{-1} \end{aligned}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $k \in \mathbb{R}$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers).

So

$$X = n^{-\sigma} (\cos(\ln(n)) - i \sin(\ln(n))),$$

$$Y = n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n))),$$

$$G=[1 - (p^{-\sigma})(\cos(\text{tln}p) - \text{isin}(\text{tln}p))]^{-1},$$

$$H=[1 - (p^{-\sigma})(\cos(\text{tln}p) + \text{isin}(\text{tln}p))]^{-1},$$

X and Y are complex conjugates of each other, that is

$X=\bar{Y}$ , and G and H are complex conjugates of each other, that is

$$G=\bar{H}, \text{ so } \zeta(s)=\sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} X = \prod_{p=1}^{\infty} G \text{ (} s \in \mathbb{C} \text{ and } s \neq 1 \text{)}, \text{ and } \zeta(\bar{s})=\sum_{n=1}^{\infty} \frac{1}{n^{\bar{s}}} = \sum_{n=1}^{\infty} Y =$$

$$\prod_{p=1}^{\infty} H \text{ (} s \in \mathbb{C} \text{ and } s \neq 1 \text{)}, \text{ so } \zeta(s)=\overline{\zeta(\bar{s})} \text{ (} s \in \mathbb{C} \text{ and } s \neq 1 \text{)},$$

and only when  $\sigma = \frac{1}{2}$  then  $\zeta(1-s)=\zeta(\bar{s}) \text{ (} s \in \mathbb{C} \text{ and } s \neq 1 \text{)},$

and only when  $\sigma = \frac{k}{2} \text{ (} k \in \mathbb{R} \text{)},$  then  $\zeta(k-s)=\zeta(\bar{s}) \text{ (} s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R} \text{)},$  so

only  $k=1$  then  $\zeta(1-s)=\zeta(\bar{s}) = \zeta(k-s) \text{ (} s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R} \text{)},$

only  $k=1 \text{ (} k \in \mathbb{R} \text{)}$  is true, and when  $\zeta(s)=0,$  then

$$\zeta(1-s)=\zeta(k-s) = \zeta(\bar{s})=\zeta(s)=0 \text{ (} s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R} \text{)}.$$

Because

$$\text{GRH} \left( s, \chi(n) \right) = L \left( s, \chi(n) \right) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \chi(n) \sum_{n=1}^{\infty} \frac{1}{n^s} = \chi(n) \sum_{n=1}^{\infty} \frac{1}{n^{\sigma+ti}} = \chi(n) \sum_{n=1}^{\infty} \left( \frac{1}{n^{\sigma}} \frac{1}{n^{ti}} \right) =$$

$$\chi(n) \sum_{n=1}^{\infty} (n^{-\sigma}) \frac{1}{(\cos(\ln(n))+\text{isin}(\ln(n)))^t} = \chi(n) \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) + \text{isin}(\ln(n)))^{-t}) =$$

$$\chi(n) \sum_{n=1}^{\infty} n^{-\sigma} (\cos(\text{tln}(n)) - \text{isin}(\text{tln}(n))) \text{ (} t \in \mathbb{C} \text{ and } t \neq 0, s \in \mathbb{C} \text{ and } s \neq 1, n \in$$

$Z^+$  and  $n$  goes through all positive integers), because  $\zeta(s)=2^s \pi^{s-1} \text{Sin}(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s) \text{ (} s \in \mathbb{C} \text{ and}$

$s \neq 1)$  (Formula 7).  $s=-2n \text{ (} n \in Z^+)$  is the trivial zero of the  $\zeta(s)$  function, so  $s=-2n \text{ (} n \in Z^+)$  is

the trivial zero of the Landau-Siegel function  $L(\beta, \chi(n)) \text{ (} \beta \in \mathbb{R}, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0, n \in Z^+$

and  $n$  traverses all positive integers). so if  $\beta \in \mathbb{R}$  and  $\beta = -2n \text{ (} n \in Z^+)$ , then  $\zeta(s)=0$ . So

$$L(\beta, \chi(n))=$$

$$\chi(n) \sum_{n=1}^{\infty} (n^{-\beta} (\cos(0 \times \ln(n)) + \text{isin}(0 \times \ln(n)))) = \chi(n) \sum_{n=1}^{\infty} (n^{-\beta}) =$$

$$\left( \chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(4)4^{-\beta} + \dots \right) \left( \chi(n) \in \mathbb{R}, \beta \in \mathbb{R} \text{ and } \beta \neq -2n, n \in$$

$Z^+, "$  $\times$ " is the symbol for multiplication, because the real exponential function of the real number has a function value greater than zero, so

$n^{-\beta} > 0 \text{ (} n \in Z^+ \text{ and } n \text{ traverses all positive integers)}$  and  $1^{\beta} - 2^{\beta} < 0, 3^{\beta} - 4^{\beta} < 0, 5^{\beta} - 6^{\beta} < 0, \dots, (n-1)^{\beta} - (n)^{\beta} < 0, \dots,$  or  $1^{\beta} - 2^{\beta} > 0, 3^{\beta} - 4^{\beta} > 0, 5^{\beta} - 6^{\beta} > 0, \dots, (n-1)^{\beta} -$

$(n)^{\beta} > 0$ , it can be known that if  $\chi(n) \neq 0 \text{ (} n \in Z^+ \text{ and } n \text{ traverses all positive integers)}$  and  $\beta$

$\in \mathbb{R}$  and  $\beta \neq -2n \text{ (} n \in Z^+)$ , then  $L(\beta, \chi(n)) \neq 0 \text{ (} \beta \in \mathbb{R} \text{ and } \beta \neq -2n, n \in Z^+, \chi(n) \in \mathbb{R} \text{ and } n$

traverses all positive integers) and  $L(\beta, 1) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ , and  $n$  traverses all positive integers), so for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, s \neq -2n, n \in \mathbb{Z}^+$ ) functions, its corresponding Landau-Siegel function  $L(\beta, 1)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) of pure real zero does not exist, this means that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function does not have a zero of a pure real variable  $s$ , and the generalized Riemann conjecture  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ , and  $s \neq -2n, n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}, t \neq 0$ ) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are almost true. And if  $\chi(n) = 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) or  $\beta \in \mathbb{R}$  and  $\beta = -2n$  ( $n \in \mathbb{Z}^+$ ), then  $L(\beta, \chi(n)) = 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) and  $L(\beta, 1) = 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ , and  $n$  traverses all positive integers), so for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) functions, its corresponding Landau-Siegel function  $L(\beta, 1)$  ( $\beta \in \mathbb{R}, \chi(n) \in \mathbb{R}, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) of pure real zero exist, this means that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function has a zero of a pure real variable  $s$ , and the generalized Riemann conjecture  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, \chi(n) \in \mathbb{R}$  and  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are completely true.

According  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7) obtained by Riemann, so when  $\zeta(s) = 0$  then  $\zeta(1-s) = \zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). Because only when  $\sigma = \frac{1}{2}$ , the next three equations  $\zeta(\sigma + ti) = 0$ ,  $\zeta(1 - \sigma - ti) = 0$ , and  $\zeta(\sigma - ti) = 0$  are all true, so only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) is true.

$$\begin{aligned} \zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} \frac{1}{n^{\sigma+ti}} = \sum_{n=1}^{\infty} \left( \frac{1}{n^{\sigma}} \times \frac{1}{n^{ti}} \right) = \sum_{n=1}^{\infty} (n^{-\sigma}) \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^t} = \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n)))^{-t}) = \\ &= \sum_{n=1}^{\infty} (n^{-\sigma} (\cos(\ln(n)) - i \sin(\ln(n)))) = \prod_{p=1}^{\infty} \left( \frac{1}{1-p^{-s}} \right) = \prod_{p=1}^{\infty} (1 - p^{-s})^{-1} = \prod_{p=1}^{\infty} (1 - \\ & p^{-\rho-ti})^{-1} = \prod_{p=1}^{\infty} \left( 1 - \frac{1}{p^{\sigma+ti}} \right)^{-1} = \prod_{p=1}^{\infty} \left[ 1 - (p^{-\sigma}) \frac{1}{(\cos(\ln p) + i \sin(\ln p))^t} \right]^{-1} = \prod_{p=1}^{\infty} \left[ 1 - \right. \\ & \left. (p^{-\sigma}) (\cos(\ln p) - i \sin(\ln p)) \right]^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq \\ & 0, p \text{ is prime number, and } p \neq 1). \end{aligned}$$



When  $\sigma=1$ , then if  $1 - \frac{1}{p} \cos(t \ln p) + i \frac{1}{p} \sin(t \ln p) \neq 0$  then  $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p=1}^{\infty} (\frac{1}{1-p^{-s}}) \neq$

0 .if  $1 - \frac{1}{p} \cos(t \ln p) \neq 0$  and  $\frac{1}{p} \sin(t \ln p) \neq 0$ , then  $\sin(t \ln p) \neq 0$  and  $\frac{1}{p} \cos(t \ln p) \neq 1$ , then

$t \neq \frac{k\pi}{\ln p}$  ( $k \in \mathbb{Z}$ ,  $p$  is prime number, and  $p \neq 1$ ) and  $\cos(t \ln p) \neq p$  ( $t \in \mathbb{R}$  and  $t \neq 1$ ), so if

$p > 1$  ( $p$  is prime number, and  $p \neq 1$ ) then  $t \neq \frac{k\pi}{\ln p}$  ( $k \in \mathbb{Z}$ ,  $p$  is prime number, and  $p \neq 1$ ) and

$\cos(t \ln p) \neq p$  ( $p$  is prime and  $p > 1$ ), or  $p = 1$ , then  $|t| \neq \frac{k\pi}{\ln 1} \neq +\infty$  ( $k \in \mathbb{Z}$  and  $p = 1$ ) and

$\cos(t \ln 1) = 1$ ,  $t \in \mathbb{R}$  and  $t \neq 1$ . So if  $\sigma = \text{Re}(s) = 1$  and  $t \neq \frac{k\pi}{\ln p}$  ( $k \in \mathbb{Z}$ , and  $p \neq 1$ ) and

and  $t \in \mathbb{R}$  and  $t \neq 0$ , then  $\zeta(1 + ti) = \prod_{p=1}^{\infty} [1 - \frac{1}{p} \cos(t \ln p) + i \frac{1}{p} \sin(t \ln p)]^{-1} \neq$

0 ( $s \in \mathbb{C}$  and  $s \neq 1$ ). When  $s = 1 + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) then

$\zeta(1 + ti) = \prod_{p=1}^{\infty} [1 - \frac{1}{p} \cos(t \ln p) + i \frac{1}{p} \sin(t \ln p)]^{-1} \neq 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ). And when

$\text{Re}(s) = 1$  and  $p = 1$  ( $p$  is prime number), then  $\zeta(1 + ti) =$

$$= \prod_{p=1}^{\infty} (\frac{1}{1-p^{-s}}) = \prod_{p=1}^{\infty} [1 - \cos(t \ln p) + i \sin(t \ln p)]^{-1} = \prod_{p=1}^{\infty} \frac{1}{1-(p^{-1})(\cos(t \ln p) - i \sin(t \ln p))}$$

$$= \prod_{p=1}^{\infty} \frac{1}{1-(1^{-1})(\cos(t \ln 1) - i \sin(t \ln 1))} = \frac{1}{0} \rightarrow +\infty$$
 ( $t \in \mathbb{C}$  and  $t \neq 0$ ), then  $\zeta(1 + ti) \neq 0 \rightarrow$

$+\infty$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), diverges, without zero, so  $\zeta(1 + ti) \neq 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ). When  $\sigma = 0$ ,

then if  $1 - \cos(t \ln p) + i \sin(t \ln p) \neq 0$  then  $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p=1}^{\infty} (\frac{1}{1-p^{-s}}) \neq 0$ . if  $1 -$

$\cos(t \ln p) \neq 0$  and  $\sin(t \ln p) \neq 0$ , then  $t \ln p \neq k\pi$  ( $k \in \mathbb{Z}$ ) and  $\cos(t \ln p) \neq 1$ , then  $t \neq$

$\frac{k\pi}{\ln p}$  ( $k \in \mathbb{Z}$  and  $p \neq 1$ ) and  $\cos(t \ln p) \neq 1$ , so  $p > 1$ , then  $t \neq \frac{k\pi}{\ln p}$  ( $k \in \mathbb{Z}$  and  $p \neq 1$ ) and

$\cos(t \ln p) \neq 1$  ( $p \neq 1$ ), or  $p = 1$ , then  $|t| \neq \frac{k\pi}{\ln 1} \neq +\infty$  ( $k \in \mathbb{Z}$  and  $p = 1$ ) and  $|t| \neq +\infty$

,  $t \in \mathbb{R}$  and  $t \neq 0$ , then  $\zeta(0 + ti) = \prod_{p=1}^{\infty} [1 - \cos(t \ln p) + i \sin(t \ln p)]^{-1} \neq 0$  ( $t \in \mathbb{R}$  and  $t \neq$

0). So when  $\text{Re}(s) = 0$  and  $p \neq 1$ , then  $\zeta(0 + ti) = \prod_{p=1}^{\infty} [1 - \cos(t \ln p) + i \sin(t \ln p)]^{-1} \neq 0$ .

And when  $\sigma = \text{Re}(s) = 0$  and  $p = 1$ , then  $\prod_{p=1}^{\infty} [1 - (p^{-0})(\cos(t \ln p) - i \sin(t \ln p))]^{-1} =$

$$\prod_{p=1}^{\infty} \frac{1}{1-(p^{-0})(\cos(t \ln p) - i \sin(t \ln p))} = \prod_{p=1}^{\infty} \frac{1}{1-(1^{-0})(\cos(t \ln 1) - i \sin(t \ln 1))} = \frac{1}{0} \rightarrow +\infty$$
, then  $\zeta(0 + ti) \neq$

$0 \rightarrow +\infty$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), diverges, without zero. So  $\zeta(0 + ti) \neq 0$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ). It is a

fact that the non-trivial zeros of the Riemann  $\zeta(s)$  function (meaning zeros other than negative even numbers) exist, Riemann proved that the real part  $\text{Re}(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) of the nontrivial zero  $s$  of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function must satisfy  $\text{Re}(s) \in [0, 1]$ . It is

not easy to calculate the non-trivial zeros of the  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function by hand, and

Riemann calculated a dozen of them, all of which have a real part  $\text{Re}(s)$  equal to  $\frac{1}{2}$ , so the

non-trivial zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function (meaning zeros other than negative even numbers) exist.,and the real part  $\text{Re}(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) of the nontrivial zero  $s$  of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function must satisfy  $\text{Re}(s) \in (0,1)$ .When  $s=1+ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ),  $\text{Re}(s)=\sigma=1$ ,then  $\zeta(s) = \zeta(1 + ti) = \prod_{p=1}^{\infty} (\frac{1}{1-p^{-s}}) = \prod_{p=1}^{\infty} (1 - p^{-s})^{-1} = \prod_{p=1}^{\infty} (1 - p^{-1-ti})^{-1}$

$$= \prod_{p=1}^{\infty} [1 - (p^{-1}) \frac{1}{(\cos(\text{tln}p) + i\sin(\text{tln}p))^t}]^{-1} = \prod_{p=1}^{\infty} [1 - (p^{-1})(\cos(\text{tln}p) - i\sin(\text{tln}p))]^{-1} = \prod_{p=1}^{\infty} [1 - \frac{1}{p} \cos(\text{tln}p) + i \frac{1}{p} \sin(\text{tln}p)]^{-1} = \prod_{p=1}^{\infty} \frac{1}{[1 - \frac{1}{p} \cos(\text{tln}p) + i \frac{1}{p} \sin(\text{tln}p)]}$$

0 ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $t \neq 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  *traves all prime numbers*),When the independent variable  $s$  is extended from a positive integer to a general complex number, in the Euler product formula, the numerator of every product fraction factor is 1, and the denominator of every product fraction factor is a polynomial related to the natural logarithm

function. When  $p \in \mathbb{Z}^+$  and  $p$  *traves all prime numbers*, then  $\zeta(1+ti) \neq 0$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ),

indicating that the number of primes not greater than  $x$  is finite. From the analytic extended Euler product formula, we can see that for positive integers not greater than  $x$ , every increase of a prime  $p$  will increase a fraction factor related to  $\ln(p)$  in the Euler product formula,

indicating that the probability that there is a prime  $p$  near  $x$  (that is,  $x=p$ ) is about  $\frac{1}{\ln(p)}$ , that

is  $\frac{1}{\ln(x)}$ . If we use  $\pi(x)$  to represent the number of primes not greater than  $x$ , then for a

positive integer  $p$  not greater than  $x$ , the probability that it is prime is approximately  $\frac{\pi(x)}{x}$ ,

then  $\frac{\pi(x)}{x} \approx \frac{1}{\ln(x)}$ ,  $\pi(x) \approx \frac{x}{\ln(x)}$ ,  $\pi(x) \approx \frac{x}{\ln(x)}$  is the expression for the prime number theorem.

As Riemann said in his paper,  $n$  takes all the positive integers, so  $n=1,2,3,\dots$ , Let's just plug in all

the positive integers to  $\sum \frac{1}{n^s}$ .

Obviously,

$$\zeta(s) = \zeta(\sigma+ti) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \sum X = [1^{-\sigma} \cos(\text{tln}1) + 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) + 4^{-\sigma} \cos(\text{tln}4) + \dots] - i [1^{-\sigma} \sin(\text{tln}1) + 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) + 4^{-\sigma} \sin(\text{tln}4) + \dots] = U - Vi$$

$$(s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0),$$

$$U = [1^{-\sigma} \cos(\text{tln}1) + 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) + 4^{-\sigma} \cos(\text{tln}4) + \dots],$$

$$V = [1^{-\sigma} \sin(\text{tln}1) + 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) + 4^{-\sigma} \sin(\text{tln}4) + \dots],$$

then

$$\zeta(\bar{s}) = \zeta(\sigma-ti) = \sum_{n=1}^{\infty} \frac{1}{n^{\bar{s}}} = \sum Y = [1^{-\sigma} \cos(\text{tln}1) + 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) + 4^{-\sigma} \cos(\text{tln}4) + \dots] + i [1^{-\sigma} \sin(\text{tln}1) + 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) + 4^{-\sigma} \sin(\text{tln}4) + \dots] = U + Vi$$

$$(s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0),$$

$$U = [1^{-\sigma} \cos(\text{tln}1) + 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) + 4^{-\sigma} \cos(\text{tln}4) + \dots],$$

$$V = [1^{-\sigma} \sin(\text{tln}1) + 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) + 4^{-\sigma} \sin(\text{tln}4) + \dots],$$

$$\zeta(1-s) = \sum (x^{\sigma-1}) (\cos(\text{tln}x) + i \sin(\text{tln}x)) = [1^{\sigma-1} \cos(\text{tln}1) + 2^{\sigma-1} \cos(\text{tln}2) + 3^{\sigma-1} \cos(\text{tln}3) +$$

$4^{\sigma-1}\cos(\ln 4)+\dots]+i[1^{\sigma-1}\sin(\ln 1) + 2^{\sigma-1}\sin(\ln 2) + 3^{\sigma-1}\sin(\ln 3) + 4^{\sigma-1}\sin(\ln 4)$   
 $+\dots](s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0), \text{ so only when } \sigma = \frac{1}{2} \text{ and } \zeta(s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1), \text{ then it}$

must be true that  $\zeta(1-s) = \overline{\zeta(s)} = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ .

