Cosmological thermodynamics and the source of dark matter

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Abstract

The Friedmann equations and the ΛCDM model describe the adiabatic expansion of the universe. The model explains the red-shift and decrease in total energy of the photons but the equations also require energy conservation according to the first law of thermodynamics, an apparent contradiction that has proved hard to understand. Further consideration of the thermodynamics of adiabatic expansion demonstrates how to reconcile the loss of energy of the photons with overall conservation of energy, and points to the source of dark matter.

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The ΛCDM model is the leading description of the universe and assigns density ρ and pressure p to normal and dark matter, photons and dark energy, the latter inferred from the observation of late universe acceleration.[Peebles and Ratner 2003, Frieman et al 2008, Perlmutter et al 1999] The model is based on the Friedmann equations which are solutions to general relativity under the assumptions that the universe is homogeneous and isotropic, and that its components comprise ideal gases expanding adiabatically.[Romeu 2014] Text-book cosmology shows that the two Friedmann equations can be combined to give the energy continuity equation,

$$\dot{\rho} + 3H(\rho + p) = 0. \qquad (1)$$

where H is the Hubble parameter. This equation can be further rearranged as,

$$\rho = \rho_0 a^{-3(1+w)} \qquad (2)$$

where $w=p/\rho$ and is the constant equation of state parameter for each component. Derived from either of these equations is,

$$\mathrm{d}(\rho a^3) + p\mathrm{d}(a^3) = 0 \qquad (3a)$$

The total energy E in an expanding volume $V=a^3$ is $E=\rho a^3$ so that this equation is equivalently,

$$\mathrm{d}E + p\mathrm{d}V = 0, \qquad (3b)$$

which is the first law of thermodynamics for an adiabatically expanding gas, as expected for the assumptions of the model. It should be noted that although the universe as a whole expands adiabatically since there is no external source of energy, each individual component is not necessarily adiabatic as there may be an exchange of energy between components.

Equation 2 is central to the ΛCDM model and is used to determine how each component of the universe evolves. Normal matter has no pressure so $w=0$ and the energy density decreases as a^{-3}. The total energy E_M is constant, and matter simply dilutes in the expanding volume.

Photons have pressure $p=\rho/3$, hence $w=1/3$ and the photon energy density decreases as a^{-4} and the total energy E_P therefore decreases as a^{-1} reflecting the dilution of a constant number of photons and their redshift, in agreement with observation. The constant dark energy density requires $w=-1$ and its total energy increases with the expansion.

The fact that the total photon energy E_P decreases with the expansion of the universe creates an apparent contradiction. The photon energy E_P is not conserved and yet equation 3 is a statement that energy is conserved. No definitive resolution has been given previously but several suggestions have been made as to how this contradiction might be resolved. [Harrison 1995, 2000, Baryshev 2006, Yu et al 2022] Proposed explanations include that energy is not actually conserved, or perhaps that it is conserved locally but not globally. Another suggestion is that the photons do conserve energy by some undefined mechanism perhaps related to the dynamics of spacetime. However, the problem is more categorical than the question of energy conservation. Equations 2 and 3 are mathematically identical, since each can be derived from the other. The ΛCDM model obviously contains an error when one of these equations is obeyed and the other one is not obeyed. Equation 3 is not obeyed by ΛCDM because the $p\mathrm{d}V$ energy is not included in the model.

The source of the error in the ΛCDM model and its resolution lie in the thermodynamic treatment. The thermodynamics of an adiabatically expanding gas as described by the equations above invariably comprises a system with two components; one is the expanding gas and the second component opposes the expansion. The properties of the expanding gas and the system as a whole, depend critically on the properties of the second component which must be included in the model. The governing equations would be different from equations 2 and 3 if the second component did not exist. Consider the case of an adiabatically expanding photon gas with positive pressure and with a second component opposing expansion. Slow adiabatic expansion is reversible and in the limit of slow expansion, the second component has an equal and opposite pressure to the first. The second component experiences negative $p\mathrm{d}V$ work and hence an increase in its internal energy E_2. The result is the transfer of energy from the first to the second component so that,

$$\mathrm{d}E_P + \mathrm{d}E_2 = 0 \quad (4)$$

The energy lost by the photon gas is transferred to the second component. Both components change with the expansion of the universe, but their sum is constant. Hence energy is conserved and equations 2 and 3 are both satisfied provided that a corresponding equation 2 is included for the second component. This is the standard text-book description for adiabatic expansion. Obviously both components are within the universe and so the ΛCDM model is only consistent when both are included in the model.

The second component that opposes the photon expansion changes with the evolution of the universe, and so does the thermodynamic treatment. Before photon decoupling, photons interact with electrons and through that interaction with baryons. Sufficiently early in the expansion (but later than nucleosynthesis) all the $p\mathrm{d}V$ work done by the photon expansion is transferred to the electrons. However, there are also back reactions from electrons to photons that exchange heat, through normal and inverse Compton scattering, photoemissions and other mechanisms. The exchange of heat means that the photon expansion is not adiabatic and instead, the strong plasma interactions maintain the combined system, the plasma of photons and matter, in thermal equilibrium. As the combined system expands, the photon temperature drops as $1/a$

because the equilibrium blackbody temperature is related to the photon density. This is the same dependence on the scale factor as for adiabatic expansion, but the mechanism is different.

After decoupling at z=1,100 (z=1/a-1), photons free-stream and all their pdV energy is taken up entirely by the gravitational interaction, which becomes the second component. The interaction is only through the pdV work, there is no separate heat exchange, the expansion is slow, and so the interaction is properly described by adiabatic expansion of the type described by eq. 3. The pdV work of expansion is transferred from the photons to the gravitational interaction according to equation 4, by the amount E_2.

In Newtonian gravity the kinetic energy of an object moving against a force of gravity is equivalent to the pressure of a gas in cosmology and the object gains potential energy as it loses kinetic energy by gravity. The Friedmann equations have no explicit potential energy term, the first Friedman equation being,

$$H^2 = (\dot{a}/a)^2 = (8\pi G/3)\rho - k/a^2 + \Lambda/3 \qquad (5)$$

where k is the curvature parameter, Λ is the cosmological constant and ρ is a sum over all components. The energy transferred from the photons can only appear as a change in one of the three parameters ρ, k or Λ. Experiment finds that the universe is flat so that k=0, [Efstathiou and Gratton 2020] and the cosmological constant is negligible at large values of z when most of the photon energy is transferred, [Perlmutter et al. 1999] so that neither of these terms can accommodate the energy transfer. The energy E_2 of the second system (gravity) must therefore be in the form of a real energy density added to the energy densities of the other components. Whereas in Newtonian gravity potential energy is a fictitious energy, for general relativity according to the Friedmann equations, the pdV work is a real energy density.

Prior to recombination, photons had a long mean free path, so there is a period when photons interacted with both electrons and with gravity. For example, at z=10,000 the mean time between photon scattering is ~10^8 s, which is more than a year. The rate of photon energy loss to gravity is,

$$R_P = dW_{Ph}(a)/dt = W_{Ph0}a^{-2}\dot{a} = H(a)W_{Ph}(a) \qquad (6)$$

where $W_{Ph}(a)$ is the average photon energy (i.e. $\hbar\omega_{ave}$) and W_{Ph0} is the photon energy at the present time, a=1. The principal mechanism for the interaction of photons with individual electrons is Thomson/Compton scattering for which the rate that photons lose energy is,

$$R_{Th} = (W_{Ph}^2(a)/m_e\, c^2)cn_e(a)\sigma_T \qquad (7)$$

where m_e is the electron mass, $n_e(a)$ is the electron density, c is the velocity of light and σ_T is the Thomson scattering cross section. The first term in brackets on the right is the average loss of photon energy at each scattering event and the second term is the scattering rate. There is a dependence on scattering angle which varies from 0 to 2 and is approximated by unity. The parameters are all known and a calculation, shown in Figure 1, finds that the scattering rate exceeds the rate of transfer to gravity at earlier times than z~30,000. Other collective scattering mechanisms of photons with the plasma may increase the transfer of energy from photons to matter. The magnitude of E_2 in today's universe can now be found and is between 1,100 and 30,000 times the present photon energy density, the result being between 0.5 and 15x10^{-30} gm/cm^3.