$\zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  and  $\zeta(\bar{s}) (s \in \mathbb{C} \text{ and } s \neq 1)$  are complex conjugates of each other, that is  $\zeta(s) = \overline{\zeta(\bar{s})} (s \in \mathbb{C} \text{ and } s \neq 1)$ ,

if  $\zeta(s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , then must  $\zeta(\bar{s}) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , and so if  $\zeta(s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , then it must be true that  $\zeta(s) = \zeta(\bar{s}) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ .

According to Riemann's paper "On the Number of primes not Greater than x", we can obtain an expression  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  in relation to the Riemann  $\zeta(s)$

$(s \in \mathbb{C} \text{ and } s \neq 1)$  function, which has long been known to modern mathematicians, and which I

derive later. According  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  (Formula 7) obtained

by Riemann, so when  $\zeta(s) = 0$  then  $\zeta(1-s) = \zeta(s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$  because only when  $\sigma = \frac{1}{2}$ , the next

three equations  $\zeta(\sigma+ti) = 0$ ,  $\zeta(1-\sigma-ti) = 0$ , and  $\zeta(\sigma-ti) = 0$  are all true, so only  $s = \frac{1}{2} + ti (t \in \mathbb{R} \text{ and } t \neq 0)$  is

true. And when  $\zeta(s) = 0$  then according  $\zeta(1-\bar{s}) = \overline{\zeta(1-s)} = \overline{\zeta(s)} = \overline{\zeta(\bar{s})} = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , is

also say  $\zeta(s) = \zeta(\bar{s}) = \zeta(1-\bar{s}) = \zeta(1-s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , then only  $\zeta(\sigma+ti) = \zeta(\sigma-ti) = 0$  is true. Since Riemann has shown that the Riemann  $\zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  function has zero, that is, in

$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$ ,  $\zeta(s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$  (Formula 7) is true, so

when  $\zeta(s) = 0$ , in the process of the Riemann hypothesis proved about  $\zeta(s) = \zeta(1-s) = \zeta(\bar{s}) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , is refers to the  $\zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  is a functional number. Does  $\zeta(s) = \zeta(1-s) = \zeta(\bar{s}) (s \in \mathbb{C} \text{ and } s \neq 1)$  mean the symmetry of the  $\zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  function equation? Does that mean the symmetry of the equation  $s = \bar{s} = 1-s$ ? Not really. In my analyst,  $\zeta(s) (s \in$

$\mathbb{C} \text{ and } s \neq 1)$ 、 $\zeta(1-s) (s \in \mathbb{C} \text{ and } s \neq 1)$  and  $\zeta(\bar{s}) (s \in \mathbb{C} \text{ and } s \neq 1)$  function expression are both

from  $\sum_{n=1}^{\infty} n^{-s} = \prod_p (1 - p^{-s})^{-1} (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all the positive integers, } p \in \mathbb{Z}^+ \text{ and } p \text{ goes through all the prime numbers})$ , so according to  $\sum_{n=1}^{\infty} n^{-s} = \prod_p (1 - p^{-s})^{-1} (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all the positive integers, } p \in \mathbb{Z}^+ \text{ and } p \text{ goes through all the prime numbers})$ ,  $\zeta(s) (s \in \mathbb{C} \text{ and } s \neq 1)$  function of the independent variable  $s$ , the relationship between  $\bar{s}$  and  $1-s$  only  $C_3^2 = 3$  kinds, namely  $s = \bar{s}$  or  $s = 1-s$  or  $\bar{s} = 1-s$ . As follows: according  $\zeta(s) = \zeta(1-s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$  and  $\zeta(s) = \zeta(\bar{s}) = \zeta(1-s) = 0 (s \in \mathbb{C} \text{ and } s \neq 1)$ , then only  $s = \bar{s}$  or  $s = 1-s$  or  $\bar{s} = 1-s$ , so  $s \in \mathbb{R}$  and  $s = -2n (n \in \mathbb{Z}^+)$ ,

or  $\sigma+ti = 1-ti$ , or  $\sigma-ti = 1-\sigma-ti$ , so  $s \in \mathbb{R}$  and  $s = -2n (n \in \mathbb{Z}^+)$ , or  $\sigma = \frac{1}{2}$  and  $t = 0$ , or  $\sigma = \frac{1}{2}$  and  $t \in \mathbb{R}$

and  $t \neq 0$ , so  $s \in \mathbb{R}$ , or  $s = \frac{1}{2} + 0i$ , or  $s = \frac{1}{2} + ti (t \in \mathbb{R} \text{ and } t \neq 0)$ , because  $\zeta(\frac{1}{2}) \rightarrow +\infty, \zeta(1) \rightarrow +\infty$ ,

$\zeta(1)$  is divergent,  $\zeta(\frac{1}{2})$  is more divergent, so drop  $s = 1$  and  $s = \frac{1}{2}$ . According the equation  $\xi(s) =$

$\frac{1}{2}s(s-1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by Riemann, so  $\xi(s)=\xi(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ),

because  $\Gamma(\frac{s}{2}) = \overline{\Gamma(\frac{\bar{s}}{2})}$ , and  $\pi^{-\frac{s}{2}} = \overline{\pi^{-\frac{\bar{s}}{2}}}$ , and because  $\zeta(s) = \overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so

$\xi(s) = \overline{\xi(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). So when  $\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(s)=\zeta(1-s) = \zeta(\bar{s}) =$

$0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(s)=\xi(1-s)=\xi(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) must be true, so the nontrivial

zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the nontrivial zeros of the Riemann

$\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function are identical, so the complex root of Riemann  $\xi(s)=0$  ( $s \in$

$\mathbb{C}$  and  $s \neq 1$ ) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ). According to the Riemann function

$\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) defined by Riemann

and he Riemann hypothesis  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), because  $s \neq 1$ , and  $\prod_{2}^s \neq 0$ ,  $\pi^{-\frac{s}{2}} \neq 0$ , so

$\prod_{2}^s(s-1)\pi^{-\frac{s}{2}} \neq 0$ , and when  $\xi(t)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), then  $\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$  ( $t \in$

$\mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ), and  $\zeta(\frac{1}{2}+ti) = \frac{\xi(t)}{\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}} = \frac{0}{\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}} = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in$

$\mathbb{C}$  and  $s \neq 1$ ), so  $t \in \mathbb{R}$  and  $t \neq 0$ . So the root  $t$  of the equations  $\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$  ( $t \in$

$\mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and  $4 \int_1^\infty \frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2}t \ln x) dx = \xi(t)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in$

$\mathbb{C}$  and  $s \neq 1$ ) and

$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2}t \ln x) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) must be real and  $t \neq 0$ .

Riemann got  $\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = \frac{1}{2} -$

$(t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2}t \ln x) dx$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) in his paper, or

$\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and

$\xi(t) = 4 \int_1^\infty \frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2}t \ln x) dx$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ), because the root of

$\zeta(\frac{1}{2}+ti)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) is the root of  $\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq$

$1$ ), and because the root of  $\zeta(\frac{1}{2}+ti)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) is the root of

$\prod_{2}^s(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti) = 4 \int_1^\infty \frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2}t \ln x) dx = \xi(t)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ),

and because the root of  $\zeta(\frac{1}{2}+ti)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) is the root of

$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ), so the roots of

equations  $\prod \frac{s(s-1)}{2} \pi^{-\frac{s}{2}} \zeta(\frac{1}{2} + ti) = \xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ )

and  $4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2} t \ln x) dx = \xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = \frac{1}{2} - (t^2 +$

$\frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ) must all be real numbers, and the

roots are the same number, because the root of  $\zeta(\frac{1}{2} + ti) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) is  $s = \frac{1}{2} + ti$  ( $t \in$

$\mathbb{R}$  and  $t \neq 0$ ), so when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), the real part of the

root of  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) must be between 0 and  $T$ , and the real roots of  $\xi(t) = 0$  ( $t \in$

$\mathbb{C}$  and  $t \neq 0$ ) has the same number of complex roots of  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ). So when

$\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), the number of roots of  $\xi(t) = 0$  ( $t \in$

$\mathbb{C}$  and  $t \neq 0$ ) must be approximately equal to  $\frac{T}{2\pi} \ln \frac{T}{2\pi} - \frac{T}{2\pi}$ , all the roots of  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq$

0) are real numbers, so the Riemann hypothesis and the Riemann conjecture are perfectly valid.

Because the number of roots  $t$  of  $\zeta(\frac{1}{2} + it) = \sum_{n=1}^\infty (n^{-\frac{1}{2}} (\cos(t \ln(n)) - i \sin(t \ln(n)))) =$

$\sum_{n=1}^\infty (n^{-\frac{1}{2}} (\cos(\ln(n^t)) - i \sin(\ln(n^t)))) = 0$  is the number of roots of

$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) = 0$ . Because when  $t=0$ , then  $\zeta(\frac{1}{2})$  is divergent, when

$\ln(n^t) \in [0, 2\pi]$ , the numbers of the root  $t$  of

$\zeta(\frac{1}{2} + it) = \sum_{n=1}^\infty (n^{-\frac{1}{2}} (\cos(t \ln(n)) - i \sin(t \ln(n)))) =$

$\sum_{n=1}^\infty (n^{-\frac{1}{2}} (\cos(\ln(n^t)) - i \sin(\ln(n^t)))) = 0$  is  $\ln \frac{T}{2\pi} - 1$ , so when  $t \in (0, T]$ , the numbers of

## Formula 2

Let's say I have any complex number  $Z = x + yi$  ( $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ ), and I have any complex number

$s = \sigma + ui$  ( $\sigma \in \mathbb{R}$ ,  $u \in \mathbb{R}$ ). We use  $r$  ( $r \in \mathbb{R}$ , and  $r > 0$ ) to represent the module  $|Z|$  of complex  $Z = x + yi$

( $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ ), and  $\varphi$  to represent the argument  $\text{Am}(Z)$  of complex  $Z = x + yi$  ( $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ ). That is

$$|Z| = r, \text{ then } r = (x^2 + y^2)^{\frac{1}{2}},$$

so  $Z = r(\text{Cos}(\varphi) + i \text{Sin}(\varphi))$  and  $\varphi = |\arccos(\frac{x}{(x^2 + y^2)^{\frac{1}{2}}})|$ , and  $\varphi \in (-\pi, \pi]$ , then  $\varphi = \text{Am}(Z)$ .

Base on  $x^s = x^\sigma x^{ui} = x^\sigma (\cos(\ln x) + i \sin(\ln x))^u = x^\sigma (\cos(u \ln x) + i \sin(u \ln x))$  can get

$$r^s = r^\sigma r^{ui} = r^\sigma (\cos(\ln x) + i \sin(\ln x))^u = r^\sigma (\cos(u \ln x) + i \sin(u \ln x)) \quad (r > 0), \text{ then}$$

$$f(Z, s) = z^s = (r(\cos(\varphi) + i \sin(\varphi))^{\sigma+ui}) = (r(\cos(\varphi) + i \sin(\varphi))^\sigma r(\cos(\varphi) + i \sin(\varphi))^{ui}) =$$

$$r^\sigma (\cos(\sigma\varphi) + i \sin(\sigma\varphi)) (r(\cos(\varphi) + i \sin(\varphi))^{ui}) = r^\sigma (\cos(\sigma\varphi) + i \sin(\sigma\varphi)) r^{ui} (\cos(\varphi) + i \sin(\varphi))^{ui}$$

$$= r^\sigma (\cos(\sigma\varphi) + i \sin(\sigma\varphi)) (\cos(u \ln r) + i \sin(u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^{ui}$$

$$= r^\sigma (\cos(\rho\varphi) + i \sin(\rho\varphi)) (\cos(u \ln r) + i \sin(u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^{ui}$$

$$= r^\sigma (\cos(\rho\varphi + u \ln r) + i \sin(\rho\varphi + u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^{ui}.$$

Beacuse of

$$Z =$$

$$e \ln|Z| + i \text{Am}(Z) = e \ln|Z| e^{i \text{Am}(Z)} = e \ln|Z| (\cos(\text{Am}(Z)) + i \sin(\text{Am}(Z))) = r (\cos(\text{Am}(Z)) + i \sin(\text{Am}(Z))),$$

so  $\ln Z = \ln|Z| + i \text{Am}(Z) \quad (-\pi < \text{Am}(Z) \leq \pi).$

Suppose  $a > 0$ , then  $a^x = e^{\ln(a^x)} = e^{x \ln a}$ , then  $z^s = e^{s \ln z}$ .

Suppose any complex Number  $Q = \cos(u\varphi) + i \sin(u\varphi)$ , and Suppose

the complex  $\psi = i$ , then  $\ln Q = \ln|Q| + i \text{Am}(Q) \quad (-\pi < \text{Am}(Q) \leq \pi).$

Because  $0 \leq |\sin(u\varphi)| \leq 1$ ,

so

If  $-\pi < u\varphi \leq \pi$ , then  $\text{Am}(Q) = u\varphi$  and  $-\pi < \text{Am}(Q) \leq \pi$ ;

If  $u\varphi > \pi$ , then  $\text{Am}(Q) = u\varphi - 2k\pi \quad (k \in \mathbb{Z}^+)$  and  $-\pi < \text{Am}(Q) \leq \pi$ ;

if  $u\varphi < -\pi$ , then  $\text{Am}(Q) = u\varphi + 2k\pi \quad (k \in \mathbb{Z}^+)$  and  $-\pi < \text{Am}(Q) \leq \pi$ . Then

If  $\text{Am}(Q) = u\varphi$ , then

$$(\cos(u\varphi) + i \sin(u\varphi))^i = Q^\psi = e^{\psi \ln Q} = e^{\psi (\ln|Q| + i \text{Am}(Q))} = e^{i(\ln|Q| + \text{Am}(Q))} = e^{-u\varphi}.$$

then

$$f(Z, s) = z^s = r^\sigma (\cos(\sigma\varphi + u \ln r) + i \sin(\sigma\varphi + u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^i$$

$$= r^\sigma (\cos(\sigma\varphi + u \ln r) + i \sin(\sigma\varphi + u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^i$$

$$= e^{-u\varphi} r^\sigma (\cos(\rho\varphi + u \ln r) + i e^{-u\varphi} r^\sigma \sin(\rho\varphi + u \ln r)).$$

Substituting  $r = (x^2 + y^2)^{\frac{1}{2}}$  into the above equation gives:

$$f(Z, s) = z^s = e^{-u\varphi} (x^2 + y^2)^{\frac{\sigma}{2}} (\cos(\rho\varphi + u \ln(x^2 + y^2)^{\frac{1}{2}}))$$

$$+ i e^{-u\varphi} (x^2 + y^2)^{\frac{\sigma}{2}} (\sin(\sigma\varphi + u \ln(x^2 + y^2)^{\frac{1}{2}})).$$

If  $\text{Am}(Q) = u\varphi - 2k\pi \quad (k \in \mathbb{Z}^+)$ , then

$$(\cos(u\varphi) + i \sin(u\varphi))^i = Q^\psi = e^{\psi \ln Q} = e^{\psi (\ln|Q| + i \text{Am}(Q))} = e^{i(\ln|Q| + \text{Am}(Q))} = e^{2k\pi - u\varphi}, \text{ then}$$

$$f(Z, s) = z^s = r^\sigma (\cos(\sigma\varphi + u \ln r) + i \sin(\sigma\varphi + u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^i$$

$$= r^\sigma (\cos(\sigma\varphi + u \ln r) + i \sin(\sigma\varphi + u \ln r)) (\cos(u\varphi) + i \sin(u\varphi))^i$$

$$= e^{2k\pi - u\varphi} r^\sigma (\cos(\sigma\varphi + u \ln r) + i e^{2k\pi - u\varphi} r^\sigma \sin(\sigma\varphi + u \ln r)).$$

Substituting  $r = (x^2 + y^2)^{\frac{1}{2}}$  into the above equation gives:

$$f(Z, s) = z^s = e^{2k\pi - u\varphi} (x^2 + y^2)^{\frac{\sigma}{2}} (\cos(\sigma\varphi + u \ln(x^2 + y^2)^{\frac{1}{2}}))$$

$$+ie^{2k\pi-u\varphi}(x^2 + y^2)^{\frac{\sigma}{2}}(\sin(\sigma\varphi + \text{uln}(x^2 + y^2)^{\frac{1}{2}})).$$

If  $\text{Am}(Q)=u\varphi+2k\pi(k \in \mathbb{Z}^+)$ , then

$$(\cos(u\varphi) + i\sin(u\varphi))^i = Q^\psi = e^{\psi \ln Q} = e^{\psi(\ln|Q|+i\text{Am}(Q))}=e^{i(o+i(u\varphi+2k\pi))}=e^{-2k\pi-u\varphi}, \text{ then}$$

$$f(Z,s)=Z^s=r^\sigma(\cos(\sigma\varphi + \text{ulnr}) + i\sin(\sigma\varphi + \text{ulnr}))(\cos(u\varphi) + i\sin(u\varphi))^i$$

$$=r^\sigma(\cos(\sigma\varphi + \text{ulnr}) + i\sin(\sigma\varphi + \text{ulnr}))(\cos(u\varphi) + i\sin(u\varphi))^i$$

$$=e^{-2k\pi-u\varphi}r^\sigma(\cos(\sigma\varphi + \text{ulnr})+ie^{-2k\pi-u\varphi}r^\sigma\sin(\sigma\varphi + \text{ulnr})).$$

Substituting  $r=(x^2 + y^2)^{\frac{1}{2}}$  into the above equation gives:

$$f(Z,s)=z^s=e^{-2k\pi-u\varphi}(x^2 + y^2)^{\frac{\sigma}{2}}(\cos(\sigma\varphi + \text{uln}(x^2 + y^2)^{\frac{1}{2}}))$$

$$+ie^{-2k\pi-u\varphi}(x^2 + y^2)^{\frac{\sigma}{2}}(\sin(\sigma\varphi + \text{uln}(x^2 + y^2)^{\frac{1}{2}})).$$