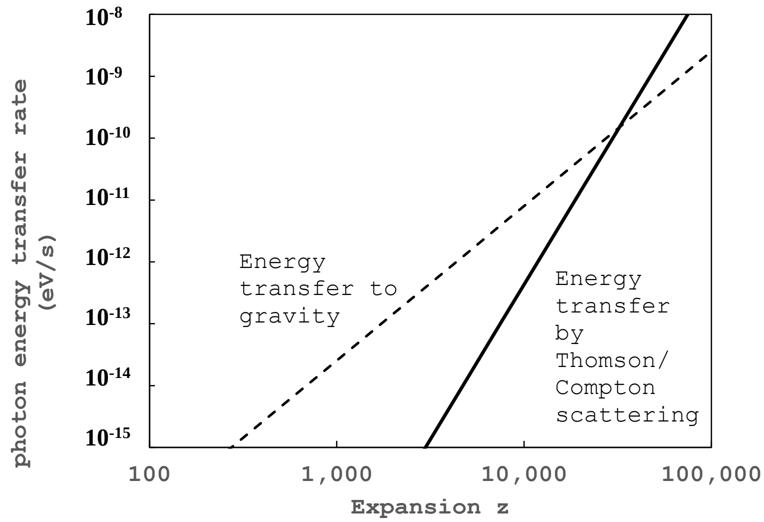

Figure 1. Rate of photon energy transfer as a function of expansion *z*. Dashed line is the transfer rate to gravity. Solid line is the transfer rates to matter by Thomson/Compton scattering.

The amount of energy that the cosmology model needs to accommodate is therefore significant in comparison with the known total energy density, which is not surprising given that photons used to dominate the energy density of the universe. One cannot ignore the fact that the photons create a source of energy density that needs accommodation in the cosmological model, while dark matter is a known energy density with no obvious source. The dark matter density according to the ΛCDM model is 2.7×10^{-30} gm/cm^3 which is in the middle of the estimated range of the energy transferred from the photons. It is therefore proposed that the energy density created by the photons is dark matter. It is difficult to reach an alternative conclusion since a model that includes both the transferred photon energy E_2 and a separate dark matter density would probably not be consistent with the measured expansion rate because the total energy density would be too large.

As with any other form of energy, the E_2 energy both contributes to and responds to the gravitational force and can therefore coalesce as described by standard dark matter models, inducing normal matter coalescence, and formation of the dark matter galaxy halos. The physical form of this gravitational energy is unclear but since photons lose their energy continuously, components of this energy could not be larger than a very small fraction of an eV and are therefore within the scope of ultra-light dark matter models. Numerous specific dark matter models have been proposed with energy elements as low as 10^{-24} eV.[Hu et al 2000, Tulin and Yu 2018, Ferreira et al 2021] Dark matter is universally agreed to be new to physics, as is the form of energy identified here.

This new analysis of the thermodynamics of the expansion of the universe has other consequences. It is obvious that dark energy has the same core problem of inconsistency with equations 2 and 3. The required source of energy for the $p\mathrm{d}V$ work needed to create a constant dark energy density by adiabatic expansion counteracts the acceleration of the universe which is the reason dark energy was proposed. A completely different model for the acceleration is required and will be described in a separate paper along with more details about the new form of energy.

The proposed mechanism for the creation of dark matter by the adiabatic expansion of the photons is not in obvious disagreement with the known quantitative data based on the ΛCDM model. The dark matter is generated early enough in the expansion to be consistent with analysis of the baryon acoustic oscillations. More detailed calculations of the photon-plasma interactions are needed to determine more accurately the magnitude of dark matter. The primary benefit of the analysis given here is to resolve an error in the ΛCDM model by the correct application of the thermodynamics of adiabatic expansion.

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