**Reasoning 1:**

For any complex number  $s$ , when  $\text{Rs}(s) > 0$  and  $(s \neq 1)$ , and if  $s=\sigma+ti(\sigma \in \mathbb{R},t \in \mathbb{R}$  and  $t \neq 0,s \in \mathbb{C})$ ,

then according to Dirichlet  $\eta(s)$ , then the relationship between the Riemann  $\zeta(s)(s \in \mathbb{C}$  and

$\text{Rs}(s)>0$  and  $s \neq 1$ ) function and the Dirichlet  $\eta(s)(s \in \mathbb{C}$  and  $\text{Rs}(s)>0$  and  $s \neq 1$ ) function is :

because

$$\eta(s)=\frac{1}{1^s}-\frac{1}{2^s}+\frac{1}{3^s}-\frac{1}{4^s}+\frac{1}{5^s}-\frac{1}{6^s}+\dots(s \in \mathbb{C} \text{ and } \text{Rs}(s) > 0 \text{ and } s \neq 1),$$

$$\zeta(s)=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\dots(s \in \mathbb{C} \text{ and } \text{Rs}(s) > 0 \text{ and } s \neq 1), \text{ so}$$

$$\eta(s) - \zeta(s) = -\left(\frac{2}{2^s} + \frac{2}{4^s} + \frac{2}{6^s} + \dots\right) = -\frac{2}{2^s} \left(\frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \dots\right) = -\frac{2}{2^s} \zeta(s) (s \in$$

$\mathbb{C}$  and  $\text{Rs} > 0$  and  $s \neq 1$ ), then

$$\eta(s) = 1 - \frac{2}{2^s} \zeta(s) = (1 - 2^{1-s}) \zeta(s) (s \in \mathbb{C} \text{ and } \text{Rs}(s) > 0 \text{ and } s \neq 1), \text{ then}$$

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} (s \in \mathbb{C} \text{ and } \text{Rs}(s) > 0 \text{ and } s \neq 1) \text{ and } \eta(s) = (1 - 2^{1-s}) \zeta(s) (s \in \mathbb{C} \text{ and } \text{Rs}(s) >$$

$0$  and  $s \neq 1$ ,  $\zeta(s)$  is the Riemann Zeta function,  $\eta(s)$  is the Dirichlet  $\eta(s)$  function,

$$\text{so Riemann } \zeta(s) = \frac{\eta(s)}{(1-2^{1-s})} = \frac{1}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})} \prod_p (1 - p^{-s})^{-1} (s \in \mathbb{C} \text{ and } \text{Rs}(s) >$$

$0$  and  $s \neq 1, n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C}$ ,  $n$  goes through all the positive integers,  $p$  goes through all the prime numbers). Let's prove that  $\zeta(s)$  and  $\zeta(\bar{s})$  are complex conjugations of each other.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = [1^{-\sigma} \cos(\text{tln}1) - 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) - 4^{-\sigma} \cos(\text{tln}4) - \dots] - i[1^{-\sigma} \sin(\text{tln}1) - 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) - 4^{-\sigma} \sin(\text{tln}4) + \dots] = U - Vi,$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} = [1^{-\sigma} \cos(\text{tln}1) - 2^{-\sigma} \cos(\text{tln}2) + 3^{-\sigma} \cos(\text{tln}3) - 4^{-\sigma} \cos(\text{tln}4) - \dots] + i[1^{-\sigma} \sin(\text{tln}1) - 2^{-\sigma} \sin(\text{tln}2) + 3^{-\sigma} \sin(\text{tln}3) - 4^{-\sigma} \sin(\text{tln}4) + \dots] = U + Vi,$$

$$\sin(\ln 2) + 3^{-\sigma} \sin(\ln 3) - 4^{-\sigma} \sin(\ln 4) + \dots = U + Vi,$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-s}} = [1^{\sigma-1} \cos(\ln 1) - 2^{\sigma-1} \cos(\ln 2) + 3^{\sigma-1} \cos(\ln 3) - 4^{\sigma-1} \cos(\ln 4) - \dots] + i[1^{-\sigma} \sin(\ln 1) - 2^{-\sigma} \sin(\ln 2) + 3^{-\sigma} \sin(\ln 3) - 4^{-\sigma} \sin(\ln 4) + \dots],$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{k-s}} = [1^{\sigma-k} \cos(\ln 1) - 2^{\sigma-k} \cos(\ln 2) + 3^{\sigma-k} \cos(\ln 3) - 4^{\sigma-k} \cos(\ln 4) - \dots] + i[1^{\sigma-k} \sin(\ln 1) - 2^{\sigma-k} \sin(\ln 2) + 3^{\sigma-k} \sin(\ln 3) - 4^{\sigma-k} \sin(\ln 4) + \dots],$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $k \in \mathbb{R}$ ),

because ,

$$\frac{(-1)^{n-1}}{(1-2^{1-s})} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}},$$

$$\prod_p (1 - p^{-s})^{-1} = \overline{\prod_p (1 - p^{-\bar{s}})^{-1}}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers),

so

$$\frac{(-1)^{n-1}}{(1-2^{1-s})} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}},$$

so

$$\frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}}},$$

$$\frac{(-1)^{n-1}}{(1-2^{1-s})} \prod_p (1 - p^{-s})^{-1} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \prod_p (1 - p^{-\bar{s}})^{-1}},$$

$$\zeta(s) = \frac{1}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})} \prod_p (1 - p^{-s})^{-1},$$

$$\zeta(\bar{s}) = \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} = \overline{\frac{(-1)^{n-1}}{(1-2^{1-s})} \prod_p (1 - p^{-s})^{-1}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer, } p \in \mathbb{Z}^+ \text{ and } p \text{ traves all prime numbers}),$$

so

only  $\zeta(s) = \overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), <sup>[2]</sup>

so

$$p^{1-s} = p^{(1-\sigma-ti)} = p^{1-\sigma} p^{-ti} = p^{1-\sigma} (\cos(\ln p) + i \sin(\ln p))^{-t} = p^{1-\sigma} (\cos(\ln p) - i \sin(\ln p)),$$

$$p^{1-\bar{s}} = p^{(1-\sigma+ti)} = p^{1-\sigma} p^{ti} = p^{1-\sigma} (p^{ti}) = p^{1-\sigma} (\cos(\ln p) + i \sin(\ln p))^t = (p^{1-\sigma} (\cos(\ln p) + i \sin(\ln p)))^t, (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } s \neq 0, p \in \mathbb{Z}^+)$$

then

$$p^{-(1-s)} = p^{(-1+\sigma+ti)} = p^{\sigma-1} p^{ti} = p^{\sigma-1} \frac{1}{(\cos(\ln p) - i \sin(\ln p))} = (p^{\sigma-1} (\cos(\ln p) + i \sin(\ln p))),$$

$$p^{-(\bar{s})} = p^{-(\sigma-ti)} = p^{-\sigma} p^{ti} = (p^{-\sigma} (\cos(\ln p) + i \sin(\ln p)))$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $s \neq 0$ ,  $p \in \mathbb{Z}^+$ ),

so

$$(1 - p^{-(1-s)}) = 1 - (p^{\sigma-1} (\cos(\ln p) + i \sin(\ln p))) = 1 - p^{\sigma-1} \cos(\ln p) - ip^{\sigma-1} \sin(\ln p),$$

$$(1 - p^{-(\bar{s})}) = 1 - (p^{-\sigma} (\cos(\ln p) + i \sin(\ln p))) = 1 - p^{-\sigma} \cos(\ln p) - ip^{-\sigma} \sin(\ln p),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $t \neq 0$ ,  $p \in \mathbb{Z}^+$ ),



$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-s}} = [1^{\sigma-1} \cos(\ln 1) - 2^{\sigma-1} \cos(\ln 2) + 3^{\sigma-1} \cos(\ln 3) - 4^{\sigma-1} \cos(\ln 4) - \dots] + i[1^{\sigma-1} \sin(\ln 1) - 2^{\sigma-1} \sin(\ln 2) + 3^{\sigma-1} \sin(\ln 3) - 4^{\sigma-1} \sin(\ln 4) + \dots],$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = [1^{-\sigma} \cos(\ln 1) - 2^{-\sigma} \cos(\ln 2) + 3^{-\sigma} \cos(\ln 3) - 4^{-\sigma} \cos(\ln 4) - \dots] + i[1^{-\sigma} \sin(\ln 1) - 2^{-\sigma} \sin(\ln 2) + 3^{-\sigma} \sin(\ln 3) - 4^{-\sigma} \sin(\ln 4) + \dots]$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $s \neq 0$ ,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer ),

when  $\sigma = \frac{1}{2}$ , then

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-s}} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer, } k \in \mathbb{R}),$$

$$(1 - p^{-(1-s)}) = (1 - p^{-\bar{s}}) \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+),$$

and

$$(1 - p^{-(1-s)})^{-1} = (1 - p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+),$$

$$\prod_p (1 - p^{-(1-s)})^{-1} = \prod_p (1 - p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, \text{ and } p \text{ traves all prime numbers, } k \in \mathbb{R}),$$

$$\prod_p (1 - p^{-(1-s)})^{-1} = \prod_p (1 - p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ traves all prime numbers, } k \in \mathbb{R}),$$

and

$$\frac{(-1)^{n-1}}{(1-2^s)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-s}} = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}},$$

$$\frac{(-1)^{n-1}}{(1-2^s)} \prod_p (1 - p^{-(1-s)})^{-1} = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \prod_p (1 - p^{-\bar{s}})^{-1}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $t \neq 0$ ,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers),

And

$$\zeta(1-s) = \frac{(-1)^{n-1}}{(1-2^s)} \prod_p (1 - p^{-(1-s)})^{-1},$$

$$\zeta(\bar{s}) = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \prod_p (1 - p^{-\bar{s}})^{-1},$$

$$\zeta(1-s) = \frac{(-1)^{n-1}}{(1-2^s)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-s}},$$

$$\zeta(\bar{s}) = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer),

so when  $\sigma = \frac{1}{2}$ , then

only  $\zeta(1-s) = \zeta(\bar{s})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) must be true.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{k-s}} = [1^{\sigma-k} \cos(\ln 1) - 2^{\sigma-k} \cos(\ln 2) + 3^{\sigma-k} \cos(\ln 3) - 4^{\sigma-k} \cos(\ln 4) - \dots] + i[1^{\sigma-k} \sin(\ln 1) - 2^{\sigma-k} \sin(\ln 2) + 3^{\sigma-k} \sin(\ln 3) - 4^{\sigma-k} \sin(\ln 4) + \dots],$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} = [1^{-\sigma} \cos(\ln 1) - 2^{-\sigma} \cos(\ln 2) + 3^{-\sigma} \cos(\ln 3) - 4^{-\sigma} \cos(\ln 4) - \dots] + i[1^{-\sigma} \sin(\ln 1) - 2^{-\sigma} \sin(\ln 2) + 3^{-\sigma} \sin(\ln 3) - 4^{-\sigma} \sin(\ln 4) + \dots],$$

$$p^{k-s} = p^{(k-\sigma-ti)} = p^{k-\sigma} p^{-ti} = p^{k-\sigma} (\cos(\ln p) + i \sin(\ln p))^{-t} = p^{k-\sigma} (\cos(\ln p) - i \sin(\ln p)),$$

$$p^{1-\bar{s}} = p^{(1-\sigma+ti)} = p^{1-\sigma} p^{ti} = p^{1-\sigma} (p^{ti}) = p^{1-\sigma} (\cos(\ln p) + i \sin(\ln p))^t = (p^{1-\sigma} (\cos(\ln p) + i \sin(\ln p)))^t,$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $k \in \mathbb{R}$ ),

Then

$$p^{-(k-s)} = p^{(-k+\sigma+ti)} = p^{\sigma-k} p^{ti} = p^{\sigma-k} \frac{1}{(\cos(\ln p) - i \sin(\ln p))} = (p^{\sigma-k} (\cos(\ln p) + i \sin(\ln p))),$$

$$p^{-(\bar{s})} = p^{-(\sigma-ti)} = p^{-\sigma} p^{ti} = (p^{-\sigma} (\cos(\ln p) + i \sin(\ln p))),$$

$$p^{-(k-s)} = (p^{\sigma-k} (\cos(\ln p) + i \sin(\ln p))),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  is a prime number  $k \in \mathbb{R}$ ),

so

$$(1 - p^{-(k-s)}) = 1 - (p^{\sigma-k} (\cos(\ln p) + i \sin(\ln p))) = 1 - p^{\sigma-k} \cos(\ln p) - ip^{\sigma-k} \sin(\ln p),$$

$$(1 - p^{-\bar{s}}) = 1 - (p^{-\sigma} (\cos(\ln p) + i \sin(\ln p))) = 1 - p^{-\sigma} \cos(\ln p) - ip^{-\sigma} \sin(\ln p),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  is a prime number  $k \in \mathbb{R}$ ),

So

when  $\sigma = \frac{k}{2}$  ( $k \in \mathbb{R}$ ) then

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-k+s}} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R}, n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer}),$$

$$(1 - p^{-(k-s)}) = (1 - p^{-\bar{s}}) \quad (s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R}, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime number}),$$

$$\text{and } (1 - p^{-(k-s)})^{-1} = (1 - p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R}, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime number}),$$

$$\prod_p (1 - p^{-(k-s)})^{-1} = \prod_p (1 - p^{-\bar{s}})^{-1}, \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ traves all prime numbers},$$

$n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $k \in \mathbb{R}$ ),

and

$$\frac{1}{(1-2^{1-k+s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{k-s}} = \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $k \in \mathbb{R}$ ),

and

$$\zeta(k-s) = \frac{(-1)^{n-1}}{(1-2^{1-k+s})} \prod_p (1 - p^{-(k-s)})^{-1},$$

$$\zeta(\bar{s}) = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \prod_p (1 - p^{-\bar{s}})^{-1},$$

$$\zeta(k-s) = \frac{1}{(1-2^{1-k+s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{k-s}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R}),$$

$$\zeta(\bar{s}) = \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} \quad (s \in \mathbb{C} \text{ and } s \neq 1),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  traves all prime numbers,  $n \in \mathbb{Z}^+$  and  $n$  traves all positive integer,  $k \in \mathbb{R}$ ),

so when  $\sigma = \frac{k}{2}$  ( $k \in \mathbb{R}$ ) then only  $\zeta(k-s) = \zeta(\bar{s})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) ( $s \in \mathbb{C}$  and  $s \neq 1, k \in \mathbb{R}$ ).

According to the equation  $\zeta(1-s)=2^{1-s}\pi^{-s}\text{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by

Riemann, since Riemann has shown that the Riemann  $\zeta(s)$  function has zero, that is, in

$$\zeta(1-s)=2^{1-s}\pi^{-s}\text{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1), \quad \zeta(s)=0 \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ is true.}$$

When  $\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then only  $\zeta(k-\bar{s})=\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), and

When  $\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(k-s)=\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). And because

when  $\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then only  $\zeta(1-s)=\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), which is  $\zeta(k-s)=\zeta(\bar{s})$  ( $s \in \mathbb{C}$  and  $s \neq 1, k \in \mathbb{R}$ ), so only  $k=1$  be true. According  $\zeta(s)=\zeta(1-s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\zeta(s)=\zeta(\bar{s})=\zeta(1-\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $s=\bar{s}$  or  $s=1-s$  or  $\bar{s}=1-s$ , so  $s \in \mathbb{R}$  and  $s=-2n$  ( $n \in \mathbb{Z}^+$ ),

or  $\sigma+ti=1-\sigma-ti$ , or  $\sigma-ti=1-\sigma-ti$ , so  $s \in \mathbb{R}$ , or  $\sigma=\frac{1}{2}$  and  $t=0$ , or  $\sigma=\frac{1}{2}$  and  $t \in \mathbb{R}$  and  $t \neq 0$ ,

so  $t \in \mathbb{R}$ , or  $s=\frac{1}{2}+0i$ , or  $s=\frac{1}{2}+ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), because  $\zeta(\frac{1}{2}) \rightarrow +\infty, \zeta(1) \rightarrow +\infty, \zeta(1)$  is

divergent,  $\zeta(\frac{1}{2})$  is more divergent, so drop them. Because only when  $\sigma=\frac{1}{2}$ , the next three

equations,  $\zeta(\sigma+ti)=0$ ,

$\zeta(1-\sigma-ti)=0$ , and  $\zeta(\sigma-ti)=0$  are all true, because  $\zeta(\frac{1}{2}) \rightarrow +\infty, \zeta(1) \rightarrow +\infty, \zeta(1)$  is

divergent,  $\zeta(\frac{1}{2})$  is more divergent, so drop  $s=1$  and  $s=\frac{1}{2}$ , so only  $s=\frac{1}{2}+ti$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$ ) is

true. Since Riemann has shown that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function has zero,

that is, in  $\zeta(1-s)=2^{1-s}\pi^{-s}\text{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ),  $\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) is true.

According to the equation  $\xi(s)=\frac{1}{2}s(s-1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(\frac{s}{2})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by Riemann, so

$\xi(s)=\xi(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), because  $\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\bar{s}}{2})}$ , and  $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\bar{s}}{2}}}$ , and because

$\zeta(s)=\overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so  $\xi(s)=\overline{\xi(\bar{s})}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). So when  $\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then

$\xi(s)=\zeta(1-s)=\zeta(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(s)=\xi(1-s)=\xi(\bar{s})=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) must be true, so the zeros of the Riemann  $\zeta(s)$  function and the nontrivial zeros of the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function are identical, so the complex root of Riemann  $\xi(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

satisfies  $s=\frac{1}{2}+ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), according to the Riemann function  $\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{s}{2})=\xi(t)$  ( $s \in \mathbb{C}$  and

$s \neq 1, t \in \mathbb{C}$  and  $t \neq 0$ ) and the Riemann hypothesis  $s=\frac{1}{2}+ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), because  $s \neq 1$ , and

$\prod_{\frac{s}{2}} \neq 0, \pi^{-\frac{s}{2}} \neq 0$ , so  $\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}} \neq 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), and when  $\xi(t)=0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), then

$$\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0), \quad \text{and} \quad \zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}}}=\frac{0}{\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}}}$$

$=0$  ( $s \in \mathbb{C}$  and  $s \neq 1, s \in \mathbb{C}$  and  $t \neq 0$ ), so  $t \in \mathbb{R}$  and  $t \neq 0$ . So the root  $t$  of the equations

$$\prod_{\frac{s}{2}}(s-1)\pi^{-\frac{s}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0) \text{ and}$$

$$4 \int_1^{\infty} \frac{d(x^2 \Psi'(x))}{dx} x^{-\frac{1}{4}} \cos\left(\frac{1}{2} t \ln x\right) dx = \xi(t) = 0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, s \in \mathbb{C} \text{ and } t \neq 0) \text{ and}$$

$$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4} \int_1^{\infty} \Psi(x) x^{-\frac{3}{4}} \cos\left(\frac{1}{2} t \ln x\right) = 0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0) \text{ must be real and } t \neq$$

0. If  $\text{Re}(s) = \frac{k}{2}$  ( $k \in \mathbb{R}$ ), then  $\zeta(k-s) = 2^{k-s} \pi^{-s} \text{Cos}\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and

$$\xi(k-s) = \frac{1}{2} s(s-k) \Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1, k \in \mathbb{R}) \text{ are true, so when } \zeta(s) = 0 \quad (s \in \mathbb{C} \text{ and } s \neq$$

1), then  $\zeta(s) = \zeta(k-s) = \zeta(\bar{s}) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, s \in \mathbb{C}$ ) and  $\xi(s) = \xi(k-s) = \xi(\bar{s}) = 0$  ( $s \in \mathbb{C}$  and

$s \neq 1, s \in \mathbb{C}$ ) must be true, and  $s = \frac{k}{2} + ti$  ( $k \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0$ ) must be true,

then

$$\prod_{\frac{s}{2}}^{s-k} (s-k) \pi^{-\frac{s}{2}} \zeta\left(\frac{k}{2} + ti\right) = \xi(t) = 0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in \mathbb{R}), \text{ and } \zeta\left(\frac{k}{2} + ti\right) =$$

$$\frac{\xi(t)}{\prod_{\frac{s}{2}}^{s-k} (s-k) \pi^{-\frac{s}{2}}} = \frac{0}{\prod_{\frac{s}{2}}^{s-k} (s-k) \pi^{-\frac{s}{2}}} = 0 \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in \mathbb{R}), \text{ so } t \in \mathbb{R} \text{ and } t \neq 0. \text{ So the root } t$$

of the equations  $\prod_{\frac{s}{2}}^{s-k} (s-k) \pi^{-\frac{s}{2}} \zeta\left(\frac{k}{2} + ti\right) = \xi(t) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, t \in \mathbb{C}$  and  $t \neq 0, k \in \mathbb{R}$ ) must be real

and  $t \neq 0$ . But the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function only satisfies

$$\zeta(1-s) = 2^{1-s} \pi^{-s} \text{Cos}\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 7) and } \xi(s) = \frac{1}{2} s(s-1) \Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s) \quad (s \in \mathbb{C}$$

and  $s \neq 1$ ), is also say that only  $\zeta(1-s) = 2^{1-s} \pi^{-s} \text{Cos}\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7) is true,

so only  $\text{Re}(s) = \frac{k}{2} = \frac{1}{2}$  is true, so only  $k=1$  is true. The Riemann hypothesis and the Riemann

conjecture must satisfy the properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the

Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, The properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function are fundamental, the Riemann

hypothesis and the Riemann conjecture must be correct to reflect the properties of the Riemann

$\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, that is, the roots of

the Riemann  $\xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) function can only be real, that is,  $\text{Re}(s)$  can only be equal to  $\frac{1}{2}$ ,

and  $\text{Im}(s)$  must be real, and  $\text{Im}(s)$  is not equal to zero. So the Riemann hypothesis and the

Riemann conjecture must be correct.

For any complex number  $s$ , when  $\text{Re}(s)$  is any real number, including  $\text{Re}(s) > 0$  and ( $s \neq 1$ ) and  $\text{Re}(s) \leq 0$  and  $s \neq 0$ ), then

Riemann  $\zeta(s)$  function is  $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7). Suppose

$s = \sigma + ti$  ( $\sigma \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$ ), let's prove that  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\zeta(\bar{s})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) are

complex conjugations of each other and get the equation  $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$

and  $s \neq 1$ ) (Formula 7).

### Reasoning 2:

The reasoning in Riemann's paper goes like:

$2\sin(\pi s)\prod(s-1)\zeta(s)=(2\pi)^s \sum n^{s-1}((-i)^{s-1}+i^{s-1})^{[1]}$  (Formula 3),

based on euler's  $e^{ix}=\cos(x) + i \sin(x)$  ( $x \in \mathbb{R}$ ) can get

$$e^{i(-\frac{\pi}{2})}=\cos(\frac{-\pi}{2}) +i\sin(\frac{-\pi}{2})=0-i=-i,$$

$$e^{i(\frac{\pi}{2})}=\cos(\frac{\pi}{2})+i\sin(\frac{\pi}{2})=0+i=i,$$

then

$$(-i)^{s-1} + i^{s-1} = (-i)^{-1}(-i)^s+(i)^{-1}(i)^s=(-i)^{-1}e^{i(-\frac{\pi}{2})s} + i^{-1}e^{i(\frac{\pi}{2})s} =$$

$$ie^{i(-\frac{\pi}{2})s} -ie^{i(\frac{\pi}{2})s} =i(\cos\frac{-\pi s}{2}+i\sin\frac{-\pi s}{2})-i(\cos\frac{\pi s}{2}+i\sin\frac{\pi s}{2})=i\cos(\frac{\pi s}{2})-i\cos(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})$$

$$=2\sin(\frac{\pi s}{2}) \text{ (Formula 4).}$$

According to the property of  $\prod(s-1)=\Gamma(s)$  of the gamma function, and

$$\sum_{n=1}^{\infty} n^{s-1}=\zeta(1-s) \text{ (} n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer, } s \in \mathbb{C}, \text{ and } s \neq 1),$$

Substitute the above (Formula 4) into the above (Formula 3), will get

$$2\sin(\pi s)\Gamma(s)\zeta(s)=(2\pi)^s \zeta(1-s)2\sin\frac{\pi s}{2} \text{ (Formula 5),}$$

If I substitute it into (Formula5), according to the double Angle formula  $\sin(\pi s)=2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})$ ,

we Will get  $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 6),

because  $\pi^{-\frac{1-s}{2}} \neq 0 \neq 0$  and  $\Gamma(\frac{1-s}{2}) \neq 0$ , so when  $\zeta(s)=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(1-s)=0$  ( $s \in$

$\mathbb{C}$  and  $s \neq 1$ ),

Substituting  $s \rightarrow 1-s$ , that is taking  $s$  as  $1-s$  into Formula 6, we will get

$$\zeta(s)=2^s \pi^{s-1} \sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s) \text{ (} s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 7),}$$

This is the functional equation for  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). To rewrite it in a symmetric form, use the residual formula of the gamma function<sup>[3]</sup>

$$\Gamma(Z)\Gamma(1-Z)=\frac{\pi}{\sin(\pi Z)} \text{ (Formula 8)}$$

and Legendre's formula

$$\Gamma(\frac{Z}{2})\Gamma(\frac{Z+1}{2})=2^{1-Z}\pi^{\frac{1}{2}}\Gamma(Z) \text{ (Formula 9) ,}$$

Take  $z=\frac{s}{2}$  in (Formula 8) and substitute it to get

$$\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})} \text{ (Formula 10) ,}$$

In (Formula 9), let  $z=1-s$  and substitute it in to get

$$\Gamma(1-s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1-\frac{s}{2}) \text{ (Formula 11)}$$

By substituting (Formula 10) and (Formula 11) into (Formula 7), we get

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s) \text{ (} s \in \mathbb{C} \text{ and } s \neq 1),$$

also

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$  is invariant under the transformation  $s \rightarrow 1-s$ ,

And that's exactly what Riemann said in his paper.

That is to say:

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$  is invariant under the transformation  $s \rightarrow 1-s$ ,

also

$$\prod(\frac{s}{2} - 1)\pi^{-\frac{s}{2}}\zeta(s) = \prod(\frac{1-s}{2} - 1)\pi^{-\frac{1-s}{2}}\zeta(1-s) \quad (s \in \mathbb{C} \text{ and } s \neq 1),$$

or

$$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s) = \pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s) \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 2),}$$

Then  $\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ),

under the transformation  $s \rightarrow 1-s$ , will get

$$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1) \text{ (Formula 1). Then } \zeta(1-s) = \frac{\zeta(s)}{2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s)} \quad (s \in \mathbb{C}$$

and  $s \neq 1$ ), when  $\zeta(s) = 0$ , then if  $\zeta(1-s) = \frac{\zeta(s)}{2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s)}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) is going to make

sense, then the denominator  $2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \neq 0$ , Clearly indicates  $2^s \neq 0$  ( $s \in \mathbb{C}$  and

$s \neq 1$ ),  $\pi^{s-1} \neq 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ),  $\Gamma(1-s) \neq 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so  $\sin(\frac{\pi s}{2})$  can not equal to zero,

so  $\sin(\frac{\pi s}{2}) \neq 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so  $s \neq 2n$  ( $n \in \mathbb{Z}^+$ ), and  $s \neq 0$ . So when  $\zeta(s) = 0$ , then

$$\zeta(1-s) = \zeta(s) = \zeta(\bar{s}) = 0 \quad (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq 0, \text{ and } s \neq -2n, n \in \mathbb{Z}^+).$$

Because

$$L(s, \chi(n)) = \chi(n) \zeta(s) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}_+ \text{ and } n \text{ goes through all the positive integer) and}$$

$$L(1-s, \chi(n)) = \chi(n) \zeta(1-s) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all the positive integer),}$$

and according to  $\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7), So

$$\text{only } L(s, \chi(n)) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) L(1-s, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+) \text{ (Formula 12).}$$

According to the property that Gamma function  $\Gamma(s)$  and exponential function are nonzero, is also

that  $\Gamma(\frac{1-s}{2}) \neq 0$ , and  $\pi^{-\frac{1-s}{2}} \neq 0$ , according to  $\pi^{-\frac{s}{2}} \Gamma(\frac{s}{2}) \zeta(s) = \pi^{-\frac{1-s}{2}} \Gamma(\frac{1-s}{2}) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

(Formula 2),

Mathematicians have shown that the real part of the complex independent variable  $s$  of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function will have zero only if  $0 < \text{Re}(s) < 1$  and  $\text{Im}(s) \neq 0$ , so we agree

on Riemann  $\zeta(s) = \frac{\eta(s)}{(1-2^{1-s})} = \frac{1}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1-2^{1-s})} \prod_p (1-p^{-s})^{-1}$  ( $s \in \mathbb{C}$  and  $0 <$

$\text{Rs}(s) < 1$  and  $s \neq 1$  and  $\text{Im}(s) \neq 0$ ,  $n \in \mathbb{N}^+, p \in \mathbb{P}, s \in \mathbb{C}$ ,  $n$  goes through all the positive integers,  $p$  goes through all the prime numbers).

According the equation  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by Riemann, since Riemann has shown that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function has zero, that is, in  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) is true, and so we agree on

$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $0 < \text{Rs}(s) < 1$  and  $s \neq 1$  and  $\text{Im}(s) \neq 0$ ,  $n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C}$ ,  $n$  goes through all the positive integers,  $p$  goes through all the prime numbers).

According to the property that Gamma function  $\Gamma(s)$  and exponential function are nonzero, is also that  $\Gamma(\frac{1-s}{2}) \neq 0$ , and  $\pi^{-\frac{1-s}{2}} \neq 0$ ,

So when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), also must  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ).

Because  $e = \lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x = \sum_{n=0}^{\infty} \frac{1}{n!} \approx 2.7182818284\dots$ ,

and because  $\sin(Z) = \frac{e^{iZ} - e^{-iZ}}{2i}$ , Suppose  $Z = s = \sigma + ti$  ( $\sigma \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0$ ), then

$$\sin(s) = \frac{e^{is} - e^{-is}}{2i} = \frac{e^{i(\sigma+ti)} - e^{-i(\sigma+ti)}}{2i},$$

$$\sin(\bar{s}) = \frac{e^{i\bar{s}} - e^{-i\bar{s}}}{2i} = \frac{e^{i(\sigma-ti)} - e^{-i(\sigma-ti)}}{2i},$$

according  $x^s = x^{(\sigma+ti)} = x^\sigma x^{ti} = x^\sigma (\cos(\ln x) + i \sin(\ln x))^t = x^\sigma (\cos(t \ln x) + i \sin(t \ln x))$  ( $x > 0$ ), then

$$e^s = e^{(\sigma+ti)} = e^\sigma e^{ti} = e^\sigma (\cos(t) + i \sin(t)) = e^\sigma (\cos(t) + i \sin(t)),$$

$$e^{is} = e^{i(\sigma+ti)} = e^{\sigma i} (\cos(it) + i \sin(it)) = (\cos(\sigma) + i \sin(\sigma)) (\cos(it) + i \sin(it)),$$

$$e^{i\bar{s}} = e^{i(\sigma-ti)} = e^{\sigma i} (\cos(-it) + i \sin(-it)) = (\cos(\sigma) + i \sin(\sigma)) (\cos(it) - i \sin(it)),$$

$$e^{-is} = e^{-i(\sigma+ti)} = e^{-\sigma i} (\cos(-it) + i \sin(-it)) = (\cos(\sigma) - i \sin(\sigma)) (\cos(it) - i \sin(it)),$$

$$e^{-i\bar{s}} = e^{-i(\sigma-ti)} = e^{-\sigma i} (\cos(it) + i \sin(it)) = (\cos(\sigma) - i \sin(\sigma)) (\cos(it) + i \sin(it)),$$

$$2^s = 2^{(\sigma+ti)} = 2^\sigma 2^{ti} = 2^\sigma (\cos(\ln 2) + i \sin(\ln 2))^t = 2^\sigma (\cos(t \ln 2) + i \sin(t \ln 2)),$$

$$2^{\bar{s}} = 2^{(\sigma-ti)} = 2^\sigma 2^{-ti} = 2^\sigma (\cos(\ln 2) + i \sin(\ln 2))^{-t} = 2^\sigma (\cos(t \ln 2) - i \sin(t \ln 2)),$$

$$\pi^{s-1} = \pi^{(\sigma-1+ti)} = \pi^{\sigma-1} \pi^{ti} = \pi^{\sigma-1} (\cos(\ln \pi) + i \sin(\ln \pi))^t = \pi^{\sigma-1} (\cos(t \ln \pi) + i \sin(t \ln \pi)),$$

$$\pi^{\bar{s}-1} = \pi^{(\sigma-1-ti)} = \pi^{\sigma-1} \pi^{-ti} = \pi^{\sigma-1} (\cos(\ln \pi) + i \sin(\ln \pi))^{-t} = \pi^{\sigma-1} (\cos(t \ln \pi) - i \sin(t \ln \pi)),$$

So

$$2^s = \overline{2^{\bar{s}}}, \quad \pi^{s-1} = \overline{\pi^{\bar{s}-1}},$$

and

$$\frac{e^{is} - e^{-is}}{2i} = \overline{\frac{e^{i\bar{s}} - e^{-i\bar{s}}}{2i}},$$

So

$$\sin(s) = \overline{\sin(\bar{s})} ,$$

and

$$\sin\left(\frac{\pi s}{2}\right) = \overline{\sin\left(\frac{\pi \bar{s}}{2}\right)} .$$

And the gamma function on the complex field is defined as:

$$\Gamma(s) = \int_0^{+\infty} t^{s-1} e^{-t} dt,$$

Among  $\text{Re}(s) > 0$ , this definition can be extended by the analytical continuation principle to the entire field of complex numbers, except for non-positive integers,

So

$$\Gamma(s) = \overline{\Gamma(\bar{s})} ,$$

and

$$\Gamma(1-s) = \overline{\Gamma(1-\bar{s})} . \text{When } \zeta(1-\bar{s}) = \overline{\zeta(1-s)} = 0 = \zeta(s) = \zeta(1-s) = 0 \ (s \in \mathbb{C} \text{ and } s \neq 1), \text{ and according}$$

$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) \ (s \in \mathbb{C} \text{ and } s \neq 1)$ , then only  $\zeta(s) = \overline{\zeta(\bar{s})} = 0 \ (s \in \mathbb{C} \text{ and } s \neq 1)$ , is also say

$\zeta(s) = \zeta(\bar{s}) = \zeta(1-\bar{s}) = 0 \ (s \in \mathbb{C} \text{ and } s \neq 1)$ . so only  $\zeta(\sigma+ti) = \zeta(\sigma-ti) = 0$  is true. According the equation

$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s) \ (s \in \mathbb{C}, \text{ and } s \neq 1)$  obtained by Riemann, since Riemann has

shown that the Riemann  $\zeta(s) \ (s \in \mathbb{C}, \text{ and } s \neq 1)$  function has zero, that is, in

$$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s) \ (s \in \mathbb{C}, \text{ and } s \neq 1) \text{ (Formula 7), } \zeta(s) = 0 \ (s \in \mathbb{C}, \text{ and } s \neq 1) \text{ is true,}$$

so when  $\zeta(s) = 0 \ (s \in \mathbb{C}, \text{ and } s \neq 1)$ , then only  $\zeta(s) = \zeta(1-s) = 0 \ (s \in \mathbb{C}, \text{ and } s \neq 1)$  is true. in the

process of the Riemann hypothesis proved about  $\zeta(s) = \zeta(1-s) = \overline{\zeta(\bar{s})} = 0$ , is refers to the  $\zeta(s)$  is a

functional number? It's not. Does  $\zeta(s) = \zeta(1-s) = \overline{\zeta(\bar{s})} \ (s \in \mathbb{C}, \text{ and } s \neq 1)$  mean the symmetry of

the  $\zeta(s)$  function equation? Does that mean the symmetry of the equation  $s = \bar{s} = 1-s$ ? Not really.

In my analyst,  $\zeta(s)$ 、 $\zeta(1-s)$  and  $\zeta(\bar{s})$  function expression is the same, are  $\sum_{n=1}^{\infty} n^{-s} \ (n \in \mathbb{Z}^+ \text{ and } n$

traves all positive integer,  $s \in \mathbb{C}, \text{ and } s \neq 1)$ , so according

to  $\sum_{n=1}^{\infty} n^{-s} \ (n \in \mathbb{Z}^+ \text{ and } n \text{ traves all positive integer, } s \in \mathbb{C}, \text{ and } s \neq 1)$ ,  $\zeta(s) \ (s \in \mathbb{C}, \text{ and } s \neq$

$1)$  function of the independent variable  $s$ , the relationship between  $\bar{s}$  and  $1-s$  only  $\mathbb{C}_3^2 = 3$

kinds, namely  $s = \bar{s}$  or  $s = 1-s$  or  $\bar{s} = 1-s$ . As follows:

According  $\zeta(s) = \zeta(1-s) = 0 \ (s \in \mathbb{C}, \text{ and } s \neq 1)$  and  $\zeta(s) = \zeta(\bar{s}) = \zeta(1-s) = 0 \ (s \in \mathbb{C}, \text{ and } s \neq 1)$ , then only

$s = \bar{s}$  or  $s = 1-s$  or  $\bar{s} = 1-s$ , so  $s \in \mathbb{R}$ , or  $\sigma+ti = 1-\sigma-ti$ , or  $\sigma-ti = 1-\sigma-ti$ , so  $s \in \mathbb{R}$  and  $s = -2n \ (n \in$

$\mathbb{Z}^+)$ , or  $\sigma = \frac{1}{2}$  and  $t = 0$ , or  $\sigma = \frac{1}{2}$  and  $t \in \mathbb{R}$  and  $t \neq 0$ , so  $s \in \mathbb{R}$ , or  $s = \frac{1}{2} + oi$ , or  $s = \frac{1}{2} + ti \ (t \in \mathbb{R} \text{ and } t \neq$

$0)$ , because  $\zeta\left(\frac{1}{2}\right) \rightarrow +\infty, \zeta(1) \rightarrow +\infty, \zeta(1)$  is divergent,  $\zeta\left(\frac{1}{2}\right)$  is more divergent, so drop

them. Because only when  $\rho = \frac{1}{2}$ , the next three equations,  $\zeta(\sigma+ti) = 0, \zeta(1-\sigma-ti) = 0$ , and

$\zeta(\sigma-ti) = 0$  are all true, because  $\zeta\left(\frac{1}{2}\right) \rightarrow +\infty, \zeta(1) \rightarrow +\infty, \zeta(1)$  is divergent,  $\zeta\left(\frac{1}{2}\right)$  is more



divergent, so only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) is true, or say only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$ ) is true. Since Riemann has shown that the Riemann  $\zeta(s)$   $s \in \mathbb{C}$ , and  $s \neq 1$  function has zero, that is, in  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7),  $\zeta(s) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) is true.

According to the equation  $\xi(s) = \frac{1}{2} s(s-1) \Gamma(\frac{s}{2}) \pi^{-\frac{s}{2}} \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by Riemann, so

$\xi(s) = \xi(1-s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ), because  $\Gamma(\frac{s}{2}) = \overline{\Gamma(\frac{\bar{s}}{2})}$ , and  $\pi^{-\frac{s}{2}} = \overline{\pi^{-\frac{\bar{s}}{2}}}$ , and because  $\zeta(s) = \overline{\zeta(\bar{s})}$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ), so  $\xi(s) = \overline{\xi(\bar{s})}$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ). So when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ), then

$\xi(s) = \zeta(1-s) = \zeta(\bar{s}) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(s) = \xi(1-s) = \xi(\bar{s}) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) must be true, so the zeros of the Riemann  $\zeta(s)$  function and the nontrivial zeros of the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) function are identical, so the complex root of Riemann  $\xi(s) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ). According to the Riemann function

$\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and the Riemann hypothesis

$s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), because  $s \neq 1$ , and  $\prod_{\frac{s}{2}}^s \neq 0$ ,  $\pi^{-\frac{s}{2}} \neq 0$ , so  $\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \neq 0$ , and when

$\xi(t) = 0$ , then  $\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(\frac{1}{2} + ti) = \xi(t) = 0$ , and

$\zeta(\frac{1}{2} + ti) = \frac{\xi(t)}{\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}}} = \frac{0}{\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}}} = 0$ , so  $t \in \mathbb{R}$  and  $t \neq 0$ . So the root  $t$  of the equations

$\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(\frac{1}{2} + ti) = \xi(t) = 0$  and  $4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2} t \ln x) dx = \xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) and

$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) must be real and  $t \neq 0$ . If

$\text{Re}(s) = \frac{k}{2}$  ( $k \in \mathbb{R}$ ), then  $\zeta(k-s) = 2^{k-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, k \in \mathbb{R}$ ) and  $\xi(k-s) =$

$\frac{1}{2} s(s-k) \Gamma(\frac{s}{2}) \pi^{-\frac{s}{2}} \zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1, k \in \mathbb{R}$ ) are true, so when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ), then

$\zeta(s) = \zeta(k-s) = \zeta(\bar{s}) = 0$  ( $s \in \mathbb{C}$ , and  $s \neq 1, k \in \mathbb{R}$ ) and  $\xi(s) = \xi(k-s) = \xi(\bar{s}) = 0$  ( $s \in \mathbb{C}$  and  $s \neq$

$1, k \in \mathbb{R}$ ) must be true, and  $s = \frac{k}{2} + ti$  ( $k \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0$ ) must be true, then

$\prod_{\frac{s}{2}}^s (s-k) \pi^{-\frac{s}{2}} \zeta(\frac{k}{2} + ti) = \xi(t) = 0$  ( $k \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0, k \in \mathbb{R}$ ), and

$\zeta(\frac{k}{2} + ti) = \frac{\xi(t)}{\prod_{\frac{s}{2}}^s (s-k) \pi^{-\frac{s}{2}}} = \frac{0}{\prod_{\frac{s}{2}}^s (s-k) \pi^{-\frac{s}{2}}} = 0$  ( $k \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ), so  $t \in \mathbb{R}$  and  $t \neq 0$ . So

the root of the equations  $\prod_{\frac{s}{2}}^s (s-k) \pi^{-\frac{s}{2}} \zeta(\frac{k}{2} + ti) = \xi(t) = 0$  ( $k \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ )

must be real and  $t \neq 0$ . But the Riemann  $\zeta(s)$  function only satisfies

$\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(s) = \frac{1}{2} s(s-1) \Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s)$  ( $s \in$

$\mathbb{C}$  and  $s \neq 1$ ), is also say that only  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7)

is true, so only  $\text{Re}(s) = \frac{k}{2} = \frac{1}{2}$  ( $k \in \mathbb{R}$ ) is true, so only  $k=1$  is true. The Riemann hypothesis and

the Riemann conjecture must satisfy the properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ )

function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) function, The properties of the Riemann

$\zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) function are

fundamental, the Riemann hypothesis and the Riemann conjecture must be correct to reflect

the properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in$

$\mathbb{C}$ , and  $s \neq 1$ ) function, that is, the roots of the Riemann  $\xi(t)$  ( $t \in \mathbb{C}$ , and  $t \neq 0$ ) function can

only be real, that is,  $\text{Re}(s)$  can only be equal to  $\frac{1}{2}$ , and  $\text{Im}(s)$  must be real, and  $\text{Im}(s)$  is not

equal to zero. So the Riemann hypothesis and the Riemann conjecture must be correct.

Riemann found in his paper that

$$\begin{aligned} \prod\left(\frac{s}{2} - 1\right) \pi^{-\frac{s}{2}} \zeta(s) &= \int_1^\infty \psi(x) x^{\frac{s}{2}-1} dx + \int_1^\infty \psi\left(\frac{1}{x}\right) x^{\frac{s-3}{2}} dx + \frac{1}{2} \int_0^1 (x^{\frac{s-3}{2}} - x^{\frac{s}{2}-1}) dx \\ &= \frac{1}{s(s-1)} + \int_1^\infty \psi(x) (x^{\frac{s}{2}-1} + x^{-\frac{1+s}{2}}) dx \end{aligned}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ) ( $s \in \mathbb{C}$  and  $s \neq 1$ ), Because  $\frac{1}{s(s-1)}$  and

$\int_1^\infty \psi(x) (x^{\frac{s}{2}-1} + x^{-\frac{1+s}{2}}) dx$  are all invariant under the transformation  $s \rightarrow 1-s$  If I introduce the

auxiliary function  $\psi(s) = \prod\left(\frac{s}{2} - 1\right) \pi^{-\frac{s}{2}} \zeta(s)$  ( $s \in \mathbb{C}$ , and  $s \neq 1$ ), So I can just write it as

$\psi(s) = \psi(1-s)$ . But it would be more convenient to add the factor  $s(s-1)$  to  $\psi(s)$  and introduce

the coefficient  $\frac{1}{2}$ , which is exactly what Riemann did, is that to take

$\xi(s) = \frac{1}{2} s(s-1) \Gamma\left(\frac{s}{2}\right) \pi^{-\frac{s}{2}} \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). Because the factor  $(s-1)$  cancels out the first pole of

$\zeta(s)$  at  $s=1$ , And the factor  $s$  cancels out the pole of  $\Gamma\left(\frac{s}{2}\right)$  at  $s=0$ , and  $s$  is equal to  $-2, -4,$

$-6, \dots$ , the rest of the poles of  $\Gamma\left(\frac{s}{2}\right)$  cancel out. So  $\xi(s)$  is an integral function. And the factor

$s(s-1)$  obviously doesn't change under the transformation  $s \rightarrow 1-s$ , so we also have the

function  $\xi(s) = \xi(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), base on  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in$

$\mathbb{C}$  and  $s \neq 1$ ) (Formula 7). At the same time, according to  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos\left(\frac{\pi s}{2}\right) \Gamma(s) \zeta(s)$  ( $s \in$

$\mathbb{C}$  and  $s \neq 1$ ), if  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then must  $\zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), is that to say

$\zeta(s) = \zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ). According to Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$ , and  $t \neq 0$ ),  $s$  and  $t$

differ by a linear transformation. It's a 90 degree rotation plus a translation of  $\frac{1}{2}$ . So line  $\text{Re}(s) = \frac{1}{2}$

in the  $s$  plane corresponds to the real number line in the  $t$  plane, the zero of Riemann  $\zeta(s)$  ( $s \in$

$C$  and  $s \neq 1$ ) on the critical line  $\text{Re}(s) = \frac{1}{2}$  corresponds to the real root of  $\xi(t)$  ( $t \in C$  and  $t \neq 0$ ). In Riemann function  $\xi(t)$  ( $t \in C$  and  $t \neq 0$ ), the function equation  $\xi(s) = \xi(1-s)$  ( $s \in C$  and  $s \neq 1$ ) becomes equation  $\xi(t) = \xi(-t)$  ( $t \in C$  and  $t \neq 0$ ) is an even function, an even function is a symmetric function, it's zeros are distributed symmetrically with respect to  $t=0$ . The function  $\xi(t)$  ( $t \in C$ , and  $t \neq 0$ ) designed by Riemann and Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in C$  and  $t \neq 0$ ,  $s \in C$ , and  $s \neq 1$ ) and  $\xi(s) = \xi(1-s)$  ( $s \in C$  and  $s \neq 1$ ) are equivalent to  $\xi(t) = \xi(-t)$  ( $t \in C$  and  $t \neq 0$ ). So the function  $\xi(s)$  ( $s \in C$  and  $s \neq 1$ ) is also an even function. The zero points on the graph of an even function  $\xi(s)$  ( $s \in C$  and  $s \neq 1$ ) with respect to the coordinates of its argument on the real number line equal to some value are symmetrically distributed on the line perpendicular to the real number line of the complex plane. When  $\xi(t) = 0$  ( $t \in C$  and  $t \neq 0$ ), is also that  $\xi(t) = \xi(-t) = 0$  ( $t \in C$  and  $t \neq 0$ ), the zeros of  $\xi(t)$  ( $t \in C$  and  $t \neq 0$ ) are symmetrically distributed with respect to  $t$  equals  $0$ . When  $\xi(s) = 0$  ( $s \in C$  and  $s \neq 1$ ), is also that  $\xi(s) = \xi(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ), the zeros of  $\xi(s)$  ( $s \in C$  and  $s \neq 1$ ) are symmetrically distributed with respect to point  $(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line of the complex plane. So when  $\xi(s) = \xi(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ),  $s$  and  $1-s$  are pair of zeros of the function  $\xi(s)$  ( $s \in C$  and  $s \neq 1$ ) symmetrically distributed in the complex plane with respect to point  $(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line of the complex plane. When  $\zeta(s) = 0$  ( $s \in C$  and  $s \neq 1$ ), then  $\zeta(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ), is also that  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ). We find  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ) and  $\xi(s) = \xi(1-s) = 0$  ( $s \in C$  and  $s \neq 1$ ) are just the name of the function is different, the independent variable  $s$  is equal to  $\frac{1}{2} + ti$  ( $t \in C, s \in C$ ), that means that the zero arguments of function  $\zeta(s)$  ( $s \in C$  and  $s \neq 1$ ) and function  $\xi(s)$  ( $s \in C$  and  $s \neq 1$ ) are exactly the same, so the zeros of the  $\zeta(s)$  ( $s \in C$  and  $s \neq 1$ ) function in the complex plane also correspond to the symmetric distribution of point  $(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line in the complex plane, so When  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in C$ , and  $s \neq 1$ ),  $s$  and  $1-s$  are pair of zeros of the function  $\zeta(s)$  ( $s \in C$  and  $s \neq 1$ ) symmetrically distributed in the complex plane with respect to point  $(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line of the complex plane. We got  $\overline{\zeta(s)} = \zeta(\bar{s})$  ( $s = \sigma + ti$ ,  $\sigma \in R$ ,  $t \in R$  and  $t \neq 0$ ) before, When  $t$  in Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in C, s \in C$  and  $t \neq 0$ ) is a complex number, and  $s = \frac{1}{2} + ti = \sigma + ti$ , then  $s$  in  $\overline{\zeta(s)} = \zeta(\bar{s})$  ( $s = \sigma + ti$ ,  $\sigma \in R$ ,  $t \in R$  and  $t \neq 0$ ) is consistent with  $s$  in Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in C, s \in C$  and  $t \neq 0$ ). If  $\zeta(s) = \zeta(\bar{s}) = 0$  ( $s = \sigma + ti, \sigma \in R, t \in R$  and  $t \neq 0$ ), Since  $s$  and  $\bar{s}$  are a pair of conjugate complex numbers, So  $s$  and  $\bar{s}$  must be a pair of zeros of the function  $\zeta(s)$  ( $s \in C$  and  $s \neq 1$ ) in the complex plane with respect to point  $(\sigma, 0i)$  on a line perpendicular to the real number line.  $s$  is a symmetric zero of  $1-s$ , and a symmetric zero of  $\bar{s}$ . By the definition of

complex numbers, how can a symmetric zero of the same function  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) of the same zero independent variable  $s$  on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of  $1-s$  on a line perpendicular to the real number axis of the complex plane with respect to point  $(\frac{1}{2}, 0i)$  and a symmetric zero of  $\bar{s}$  on a line perpendicular to the real number axis of the complex plane with respect to point  $(\sigma, 0i)$ ? Unless  $\sigma$  and  $\frac{1}{2}$  are the same value, is also that  $\sigma = \frac{1}{2}$ , and only  $1-s=\bar{s}$  is true, and  $1-s=s$  is wrong. Otherwise it's impossible, this is determined by the uniqueness of the zero of the function  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane, only one line can be drawn perpendicular from the zero independent variable  $s$  of the function  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) to the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of the function  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane. Because  $\overline{\zeta(s)} = \zeta(\bar{s})$  ( $s = \sigma + ti, \sigma \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0$ ), then if  $\zeta(\sigma + ti) = 0$ , then  $\zeta(\sigma - ti) = 0$ , and because  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(1-\sigma-ti) = 0$ , and because  $\zeta(s) = \zeta(1-s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), then  $\zeta(1-\sigma+ti) = 0$ . The next three equations,  $\zeta(\sigma + ti) = 0$ ,  $\zeta(\sigma - ti) = 0$ , and  $\zeta(1-\sigma-ti) = 0$ , are all true, so only  $1-\sigma = \sigma$  is true, only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) is true. Since the harmonic series  $\zeta(1)$  diverges, it has been proved by the late medieval French scholar Orem (1323-1382). The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, The properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function are fundamental, the Riemann hypothesis and the Riemann conjecture must be correct to reflect the properties of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the Riemann  $\xi(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, that is, the roots of the Riemann  $\xi(t)$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) function must only be real, that is,  $\text{Re}(s)$  can only be equal to  $\frac{1}{2}$ , and  $\text{Im}(s)$  must be real, and  $\text{Im}(s)$  is not equal to zero. So the Riemann hypothesis and the Riemann conjecture must be correct. Riemann got  $\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \xi(t)$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ), and  $\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) dx$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) in his paper, or  $\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(s) = \xi(t)$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = 4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2} t \ln x) dx$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) [1]. Because  $\zeta(\frac{1}{2} + ti) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), so the roots of equations  $\prod_{\frac{s}{2}}^s (s-1) \pi^{-\frac{s}{2}} \zeta(\frac{1}{2} + ti) = \xi(t) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and  $4 \int_1^\infty \frac{d(x^{\frac{3}{2}} \Psi'(x))}{dx} x^{-\frac{1}{4}} \cos(\frac{1}{2} t \ln x) dx = \xi(t) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) and

$\xi(t) = \frac{1}{2} - (t^2 + \frac{1}{4}) \int_1^\infty \Psi(x) x^{-\frac{3}{4}} \cos(\frac{1}{2} t \ln x) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$  and  $s \neq 1$ ) must all be real numbers. When  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), the real part of the equation  $\xi(t) = 0$  ( $t \in \mathbb{C}$ ) must be real between 0 and T. Because the real part of the equation  $\xi(t) = 0$  has the number of complex roots between 0 and T approximately equal to  $\frac{T}{2\pi} \ln \frac{T}{2\pi} - \frac{T}{2\pi}$  [1], This result of Riemann's estimate of the number of zeros was rigorously proved by Mangoldt in 1895. Then, when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), the number of real roots of the real part of the equation  $\xi(t) = 0$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) between 0 and T must be approximately equal to  $\frac{T}{2\pi} \ln \frac{T}{2\pi} - \frac{T}{2\pi}$  [1], so when the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function has nontrivial zeroes, then the Riemann hypothesis and the Riemann conjecture are perfectly valid. According to the  $2\sin(\pi s) \prod (s-1)\zeta(s) = \int_0^\infty \frac{x^{s-1} dx}{e^x - 1}$  Riemann got in his paper and the  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), We know that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function is a two-to-one mapping, or even a many-to-one mapping deterministic universal function, or a one-to-two mapping, or even a one-to-many mapping deterministic universal function. If we consider the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function as a general complex number whose domain includes real numbers, then  $s = -2n$  ( $n$  is a positive integer) is the only class of real zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function at the root, If we consider the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function as a general complex number whose domain does not include real numbers, then  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) is the only class of complex zeros of the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function at the root, so the zero real root of the Landau-Siegel function  $L(\beta, 1)$  ( $\beta \in \mathbb{R}$ ) does not exist.

Definition: Assuming that  $a(n)$  is a uniprimitive function, then the Dirichlet series  $\sum_{n=1}^\infty a(n)n^{-s}$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  goes through all the positive numbers) is equal to the Euler product

$\prod_p P(p, s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers). Where the product is applied to all prime numbers  $p$ , it can be expressed as:  $1 + a(p)p^{-s} + a(p^2)p^{-2s} + \dots$ , this can be seen as a formal generating function, where the existence of a formal Euler product expansion and  $a(n)$  being a product function are mutually sufficient and necessary conditions. When  $a(n)$  is a completely integrative function, an important special case is obtained, where  $P(p, s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers) is a geometric series, and

$$P(p, s) = \frac{1}{1 - a(p)p^{-s}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ goes through all the prime numbers}).$$

When  $a(n) = 1$ , it is the Riemann zeta function, and more generally the Dirichlet feature.

Euler's product formula: for any complex number  $s$ ,  $\text{Re}(s) > 1$  and  $s \neq 1$ , then  $\sum_{n=1}^\infty n^{-s} = \prod_p (1 - p^{-s})^{-1}$  ( $s \in \mathbb{C}$  and  $s \neq 1, p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive numbers), and when  $\text{Re}(s) > 1$

$$1 \text{ Riemann Zeta function } \zeta(s) = \sum_{n=1}^\infty n^{-s} = \prod_p (1 - p^{-s})^{-1} \quad (s \in \mathbb{C} \text{ and } \text{Re}(s) > 1 \text{ and } s \neq 1,$$

$n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C}$ ,  $n$  goes through all the positive numbers,  $p$  goes through all the prime numbers).

Riemann zeta function expression:

$$\zeta(s) = 1/1^s + 1/2^s + 1/3^s + \dots + 1/m^s \quad (m \text{ tends to infinity, and } m \text{ is always even}).$$

(1) Multiply both sides of the expression by  $(1/2^s)$ ,

$$(1/2^s)\zeta(s) = 1/1^s(1/2^s) + 1/2^s(1/2^s) + 1/3^s(1/2^s) + \dots + 1/m^s(1/2^s) = 1/2^s + 1/4^s + 1/6^s + \dots + 1/(2m)^s$$

This is given by (1) - (2)

$$\zeta(s) - (1/2^s)\zeta(s) = 1/1^s + 1/2^s + 1/3^s + \dots + 1/m^s - [1/2^s + 1/4^s + 1/6^s + \dots + 1/(2m)^s]$$

The derivation of Euler product formula is as follows:

$$\zeta(s) - (1/2^s)\zeta(s) = 1/1^s + 1/3^s + 1/5^s + \dots + 1/(m-1)^s.$$

Generalized Euler product formula:

Suppose  $f(n)$  is a function that satisfies  $f(n_1)f(n_2) = f(n_1n_2)$  and  $\sum_n |f(n)| < +\infty$  ( $n_1$  and  $n_2$  are both natural numbers), then  $\sum_n f(n) = \prod_p [1 + f(p) + f(p^2) + f(p^3) + \dots]$ .

Proof:

The proof of Euler product formula is very simple, the only caution is to deal with infinite series and infinite products, can not arbitrarily use the properties of finite series and finite products. What I prove below is a more general result, and the Euler product formula will appear as a special case of this result.

Due to  $\sum_{n=1}^{\infty} |f(n)| < +\infty$ , so  $1 + f(p) + f(p^2) + f(p^3) + \dots$  absolute convergence. Consider the part of  $p < N$  in the continued product (finite product), since the series is absolutely convergent and the product has only finite terms, the same associative and distributive laws can be used as ordinary finite summations and products.

Using the product property of  $f(n)$ , we can obtain:

$\prod_{p < N} [1 + f(p) + f(p^2) + f(p^3) + \dots] = \sum f(n)$ . The right end of the summation is performed on all natural numbers with only prime factors below  $N$  (each such natural number occurs only once in the summation, because the prime factorization of the natural numbers is unique). Since all natural numbers that are themselves below  $N$  obviously contain only prime factors below  $N$ , So  $\sum f(n) = \sum_{n < N} f(n) + R(N)$ , where  $R(N)$  is the result of summing all natural numbers that are greater than or equal to  $N$  but contain only prime factors below  $N$ . From this we get:  $\prod_{p < N} [1 + f(p) + f(p^2) + f(p^3) + \dots] = \sum_{n < N} f(n) + R(N)$ . For the generalized Euler product formula to hold, it is only necessary to prove  $\lim_{N \rightarrow \infty} R(N) = 0$ , and this is obvious, because  $|R(N)|$

$$\leq \sum_{n \geq N} |f(n)|, \text{ and } \sum_n |f(n)| < +\infty \text{ sign of}$$

$\lim_{N \rightarrow \infty} \sum_{n \geq N} |f(n)| = 0$ , thus  $\lim_{N \rightarrow \infty} R(N) = 0$ . Because  $1 + f(p) + f(p^2) + f(p^3) + \dots = 1 + f(p) + f(p)^2 + f(p)^3 + \dots = [1 - f(p)]^{-1}$ , so the generalized Euler product formula can also be written as:

$\sum_n f(n) = \prod_p [1 - f(p)]^{-1}$ . In the generalized Euler product formula, take  $f(n) = n^{-s}$ , then obviously  $\sum_n |f(n)| < +\infty$  corresponds to the condition  $\text{Re}(s) > 1$  in the Euler product formula, and the generalized Euler product formula is reduced to the Euler product formula.

From the above proof, we can see that the key to the Euler product formula is the basic property that every natural number has a unique prime factorization, that is, the so-called fundamental theorem of arithmetic.

For any complex number  $s$ ,  $\chi(n)$  is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer  $q$  such that  $\chi(n+q) = \chi(n)$ ;

2: when  $n$  and  $q$  are not mutual prime,  $\chi(n) = 0$ ;

3:  $\chi(a) \cdot \chi(b) = \chi(ab)$  for any integer  $a$  and  $b$ ;

Reasoning 3:

If  $\text{Re}(s) > 1$  and  $s \neq 1$ , then

$$L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} \quad (n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C} \text{ and } s \neq 1, n \text{ goes through all the positive numbers, } p$$

goes through all the prime numbers,  $\chi(n) \in \mathbb{R}$

$$\text{and } (\chi(n) \neq 0), a(n) = a(p) = \chi(n), P(p, s) = \frac{1}{1 - a(p)p^{-s}}.$$

Next we prove the generalized Riemann conjecture when the Dirichlet eigen function  $\chi(n)$  is any real number that is not equal to zero, and

$$\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \quad (s \in \mathbb{C} \text{ and } \text{Re}(s) > 0 \text{ and } (s \neq 1)) \text{ and } (\eta s) = (1 - 2^{1-s})\zeta(s) \quad (s \in \mathbb{C} \text{ and } \text{Re}(s) >$$

$$0 \text{ and } s \neq 1), \zeta(s) \text{ is the Riemann } \zeta(s) = \frac{\eta(s)}{(1 - 2^{1-s})} = \frac{1}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} = \frac{(-1)^{n-1}}{(1 - 2^{1-s})} \prod_p (1 -$$

$p^{-s})^{-1} \quad (s \in \mathbb{C} \text{ and } \text{Re}(s) > 0 \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all the$

positive integers,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers), so

$$\text{GRH}(s, \chi(n)) = L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \sum_{n=1}^{\infty} a(n)n^{-s} = \prod_p P(p, s) = \prod_p \left( \frac{1}{1 - a(p)p^{-s}} \right) \quad (n \in$$

$\mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C} \text{ and } s \neq 1, n \text{ goes through all the positive integers, } p \text{ goes through all the prime$

$$\text{numbers, } \chi(n) \in \mathbb{R} \text{ and } (\chi(n) \neq 0), a(n) = a(p) = \chi(n), P(p, s) = \frac{1}{1 - a(p)p^{-s}}.$$

$$a(p)p^{-s} = a(p)p^{-\sigma} \frac{1}{(\cos(t \ln p) + i \sin(t \ln p))} = a(p)(p^{-\sigma}(\cos(t \ln p) - i \sin(t \ln p))) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in$$

$\mathbb{C} \text{ and } t \neq 0),$

$$(1 - a(p)p^{-s}) = 1 - a(p)(p^{-\sigma}(\cos(t \ln p) - i \sin(t \ln p))) = 1 - a(p)p^{-\sigma} \cos(t \ln p) + ia(p)p^{-\sigma} \sin(t \ln p) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0),$$

$$a(p)p^{-\bar{s}} = a(p)p^{-\sigma} \frac{1}{(\cos(t \ln p) - i \sin(t \ln p))} = a(p)(p^{-\sigma}(\cos(t \ln p) + i \sin(t \ln p))) \quad (s \in \mathbb{C} \text{ and } s \neq$$

$1, t \in \mathbb{C} \text{ and } t \neq 0),$

$$(1 - a(p)p^{-\bar{s}}) = 1 - a(p)p^{-\sigma} \cos(t \ln p) - ia(p)p^{-\sigma} \sin(t \ln p) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0),$$

because

$$(1 - a(p)p^{-s}) = \overline{(1 - a(p)p^{-\bar{s}})} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime integer}),$$

so

$$(1 - a(p)p^{-s})^{-1} = \overline{(1 - a(p)p^{-\bar{s}})^{-1}} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime number}),$$

so

$$\prod_p(1 - a(p)p^{-s})^{-1} = \overline{\prod_p(1 - a(p)p^{-\bar{s}})^{-1}}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers)) .

because  $L(s, \chi(n)) = \sum_{n=1}^{\infty} a(n)n^{-s} = \prod_p(1 - a(p)p^{-s})^{-1}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and

$$L(\bar{s}, \chi(n)) = \sum_{n=1}^{\infty} a(n)n^{-\bar{s}} = \prod_p(1 - a(p)p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers)). For the Generalized Riemann function

$$L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \sum_{n=1}^{\infty} a(n)n^{-s} = \prod_p \frac{1}{1 - a(p)p^{-s}}$$

( $\chi(n) \in \mathbb{R}$  and ( $\chi(n) \neq 0$ ,  $a(n) = a(p) = \chi(n)$ ),  $P(p, s) = \frac{1}{1 - a(p)p^{-s}}$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in$

$\mathbb{Z}^+$  and  $n$  goes through all the positive integers,  $p \in$

$\mathbb{Z}^+$  and  $p$  goes through all the prime numbers)) .so  $L\left(s, \chi(n)\right) = \overline{L\left(\bar{s}, \chi(n)\right)}$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers).

$$a(p)p^{1-s} = a(p)p^{(1-\sigma-ti)} = a(p)p^{1-\sigma}x^{-ti} = a(p)p^{1-\sigma}(\cos(\ln p) + i \sin(\ln p))^{-t} = a(p)p^{1-\sigma}(\cos(t \ln p) - i \sin(t \ln p)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0)$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $t \neq 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers),

$$a(p)p^{1-\bar{s}} = a(p)p^{(1-\sigma+ti)} = a(p)p^{1-\sigma}p^{ti} = a(p)p^{1-\sigma}(p^{ti}) = a(p)p^{1-\sigma}(\cos(\ln p) + i \sin(\ln p))^t = a(p)p^{1-\sigma}(\cos(t \ln p) - i \sin(t \ln p)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, p \in \mathbb{Z}^+ \text{ and } p \text{ goes through all the prime numbers),$$

then

$$a(p)p^{-(1-s)} = a(p)p^{\sigma-1} \frac{1}{(\cos(t \ln p) - i \sin(t \ln p))} = a(p)(p^{\sigma-1}(\cos(t \ln p) + i \sin(t \ln p))) \quad (s \in \mathbb{C} \text{ and } s \neq$$

$1, t \in \mathbb{C}$  and  $t \neq 0, p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers),

$$(1 - a(p)p^{-(1-s)})^{-1} = \frac{1}{1 - a(p)p^{\sigma-1}(\cos(t \ln p) + i \sin(t \ln p))} = \frac{1 - a(p)p^{\sigma-1} \cos(t \ln p) - a(p)p^{\sigma-1} i \sin(t \ln p)}{1 - a(p)p^{\sigma-1}(\cos(t \ln p) + i \sin(t \ln p))}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $t \in \mathbb{C}$  and  $t \neq 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers),

$$(1 - a(p)p^{-\bar{s}})^{-1} = \frac{1}{1 - a(p)(p^{-\sigma}(\cos(t \ln p) + i \sin(t \ln p)))} = \frac{1 - a(p)p^{-\sigma} \cos(t \ln p) - i a(p)p^{-\sigma} \sin(t \ln p)}{1 - a(p)p^{-\sigma}(\cos(t \ln p) + i \sin(t \ln p))} \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, p \in$$

$\mathbb{Z}^+$  and  $p$  goes through all the prime numbers),

When  $\sigma = \frac{1}{2}$ , then

$$(1 - a(p)p^{-(1-s)}) = (1 - a(p)p^{-\bar{s}}) \quad (s \in \mathbb{C} \text{ and } s \neq 1),$$

$$(1 - a(p)p^{-(1-s)})^{-1} = (1 - a(p)p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1),$$

so

$$\prod_p(1 - a(p)p^{-(1-s)})^{-1} = \prod_p(1 - a(p)p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1), \text{ because } L(1-s, \chi(n)) = \prod_p(1 -$$

$$a(p)p^{-(1-s)})^{-1} \text{ and } L(\bar{s}, \chi(n)) = \prod_p(1 - a(p)p^{-\bar{s}})^{-1}, n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C} \text{ and } s \neq 1, n \text{ goes}$$



through all the positive integers,  $p$  goes through all the prime numbers,  $\chi$

$$(n) \in \mathbb{R} \text{ and } (\chi(n) \neq 0), a(n) = a(p) = \chi(n), P(p, s) = \frac{1}{1 - a(p)p^{-s}}.$$

so

Only

$$L(1 - s, \chi(n)) = L(\bar{s}, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$$

and

$$\text{Only } L(1 - \bar{s}, \chi(n)) = L(s, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1)$$

( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers),

Because  $L(s, \chi(n)) = \chi(n)\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  goes through all the

positive integers), and  $L(1 - s, \chi(n)) = \chi(n)\zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  goes through

all the positive integers), so When only  $\sigma = \frac{1}{2}$ , it must be true that  $L(s, \chi(n)) = \overline{L(\bar{s}, \chi(n))}$  ( $s \in \mathbb{C}$

and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers), and it must be true that

$$L(1 - s, \chi(n)) = L(\bar{s}, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all the}$$

positive integers), Suppose  $k \in \mathbb{R}$ ,

$$a(p)p^{k-s} = a(p)p^{(k-\sigma-ti)} = a(p)p^{k-\sigma} x^{-ti} = a(p)p^{k-\sigma} (\cos(\ln p) + i \sin(\ln p))^{-t} = a(p)p^{k-\sigma} (\cos(\ln p) - i \sin(\ln p)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in \mathbb{R}),$$

$$a(p)p^{k-\bar{s}} = a(p)p^{(k-\sigma+ti)} = a(p)p^{k-\sigma} p^{ti} = a(p)p^{k-\sigma} (p^{ti}) = a(p)p^{k-\sigma} (\cos(\ln p) + i \sin(\ln p))^t =$$

$$a(p)(p^{k-\sigma} (\cos(\ln p) + i \sin(\ln p))) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in \mathbb{R}),$$

then

$$a(p)p^{-(k-s)} = a(p)p^{\sigma-k} \frac{1}{(\cos(\ln p) - i \sin(\ln p))} = a(p)$$

$$(p^{\sigma-k} (\cos(\ln p) + i \sin(\ln p))) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in \mathbb{R}),$$

$$(1 - a(p)p^{-(k-s)}) = 1 - (a(p)p^{\sigma-k} (\cos(\ln p) + i \sin(\ln p))) = 1 -$$

$$a(p)p^{\sigma-k} \cos(\ln p) - ip^{\sigma-k} \sin(\ln p) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, p \in$$

$\mathbb{Z}^+$  and  $p$  is a prime number,  $k \in \mathbb{R}$ ),

$$(1 - a(p)p^{-\bar{s}}) = 1 - (a(p)p^{-\sigma} (\cos(\ln p) + i \sin(\ln p))) = 1 -$$

$$a(p)p^{-\sigma} \cos(\ln p) - ia(p)p^{-\sigma} \sin(\ln p) \quad (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, p \text{ is a prime number}),$$

When  $\sigma = \frac{k}{2}$  ( $k \in \mathbb{R}$ ),

then

$$(1 - a(p)p^{-(k-s)}) = (1 - a(p)p^{-\bar{s}}) \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime integer, } k \in \mathbb{R}),$$

$$(1 - a(p)p^{-(k-s)})^{-1} = (1 - a(p)p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1, p \in \mathbb{Z}^+ \text{ and } p \text{ is a prime integer, } k \in \mathbb{R}),$$

so

$$\prod_p (1 - a(p)p^{-(k-s)})^{-1} = \prod_p (1 - a(p)p^{-\bar{s}})^{-1} \quad (s \in \mathbb{C} \text{ and } s \neq 1)$$

( $s \in \mathbb{C}$  and  $s \neq 1, k \in \mathbb{R}, p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers,  $k \in \mathbb{R}$ ),

because  $L(k - s, \chi(n)) = \prod_p (1 - a(p)p^{-(k-s)})^{-1}$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $p$  is a prime number,  $k \in \mathbb{R}$ ),

and  $L(\bar{s}, \chi(n)) = \prod_p (1 - a(p)p^{-\bar{s}})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$ ,  $n$  goes through all positive integers,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers), for the generalized Riemann

function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all the positive integers,  $p \in \mathbb{Z}^+$  and  $p$  goes through all the prime numbers,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 0$ ,  $a(n) =$

$a(p) = \chi(p)$ ),  $P(p, s) = \frac{1}{1 - a(p)p^{-s}}$ .

so

$$\text{Only } L(k - s, \chi(n)) = L(\bar{s}, \chi(n))$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers,  $k \in \mathbb{R}$ ),

and

$$\text{Only } L(k - \bar{s}, \chi(n)) = L(s, \chi(n)),$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers,  $k \in \mathbb{R}$ ),

$$\text{And because Only } L(1 - s, \chi(n)) = L(\bar{s}, \chi(n))$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers), so only  $k=1$  be true.

$$\begin{aligned} \text{GRH}(s, \chi(n)) &= L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \frac{\chi(n)\eta(s)}{(1 - 2^{1-s})} = \frac{\chi(n)}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \\ &= \frac{\chi(n)}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\sigma+ti}} = \frac{(-1)^{n-1}}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \chi(n) \left( \frac{1}{n^{\sigma}} \frac{1}{n^{ti}} \right) = \\ &= \frac{(-1)^{n-1}}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \chi(n) (n^{-\sigma}) \frac{1}{(\cos(\ln(n)) + i\sin(\ln(n)))^t} \\ &= \frac{(-1)^{n-1}}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \chi(n) (n^{-\sigma} (\cos(\ln(n)) + i\sin(\ln(n)))^{-t}) \\ &= \frac{(-1)^{n-1}}{(1 - 2^{1-s})} \sum_{n=1}^{\infty} \chi(n) n^{-\sigma} (\cos(\ln(n)) - i\sin(\ln(n))) \end{aligned}$$

( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers),

$$\begin{aligned}
 \text{GRH}(\bar{s}, \chi(n)) &= L(\bar{s}, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{\bar{s}}} = \frac{\chi(n)\eta(\bar{s})}{(1-2^{1-\bar{s}})} = \frac{\chi(n)}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}} \\
 &= \frac{\chi(n)}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\sigma-ti}} = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \chi(n) \left( \frac{1}{n^{\sigma}} \frac{1}{n^{-ti}} \right) \\
 &= \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \left( \chi(n) \frac{1}{n^{\sigma}} \frac{1}{(\cos(\ln(n)) + i \sin(\ln(n)))^{-t}} \right) \\
 &= \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \left( \chi(n) n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n)))^t \right) = \\
 &\quad \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \left( \chi(n) n^{-\sigma} (\cos(\ln(n)) + i \sin(\ln(n))) \right)
 \end{aligned}$$

( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$ ,  $n$  goes through all positive integers).

$$\begin{aligned}
 \text{GRH}(1-s, \chi(n)) &= L(1-s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \frac{\chi(n)\eta(1-s)}{(1-2^s)} = \frac{\chi(n)}{(1-2^s)} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-\sigma-ti}} \\
 &= \frac{(-1)^{n-1}}{(1-2^s)} \sum_{n=1}^{\infty} \chi(n) \left( \frac{1}{n^{1-\sigma}} \frac{1}{n^{-ti}} \right) \\
 &= \frac{(-1)^{n-1}}{(1-2^s)} \sum_{n=1}^{\infty} \left( \chi(n) n^{\sigma-1} (\cos(\ln(n)) + i \sin(\ln(n))) \right),
 \end{aligned}$$

( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$ ,  $n$  goes through all positive integers).

Suppose

$$U = [\chi(n)1^{-\sigma} \cos(\ln 1) - \chi(n)2^{-\sigma} \cos(\ln 2) + \chi(n)3^{-\sigma} \cos(\ln 3) - \chi(n)4^{-\sigma} \cos(\ln 4) + \dots],$$

$$V = [\chi(n)1^{-\sigma} \sin(\ln 1) - \chi(n)2^{-\sigma} \sin(\ln 2) + \chi(n)3^{-\sigma} \sin(\ln 3) - \chi(n)4^{-\sigma} \sin(\ln 4) + \dots],$$

then

$$L(s, \chi(n)) = L(\bar{s}, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}).$$

And  $n$  goes through all the positive numbers, so  $n=1,2,3,\dots$ , let's just plug in, so

$$\begin{aligned}
 L(s, \chi(n)) &= \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = [\chi(n)1^{-\sigma} \cos(\ln 1) - \chi(n)2^{-\sigma} \cos(\ln 2) + \chi(n)3^{-\sigma} \cos(\ln 3) \\
 &\quad - \chi(n)4^{-\sigma} \cos(\ln 4) + \dots] - i[\chi(n)1^{-\sigma} \sin(\ln 1) - \chi(n)2^{-\sigma} \sin(\ln 2) + \chi(n)3^{-\sigma} \sin(\ln 3) \\
 &\quad - \chi(n)4^{-\sigma} \sin(\ln 4) + \dots] = U - Vi
 \end{aligned}$$

( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers),

$$U=[\chi(n)1^{-\sigma}\cos(\ln 1)-\chi(n)2^{-\sigma}\cos(\ln 2)+\chi(n)3^{-\sigma}\cos(\ln 3)-\chi(n)4^{-\sigma}\cos(\ln 4)+\dots],$$

$$V=[\chi(n)1^{-\sigma}\sin(\ln 1)-\chi(n)2^{-\sigma}\sin(\ln 2)+\chi(n)3^{-\sigma}\sin(\ln 3)-\chi(n)4^{-\sigma}\sin(\ln 4)+\dots],$$

Then

$$L(\bar{s}, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{\bar{s}}} = [\chi(n)1^{-\sigma}\cos(\ln 1)-\chi(n)2^{-\sigma}\cos(\ln 2)+\chi(n)3^{-\sigma}\cos(\ln 3)-\chi(n)4^{-\sigma}\cos(\ln 4)+\dots] + i[\chi(n)1^{-\sigma}\sin(\ln 1)-\chi(n)2^{-\sigma}\sin(\ln 2)+\chi(n)3^{-\sigma}\sin(\ln 3)-\chi(n)4^{-\sigma}\sin(\ln 4)+\dots] =$$

$U+Vi, (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$

$$U=[\chi(n)1^{-\sigma}\cos(\ln 1)-\chi(n)2^{-\sigma}\cos(\ln 2)+\chi(n)3^{-\sigma}\cos(\ln 3)-\chi(n)4^{-\sigma}\cos(\ln 4)+\dots],$$

$$V=[\chi(n)1^{-\sigma}\sin(\ln 1)-\chi(n)2^{-\sigma}\sin(\ln 2)+\chi(n)3^{-\sigma}\sin(\ln 3)-\chi(n)4^{-\sigma}\sin(\ln 4)+\dots],$$

$L(s, \chi(n))$  and  $L(\bar{s}, \chi(n))$  are complex conjugates of each other, that is  $L(s, \chi(n)) = \overline{L(\bar{s}, \chi(n))}$

$(s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$

When  $\sigma = \frac{1}{2}$ , then

$$L(s, \chi(n)) = L(1-s, \chi(n)) = U - Vi$$

$(s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$

$$U=[\chi(n)1^{-\sigma}\cos(\ln 1)-\chi(n)2^{-\sigma}\cos(\ln 2)+\chi(n)3^{-\sigma}\cos(\ln 3)-\chi(n)4^{-\sigma}\cos(\ln 4)+\dots],$$

$$V=[\chi(n)1^{-\sigma}\sin(\ln 1)-\chi(n)2^{-\sigma}\sin(\ln 2)+\chi(n)3^{-\sigma}\sin(\ln 3)-\chi(n)4^{-\sigma}\sin(\ln 4)+\dots].$$

and When  $\sigma = \frac{1}{2}$ , then only  $L\left(1-s, \chi(n)\right) = \overline{L\left(\bar{s}, \chi(n)\right)}$

$(s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$

$$\text{GRH}(k-s, \chi(n)) = L(k-s, \chi(n)) = \frac{\chi(n)\eta(k-s)}{(1-2^{1-k+s})} = \frac{\chi(n)}{(1-2^{1-k+s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{k-\rho-ti}} =$$

$$\frac{(-1)^{n-1}}{(1-2^{1-k+s})} \sum_{n=1}^{\infty} \chi(n) \left( \frac{1}{n^{k-\sigma}} \frac{1}{n^{-ti}} \right) =$$

$$\frac{(-1)^{n-1}}{(1-2^{1-k+s})} \sum_{n=1}^{\infty} (\chi(n)n^{\sigma-k}(\cos(\ln(n)) + i\sin(\ln(n)))) (s \in \mathbb{C} \text{ and } s \neq 1, t \in \mathbb{C} \text{ and } t \neq 0, k \in$$

$\mathbb{R}, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}),$

$$W=[\chi(n)1^{\sigma-k}\cos(\ln 1)-\chi(n)2^{\sigma-k}\cos(\ln 2)+\chi(n)3^{\sigma-k}\cos(\ln 3)-\chi(n)4^{\sigma-k}\cos(\ln 4)+\dots]$$

$$U=[\chi(n)1^{\sigma-k}\sin(\ln 1)-\chi(n)2^{\sigma-k}\sin(\ln 2)+\chi(n)3^{\sigma-k}\sin(\ln 3)-\chi(n)4^{\sigma-k}\sin(\ln 4)+\dots].$$

When  $\sigma = \frac{k}{2}$  ( $k \in \mathbb{R}$ ), then

$$\text{Only } L(k-s, \chi(n)) = L(\bar{s}, \chi(n)) = W - U_i.$$

( $s \in \mathbb{C}$  and  $s \neq 1, k \in \mathbb{R}, n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers), but the Riemann

$\zeta(s)$  function only satisfies  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so when  $\zeta(s) = 0$  ( $s \in \mathbb{C}$

and  $s \neq 1$ ), then only  $\zeta(1-s) = \zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), and when  $\zeta(\bar{s}) = 0$ , then only  $\zeta(1-s) = \zeta(\bar{s}) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), which is  $\zeta(k-s) = \zeta(1-s) = \zeta(\bar{s})$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so only  $k=1$  be true. so

only  $\text{Re}(s) = \frac{k}{2} = \frac{1}{2}$  ( $k \in \mathbb{R}$ ). So Only  $L(1-s, \chi(n)) = L(\bar{s}, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$ ) is true, so

only  $k=1$  is true. According the equation  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) obtained by Riemann, since Riemann has shown that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ )

function has zero, that is, in  $\zeta(1-s) = 2^{1-s} \pi^{-s} \cos(\frac{\pi s}{2}) \Gamma(s) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ),  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and

$s \neq 1$ ) is true. So only when  $\sigma = \frac{1}{2}$  and  $\zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$ ), then

$L(s, \chi(n)) = \chi(n) \zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positives integers) is true.

Because  $L(s, \chi(n)) = \chi(n) \zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) and

$L(1-s, \chi(n)) = \chi(n) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers), so

When  $\rho = \frac{1}{2}$ , it must be true that  $L(s, \chi(n)) = \overline{L(\bar{s}, \chi(n))}$  ( $s \in \mathbb{C}$  and

$s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers), and it must be true that

$L(1-s, \chi(n)) = L(\bar{s}, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers).

According  $\zeta(1-s) = \zeta(s) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and  $\zeta(s) = \zeta(\bar{s}) = \zeta(1-\bar{s}) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ), so

$L(s, \chi(n)) = L(1-s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) and

$L(s, \chi(n)) = L(\bar{s}, \chi(n)) = L(1-\bar{s}, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive

integers), then  $s = \bar{s}$  or  $s = 1-s$  or  $\bar{s} = 1-s$ , so  $s \in \mathbb{R}$ , or  $\sigma + ti = 1 - \sigma - ti$ , or  $\sigma - ti = 1 - \sigma - ti$ , so  $s \in$

$\mathbb{R}$ , or  $\sigma = \frac{1}{2}$  and  $t = 0$ , or  $\sigma = \frac{1}{2}$  and  $t \in \mathbb{R}$  and  $t \neq 0$ , so  $s \in \mathbb{R}$ , or  $s = \frac{1}{2} + 0i$ , or  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq$

$0$ ), because  $\zeta(\frac{1}{2}) \rightarrow +\infty, \zeta(1) \rightarrow +\infty, \zeta(1)$  is divergent,  $\zeta(\frac{1}{2})$  is more divergent, so drop =

1 and  $s = 0$ . So only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$ , and  $t \neq 0, s \in \mathbb{C}$ ) is true, or say  $s = \frac{1}{2} + ti$  ( $t \in$

$\mathbb{R}$ , and  $t \neq 0, s \in \mathbb{C}$ ) is true. And because only when  $\sigma = \frac{1}{2}$ , the next three equations,

$L(\sigma + ti, \chi(n)) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers),  $L(1 - \sigma - ti, \chi(n)) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers), and  $L(\sigma - ti, \chi(n)) = 0$  ( $t \in \mathbb{R}$  and  $t \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) are all true.

And because  $L(\frac{1}{2}, \chi(n)) > 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers), so only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ) is true. The Generalized Riemann hypothesis and the Generalized Riemann

conjecture must satisfy the properties of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) function, The properties of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) function are fundamental, the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct to reflect the properties of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and

$s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) function, that is, the roots of the  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) can only be  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), that is,  $\text{Re}(s)$  must only be equal to  $\frac{1}{2}$ , and  $\text{Im}(s)$  must be real, and  $\text{Im}(s)$  is not equal to zero. So the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct.

According  $L(1 - s, \chi(n)) = L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers), so the zeros of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) function in the complex plane also correspond to the symmetric distribution of point

$(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line in the complex plane, so When  $L(1 - s, \chi(n)) = L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers),  $s$

and  $1-s$  are pair of zeros of the function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) symmetrically distributed in the complex plane with respect to point  $(\frac{1}{2}, 0i)$  on a line perpendicular to the real number line of the complex plane.

We got  $L(s, \chi(n)) = L(\bar{s}, \chi(n))$  ( $s = \sigma + ti, \sigma \in \mathbb{R}, t \in \mathbb{R}$  and  $t \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) before, When  $t$  in Generalized Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ) is a

complex number, and  $s = \frac{1}{2} + ti = \sigma + ti$ , then  $s$  in  $L(s, \chi(n)) = L(\bar{s}, \chi(n))$  ( $s = \sigma + ti$ ,  $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ) is consistent with  $s$  in Generalized Riemann's hypothesis  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{C}$  and  $t \neq 0$ ), so only  $\sigma = \frac{1}{2}$ . When  $L(s, \chi(n)) = L(\bar{s}, \chi(n)) = 0$  ( $s = \sigma + ti$ ,  $\sigma \in \mathbb{R}$ ,  $t \in \mathbb{R}$  and  $t \neq 0$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers), since  $s$  and  $\bar{s}$  are a pair of conjugate complex numbers, so  $s$  and  $\bar{s}$  must be a pair of zeros of the Generalized function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) in the complex plane with respect to point  $(\rho, 0i)$  on a line perpendicular to the real number line.  $s$  is a symmetric zero of  $1-s$ , and a symmetric zero of  $\bar{s}$ . By the definition of complex numbers, how can a symmetric zero of the same Generalized Riemann function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) of the same zero independent variable  $s$  on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of  $1-s$  on a line perpendicular to the real number axis of the complex plane with respect to point  $(\frac{1}{2}, 0i)$  and a symmetric zero of  $\bar{s}$  on a line perpendicular to the real number axis of the complex plane with respect to point  $(\sigma, 0i)$ ? Unless  $\sigma$  and  $\frac{1}{2}$  are the same value, is also that  $\sigma = \frac{1}{2}$ , and only  $1-s = \bar{s}$  is true, only  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$ ) is true. Otherwise it's impossible, this is determined by the uniqueness of the zero of Generalized Riemann function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane. Only one line can be drawn perpendicular from the zero independent variable  $s$  of Generalized Riemann function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) on the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of Generalized Riemann function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane, so I have proved the generalized Riemann conjecture when the Dirichlet eigen function  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) is any real number that is not equal to zero. Since the nontrivial zeros of the Riemannian function  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) and the generalized Riemannian function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,

$n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) are both on the critical line perpendicular to the real number line of  $\text{Re}(s)=\frac{1}{2}$  and  $\text{Im}(s) \neq 0$ , these nontrivial zeros are general complex numbers of  $\text{Re}(s)=\frac{1}{2}$  and  $\text{Im}(s) \neq 0$ , so I have proved the generalized Riemann conjecture when the Dirichlet eigen function  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) is any real number that is not equal to zero.

The Generalized Riemann hypothesis and the Generalized Riemann conjecture must satisfy the properties of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) function,

The properties of the  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) function are fundamental, the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct to reflect the properties of the  $L\left(s, \chi(n)\right)$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in$

$\mathbb{Z}^+$  and  $n$  traverse all positive integers) function, that is, the roots of the  $L(s, \chi(n))=0$  ( $s \in \mathbb{C}$  and  $s \neq 1, n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) can only be  $s=\frac{1}{2}+ti$  ( $t \in \mathbb{C}, s \in \mathbb{C}$  and  $t \neq 0$ ), that is,  $\text{Re}(s)$  can only be equal to  $\frac{1}{2}$ , and  $\text{Im}(s)$  must be real, and  $\text{Im}(s)$  is not equal to zero.

When  $L(s, \chi(n)) = 0$  ( $n \in \mathbb{Z}^+, p \in \mathbb{Z}^+, s \in \mathbb{C}$  and  $s \neq 1, n$  goes through all the positive integers,  $p$  goes through all the prime numbers,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 0$ ),  $a(n) = a(p) = \chi(n)$ ,  $P(p, s) = \frac{1}{1-a(p)p^{-s}}$ ), then the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct, and  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0, s \in \mathbb{C}$ ).

#### **Reasoning 4:**

For any complex number  $s$ , when  $\chi(n)$  is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer  $q$  such that  $\chi(n+q) = \chi(n)$  ( $n \in \mathbb{Z}^+$ );

2: when  $n$  and  $q$  are not mutual prime,  $\chi(n) = 0$  ( $n \in \mathbb{Z}^+$ );

3:  $\chi(a)\chi(b) = \chi(ab)$  ( $a \in \mathbb{Z}^+, b \in \mathbb{Z}^+$ ) for any integer  $a$  and  $b$ ;

Suppose  $q=2k$  ( $k \in \mathbb{Z}^+$ ),

if  $n$  and  $n+q$  are all prime number, and if  $\chi(Y) = 0$  ( $Y$  traverses all positive odd numbers) and

$\chi(n+q) = \chi(n) = 0$  ( $n$  and  $n+q$  traverses all positive odd numbers), because  $n$  ( $n$  traverses



all prime numbers) and  $q=2k(k \in \mathbb{Z}^+)$  are not mutual prime, then  $\chi(n)=0$  ( $n \in \mathbb{Z}^+$  and  $n$  and  $n + q$  traverses all prime numbers) and for any prime number  $a$  and  $b$ ,  $\chi(a) \cdot \chi(b) = \chi(ab)$  ( $a \in \mathbb{Z}^+, b \in \mathbb{Z}^+$ ,  $a$  traverses all prime numbers and  $b$  traverses all prime number, then the three properties described by the Dirichlet

eigenfunction  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all prime numbers). above fit the definition of the Polignac conjecture, the Polignac conjecture states that for all natural numbers  $k$ , there are infinitely many pairs of prime numbers  $(p, p+2k)$  ( $k \in \mathbb{Z}^+$ ). In 1849, the French mathematician A. Polignac proposed the conjecture. When  $k=1$ , the Polignac conjecture is equivalent to the twin prime conjecture. In other words, when  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all prime numbers,  $\chi(n) \in \mathbb{R}$ ,

$a(n) = a(p) = \chi(n)$ ),  $P(p, s) = \frac{1}{1-a(p)p^{-s}}$ , and generalized Riemann hypothesis and the generalized Riemann conjecture are true, then the Polignac conjecture must be completely true, and if the Polignac conjecture must be true, then the twin prime conjecture and Goldbach's conjecture must be true. I proved that the generalized Riemannian hypothesis and the generalized Riemannian conjecture are true, so when  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all prime numbers, and  $\chi(n) = 0$ ),  $P(p, s) = \frac{1}{1-a(p)p^{-s}}$  and  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ), I also proved that the Polignac conjecture, twin prime conjecture must be true and Goldbach conjecture are completely or almost true. The Generalized Riemann hypothesis and the Riemann conjecture are perfectly valid, so the Polignac conjecture and the twin prime conjecture and Goldbach's conjecture must satisfy the properties of the Generalized Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function and the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, so the Polignac conjecture, twin prime conjecture must be true and Goldbach conjecture is completely true. Riemann hypothesis and the Riemann conjecture are completely correct and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are completely correct and the Polignac conjecture, twin prime conjecture must be true and Goldbach conjecture are almost or completely true.

**Reasoning 5:**

In order to explain why the zero of the Landau-Siegel function exists under special conditions, we need to start with the Riemann conjecture. I have solved the Riemann conjecture for the Dirichlet

feature  $\chi(n) \equiv 1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and the generalized Riemann

conjecture for the Dirichlet feature  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), I

propose a special form of Dirichlet  $L(s, \chi(p))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0$ ,  $p \in \mathbb{Z}^+$  and  $p$

traverses all odd primes, including 1) function problem. Let me first explain to you what Landau-Siegel zero conjecture is. As you may know, the Landau-Siegel zero point problem, named after Landau and his student Siegel, boils down to solving whether there are abnormal real zeros

in the Dirichlet L function. So let's look again at what the Dirichlet L function is. Look at the above proof process, which is the expression of Dirichlet  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers)

$$L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} \quad (s \in \mathbb{C} \text{ and } s \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ goes through all positive integers}) .$$

I shall first introduce the Dirichlet  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) function and explain its relation to the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function.  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is a characteristic value of a Dirichlet function, which is all real numbers, and  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is a real function. The  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) function can be analytically extended as a meromorphic function over the entire complex plane. John Peter Dirichlet proved that  $L(1, \chi(n)) \neq 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 0$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) for all  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers), and thus proved Dirichlet's theorem. In number theory, Dirichlet's theorem states that for any positive integers  $a, d$ , there are infinitely many forms of prime numbers, such as  $a+nd$ , where  $n$  is a positive integer, i.e., in the arithmetic sequence  $a+d, a+2d, a+3d, \dots$ . There are an infinite number of prime numbers—there are an infinite number of prime modules  $d$  as well as  $a$ . If  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is the main feature, then  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) has a unipolar point at  $s=1$ . Dirichlet defined the properties of the characteristic function  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) in the Dirichlet function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) :

- 1: There is a positive integer  $q$  such that  $\chi(n+q) = \chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers);
- 2: when  $n$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all natural numbers) and  $q$  are non-mutual primes,  $\chi(n) \equiv 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers);
- 3: For any integer  $a$  and  $b$ ,  $\chi(a) \cdot \chi(b) = \chi(ab)$  ( $a$  is a positive integer,  $b$  is a positive integer);

From the expression of the Dirichlet function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  takes all positive integers), it is easy to see that when the Dirichlet characteristic real function  $\chi(n) = 1$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  takes all positive integers), Then the Dirichlet

$L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) becomes the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function, so the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function is a special function of the Dirichlet function  $L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers), when the characteristic real function  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers) is equal to 1, Also called a trivial characteristic function of the Dirichlet function  $L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers). When the eigenreal functions  $\chi(n) \neq 1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positive numbers), they are called nontrivial eigenfunctions of the Dirichlet function  $L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers).

When the independent variable  $s$  in the expression of the Dirichlet function  $L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverse all positive integers) is a real number  $\beta$ , then for all eigenfunction values  $\chi(n)$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers),  $L(\beta, \chi)$  ( $\beta \in \mathbb{R}$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is called the Landau-Siegel function. Visible landau-siegel function  $L(\beta, \chi)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$ , and  $n$  traverses all positive integers) is dirichlet function  $L(s, \chi)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$ , and  $n$  traverses all positive integers) of a special function, landau-siegel guess is landau and siegel they guess  $L(\beta, \chi)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$ , and  $n$  traverses all positive integers) is not zero, So Landau and Siegel's conjecture that  $L(\beta, \chi) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is easy to understand, right? Well, now that you know what the Landau and Siegel null conjecture is all about, let's continue to see how I'm going to solve the Landau and Siegel null conjecture. Look at the above proof process:

$$\begin{aligned}
 \text{GRH} \left( s, \chi(n) \right) &= L \left( s, \chi(n) \right) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \frac{\chi(n)\eta(s)}{(1-2^{1-s})} = \frac{\chi(n)}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^s} \\
 &= \frac{\chi(n)}{(1-2^{1-s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\sigma+ti}} = \frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_{n=1}^{\infty} \chi(n) \left( \frac{1}{n^{\sigma}} \frac{1}{n^{ti}} \right) =
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_{n=1}^{\infty} \chi(n) n^{-\sigma} \frac{1}{(\cos(\ln(n)) + i\sin(\ln(n)))^t} \\
 &= \frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_{n=1}^{\infty} \chi(n) n^{-\sigma} (\cos(\ln(n)) + i\sin(\ln(n)))^{-t} \\
 &= \frac{(-1)^{n-1}}{(1-2^{1-s})} \sum_{n=1}^{\infty} \chi(n) n^{-\sigma} (\cos(\ln(n)) - i\sin(\ln(n)))^{-t}
 \end{aligned}$$

( $t \in \mathbb{C}$  and  $t \neq 0$ ,  $s \in \mathbb{C}$  and  $s \neq 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  goes through all positive integers) ,because  $\zeta(s)=2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) (Formula 7) ,so if  $\beta \in \mathbb{R}$  and  $\beta = -2n$  ( $n \in \mathbb{Z}^+$ ) ,then  $\zeta(s)=0$ . So

$L(\beta, \chi(n))=$

$$\begin{aligned}
 & \frac{(-1)^{n-1}}{(1-2^{1-\beta})} \sum_{n=1}^{\infty} \chi(n) n^{-\beta} (\cos(0 \times \ln(n)) + i\sin(0 \times \ln(n))) = \frac{(-1)^{n-1}}{(1-2^{1-\beta})} \sum_{n=1}^{\infty} (\chi(n) n^{-\beta}) = \\
 & \frac{1}{(1-2^{1-\beta})} (\chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(4)4^{-\beta} + \dots) , \text{ " } \times \text{ " is the symbol for}
 \end{aligned}$$

multiplication, because the real exponential function of the real number has a function value greater than zero, so

$n^{-\beta} > 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \dots, (n-1)^\beta - n^\beta < 0, \dots$ , or  $1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \dots, (n-1)^\beta - n^\beta > 0$  , and  $\frac{1}{(1-2^{1-\beta})} \neq 0$  , it can be known that if  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all

positive integers), and  $\beta \in \mathbb{R}$  and  $\beta \neq -2n$  ( $n \in \mathbb{Z}^+$ ) , then  $L(\beta, \chi(n)) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in$

$\mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) and  $L(\beta, 1) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in$

$\mathbb{Z}^+$ , and  $n$  traverses all positive integers), so for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in$

$\mathbb{Z}^+$ ) functions, its corresponding landau-siegel function  $L(\beta, 1)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in$

$\mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) of pure real zero does not exist, this means that

the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$ ) function does not have a zero of a pure

real variable  $s$ , and the generalized Riemann conjecture  $L(s, \chi(n))=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$ , and  $s \neq$

$-2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $n$  traverses all positive integers) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}, t \neq 0$ ) is sufficient

to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are almost true.

And if  $\chi(n) = 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) or  $\beta \in \mathbb{R}$  and  $\beta = -2n$  ( $n \in$

$\mathbb{Z}^+$ ), then  $L(\beta, \chi(n))=0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $n$  traverses all positive

integers) and  $L(\beta, 1)=0(\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ , and  $n$  traverses all positive integers), so for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) functions, its corresponding landau-siegel function  $L(\beta, 1)(\beta \in \mathbb{R}, \chi(n) \in \mathbb{R}, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) of pure real zero exist, this means that the Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) function have a zero of a pure real variable  $s$ , and the generalized Riemann conjecture  $L(s, \chi(n))=0(s \in \mathbb{C}$  and  $s \neq 1, \chi(n) \in \mathbb{R}$  and  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are completely true.

when  $\chi(n) \neq 1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so  $n^{-\beta} > 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \dots, (n-1)^\beta - (n)^\beta < 0, \dots$ , or  $1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \dots, (n-1)^\beta - (n)^\beta > 0$  and  $|\frac{1}{(1-2^{1-\beta})}| \neq 0$ , it can be known that when  $\chi(n)=1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverse all positivel numbers), then  $L(\beta, 1) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) so for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$ ) functions, its corresponding landau -siegel function  $L(\beta, 1)(\beta \in \mathbb{R}, \chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) of pure real zero does not exist, this means that the generalized Riemann  $L(\beta, \chi(n))(\beta \in \mathbb{R}, \chi(n) \in \mathbb{R}$  and  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) function does not have a zero of a pure real variable  $s$ , and the generalized Riemann conjecture  $L(s, \chi(n))=0(s \in \mathbb{C}$  and  $s \neq 1, \chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1$  and  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}, t \neq 0$ ) is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are almost true.

When  $\chi(n) \neq 1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so  $n^{-\beta} > 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \dots, (n-1)^\beta - (n)^\beta < 0, \dots$ , or  $1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \dots, (n-1)^\beta - (n)^\beta > 0$  and  $|\frac{1}{(1-2^{1-\beta})}| \neq 0$ , it can be known that when  $\chi(n) \neq 1$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $\chi(n) \neq 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), then  $L(\beta, \chi(n)) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 1$  and  $\chi(n) \neq 0$  and  $n$

traverses all positive integers), so for generalized Riemann  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) functions, its corresponding Landau-Siegel function  $L(\beta, \chi(n))$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 1$  and  $\chi(n) \neq 0, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) of pure real zero does not exist, this means that the generalized Riemann  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) function does not have a zero of a pure real variable  $s$ . and the generalized Riemann conjecture  $L(s, \chi(n))=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 1$  and  $\chi(n) \neq 0$  and  $n$  traverses all positive integers) satisfies  $s = \frac{1}{2} + it$  ( $t \in \mathbb{R}, t \neq 0$ ) is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are all almost true.

When  $\chi(n) \equiv 0$  ( $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so

$$n^{-\beta} > 0 \quad (n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers}) \text{ and } \chi(1)1^\beta = 0, \chi(2)2^\beta = 0, \chi(3)3^\beta = 0, \chi(4)4^\beta = 0, \chi(5)5^\beta = 0, \chi(6)6^\beta = 0, \dots, \chi(n-1)(n-1)^\beta = 0, \chi(n)n^\beta = 0, \dots, \text{ and } \left| \frac{1}{(1-2^{1-\beta})} \right| \neq 0, \text{ it can be known that when } \chi(n) \equiv 0 \quad (n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers}),$$

then  $L(\beta, \chi(n)) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and  $L(\beta, 1) \neq 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+$ , and  $n$  traverses all positive integers), so for generalized Riemann  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) functions, its corresponding Landau-Siegel function  $L(\beta, 0)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 0$  and  $n$  traverses all positive integers) of pure real zero exists,

This means that the generalized Riemann  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+, n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) function has a zero of a pure real variable  $s$ , that means the twin prime conjecture, Goldbach's conjecture, Polignac's conjecture are completely true.

When  $\chi(p) \equiv 0$  ( $p \in \mathbb{Z}^+$  and  $p$  traverses all odd primes, including 1), then  $L(s, \chi(p))=0$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0, p \in \mathbb{Z}^+$  and  $p$  traverses all odd primes, including 1) was established. At the same time  $L(s, \chi(p))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0, p \in \mathbb{Z}^+$  and  $p$  traverses all odd primes, including 1) the corresponding Landau-Siegel function

$L(\beta,0)(\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  traverses all odd primes, including 1) expression as shown as follows:

$$L(\beta, \chi(p)) = \frac{(-1)^{n-1}}{(1-2^{1-\beta})} \sum_{p=1}^{\infty} \chi(p) p^{-\beta} (\cos(0 \times \ln(p)) + i \sin(0 \times \ln(p))) =$$

$$\frac{(-1)^{n-1}}{(1-2^{1-\beta})} \sum_{p=1}^{\infty} (\chi(p) p^{-\beta}) = \frac{(-1)^{n-1}}{(1-2^{1-\beta})} [\chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(5)5^{-\beta} + \chi(7)7^{-\beta} + \dots$$

$-\chi(p)p^{-\beta} + \dots]$  ( $\beta \in \mathbb{R}$ ,  $p \in \mathbb{Z}^+$  and  $p$  traverses all primes, including 1), "  $\times$  " is the symbol for multiplication.

When  $\chi(p) \equiv 0$  ( $p \in \mathbb{Z}^+$  and  $p$  traverses all odd primes, including 1), then  $L(s, \chi(p)) \equiv 0$  ( $s \in \mathbb{C}$  and

$s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(p) \equiv 0$ ,  $p$  traverses all odd primes, including 1) was

established. At the same time  $L(s, \chi(p))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  traverses all primes, including 1) the corresponding Landau-Siegel function

$L(\beta,0) = 0$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(p) \in \mathbb{R}$  and  $\chi(p) \equiv 0$ ,  $p \in \mathbb{Z}^+$  and  $p$  traverses all primes,

including 1), this means that the generalized Riemann  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) function has a zero of a pure real variable  $s$ , that means the twin prime conjecture, Goldbach's conjecture, Polignac's conjecture are all completely true.

Now I summarize the Dirichlet function  $L(s, \chi(n))$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$ , and  $n$  traverses all positive integers) as follows:

1: When  $\chi(n) \equiv 1$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers), the generalized

Riemannian hypothesis and the generalized Riemannian conjecture degenerate to the ordinary Riemannian hypothesis and the ordinary Riemannian conjecture, whose nontrivial zeros  $s$  satisfy

$$s = \frac{1}{2} + it \quad (t \in \mathbb{R} \text{ and } t \neq 0), \text{ and ordinary Riemann } \zeta(s) = L(s, \chi(n)) \quad (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in$$

$\mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) the corresponding

Landau-Siegel function  $L(\beta,1) \neq 0$  ( $\beta \in \mathbb{R}$ , and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1$  and  $n$  traverses all positive integers), ordinary Riemann hypothesis and ordinary Riemann hypothesis all hold, and for Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ) function, its corresponding

Landau-Siegel function  $L(\beta,1)$  ( $\beta \in \mathbb{R}$  and  $\beta \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) does not exist pure real zero, which also shows that Riemann  $\zeta(s)$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ) function does not exist zero when variable  $s$  is a pure real zero.

2: When  $\chi(n) \equiv 0 (n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive odd numbers, including } 1)$ , then  $\chi(p) \equiv 0 (p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd primes, including } 1)$ , a special Dirichlet function  $L(s, \chi(p)) (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R} \text{ and } \chi(p) \equiv 0, p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd primes, including } 1)$  has zero, and when zero is obtained, the independent variable  $s$  is any complex number. This special Dirichlet function  $L(s, \chi(p)) (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R} \text{ and } \chi(p) \equiv 0, p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd prime, including } 1)$  the corresponding Landau-Siegel function  $L(\beta, 0) = 0 (\beta \in \mathbb{R} \text{ and } \beta \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R} \text{ and } \chi(p) \equiv 0, p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd prime, including } 1)$  holds, so for this particular Dirichlet function  $L(s, \chi(p)) = 0 (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R} \text{ and } \chi(p) \equiv 0, p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd primes, including } 1)$  holds. The existence of a pure real zero of the corresponding Landau-Siegel function  $L(\beta, 0) (\beta \in \mathbb{R} \text{ and } \beta \neq -2n, n \in \mathbb{Z}^+, \chi(p) \in \mathbb{R} \text{ and } \chi(p) \equiv 0, p \in \mathbb{Z}^+ \text{ and } p \text{ traverses all odd prime numbers, including } 1)$  shows that the twin prime numbers, Polignac conjecture and Goldbach conjecture are all completely true.

3: When  $\chi(n) \neq 1$  and  $\chi(n) \neq 0 (n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$ , Dirichlet function  $L(s, \chi(n)) (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0 \text{ and } \chi(n) \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$  has zero, its nontrivial zero meet  $s = \frac{1}{2} + ti (t \in \mathbb{R} \text{ and } t \neq 0)$ . For Dirichlet function  $L(s, \chi(n)) (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0, n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$ , its corresponding Landau-Siegel function  $L(\beta, \chi(n)) (\beta \in \mathbb{R} \text{ and } \beta \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0 \text{ and } \chi(n) \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$  of pure real zero does not exist. In other words, it shows that the Dirichlet function  $L(s, \chi(n)) (s \in \mathbb{C} \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0 \text{ and } \chi(n) \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$  does not exist for the zero of a pure real variable  $s$ , so if  $\chi(n) \neq 0$  and  $\chi(n) \neq 1 (n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$ , then both the generalized Riemannian hypothesis and the generalized Riemannian conjecture hold and the Generalized Riemann  $L(s, \chi(n)) (s \in \mathbb{C} \text{ and } s \neq 1, \text{ and } s \neq -2n, n \in \mathbb{Z}^+, \chi(n) \in \mathbb{R} \text{ and } \chi(n) \neq 0 \text{ and } \chi(n) \neq 1, n \in \mathbb{Z}^+ \text{ and } n \text{ traverses all positive integers})$  function of nontrivial zero  $s$  also



meet  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$  and  $t \neq 0$ ). Now we know that merely proving that the nontrivial zero  $s$  of the Riemann conjecture  $L(s, 1) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \equiv 1$ ,  $n \in \mathbb{Z}^+$  and  $n$  traverses all positive integers) and the generalized Riemann conjecture  $L(s, \chi(n)) = 0$  ( $s \in \mathbb{C}$  and  $s \neq 1$  and  $s \neq -2n$ ,  $n \in \mathbb{Z}^+$ ,  $\chi(n) \in \mathbb{R}$  and  $\chi(n) \neq 1$  and  $\chi(n) \neq 0$  and  $n$  traverses all positive integers) satisfies  $s = \frac{1}{2} + ti$  ( $t \in \mathbb{R}$ ,  $t \neq 0$ ) is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are all almost true.

### **III. Conclusion**

After the Riemann hypothesis and the Riemann conjecture and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are proved to be completely valid, the research on the distribution of prime numbers and other studies related to the Riemann hypothesis and the Riemann conjecture will play a driving role. Readers can do a lot in this respect.

### **IV. Thanks**

Thank you for reading this paper.

### **V. Contribution**

The sole author, poses the research question, demonstrates and proves the question.

### **VI. Author**

Name: Teng Liao (1509135693@139.com), Sole author.



Work unit address: 237 Airport Road, Weili Community, Huli District, Xiamen City.

Zip Code: 361022

**References**

[1] Riemann : 《On the Number of Prime Numbers Less than a Given Value》 ;

[2] John Derbyshire(America): 《PRIME OBSESSION》 P218,BERHARD RIEMANN

AND THE GREATEST UNSOIVED PROBLEM IN MATHMATICS,Translated by Chen

Weifeng, Shanghai Science and Technology Education Press,

China,<https://www.doc88.com/p-54887013707687.html>;

[3] Xie Guofang: On the number of prime numbers less than a given value - Notes to Riemann's original paper proposing the Riemann conjecture,



