# Solution to Gravity Divergence and Effective Renormalization of Gravity

Hyoyoung Choi * [1]

* *Independent Researcher, Department of Physics, Seoul, Republic of Korea.*

**Abstract**

By using the fact that the mass $M$ changes to $M_{eff}$, which reflects the binding energy, I can obtain the RG flow and gravitational coupling constant $G(k)$. $G(k) = (1 - \frac{R_{gp}}{R})G_N$. When $k^* = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$, which means that gravity is zero. Therefore, we can solve the problem of gravitational divergence at high energy. This work presents an effective renormalization approach to gravity, where the running G(k) naturally leads to the resolution of gravitational divergence without requiring quantum corrections. If $R > R_{gp}$, $G(k) > 0$, and attractive force acts. If $R = R_{gp}$, $G(k) = 0$, and gravity is also zero. If $R < R_{gp}$, $G(k) < 0$, and repulsive force or antigravity acts. This repulsive force prevents the formation of a singularity at the center of the black hole. Therefore, the singularity problem is also solved.

## 1. Introduction

Gravity is basically given by the Einstein-Hilbert action, where $G$ is Newton's constant and $R$ is the scalar curvature derived from the Riemann curvature tensor.

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \tag{1}$$

However, it is known that several problems arise when trying to quantize this gravity theory.

1) The coupling constant has a dimension

Newton's constant $G$, which is the coupling constant of gravity, has the dimension of $mass^{-2}$. [1] On the other hand, the gauge theory covered in the Standard Model has a dimensionless coupling constant (scale-invariant), so it can control the flow in an appropriate way at high energies.

However, in the case of gravity, since the coupling constant has a dimension, the divergence is not controlled at high energies, and as it becomes more and more severe, it cannot be renormalized with only a finite number of terms.

2) Non-renormalizability

In two-loop and above, non-renormalizable divergence inevitably appears.

Therefore, it is known that quantum gravity based on general relativity is fundamentally not renormalized, and a new concept is needed. There are several methods to solve the divergence problem of gravity, but among them, there is a method called Asympotic Safety proposed by Weinberg. [2] [3] [4] The concept of Asympotic Safety is a hypothesis that if the coupling constant of gravity converges to a specific fixed point at high energy, the theory can be maintained finitely. [4]

In this paper, I will present a solution to the renormalization problem of gravity using a method similar to this asympotic safety. To do so, I will look at the solution to the singularity problem inside a black hole, where the idea started, and then approach the renormalization problem of gravity.

[1] E-mail:7icarus7@gmail.com

## 2. Solution of the singularity problem of a black hole [2]

### 2.1. Mass defect effect due to gravitational binding energy or gravitational potential energy

When two masses $m$ are separated by $r$, the total energy of the system is

$$E_T = 2mc^2 - \frac{Gmm}{r} \tag{2}$$

If we introduce the negative equivalent mass $-m_{gp}$ for the gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \tag{3}$$

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{4}$$

**The gravity of a composite particle composed of two objects acting on a mass $m_3$ that is relatively far away is**

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{5}$$

That is, **when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term ($-m_{gp}$) should be considered.** The total mass or equivalent mass $m^*$ of the system is less than the mass of 2m when the two objects were in a free state. The bound objects experience a mass loss (defect) due to the gravitational binding energy. This is equivalent to having a negative equivalent mass in the system.

In general, the binding energy is very small compared to the mass energy. However, as the mass increases, the ratio of binding energy to mass energy increases. Therefore, in the case of a black hole, there arises a situation where this gravitational binding energy must be considered.

### 2.2. In a gravitationally bound system such as the Sun-Earth system, when the orbit changes, stable orbit and the change in total energy

In a gravitationally bound system like the Sun-Earth system, as the orbit changes, the gravitational potential energy must change, and with it the total energy.

1) Initial state ($r_0$)
The Sun and Earth are gravitationally bound at a distance $r_0$.
The total mechanical energy is $E_{me} = K_0 + U_0$
In this case, the equivalent mass is $M_{eff,0} = M_{free} - M_{binding,0}$

2) Orbital change ($r_1 < r_0$)
When Earth moves to a lower orbit $r_1$, the gravitational potential energy decreases ($U_1 < U_0$).
To stabilize the system, the excess energy must be radiated away. As a result, the total energy of the system decreases, and so does the effective mass. That is, $M_{eff,1} < M_{eff,0}$
In the intermediate stage of the orbit change, according to the law of conservation of mechanical energy, the negative gravitational potential energy decreases, and the positive kinetic energy also increases, so the total energy is conserved.

However, in order for the system to become stable in a low orbit, the kinetic energy exceeding the kinetic energy required to move through the low orbit must be released outside the system. And, due to this energy release, the total mass or equivalent mass of the system decreases.

This process is also observed in the process of celestial bodies forming black holes through gravitational collapse. [6]

[2] Chapter 2 is almost the same as the contents of the previous paper. [5] It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

**2.3. Gravitational self-energy or total gravitational potential energy of an object**

The concept of gravitational self-energy is the total of gravitational potential energy ($U_{gp}$) possessed by a certain object $M$ itself. Since a certain object $M$ itself is a binding state of infinitesimal mass $dMs$, it involves the existence of gravitational potential energy among these $dMs$ and is the value of adding up these. $M = \sum dM$. The gravitational self-energy is equal to the minus sign of the gravitational binding energy. Only the sign is different because it defines the gravitational binding energy as the energy that must be supplied to the system to bring the bound object into a free state.

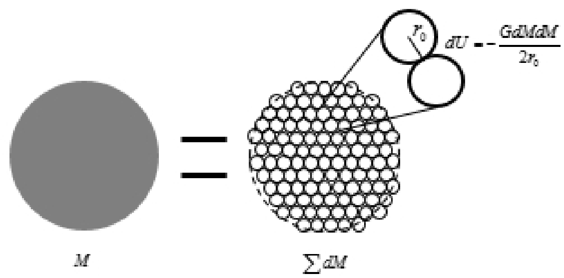


Figure 1: Since all mass $M$ is a set of infinitesimal mass $dMs$ and each $dM$ is gravitational source, too, there exists gravitational potential energy among each of $dMs$. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into $c^2$.

In the case of a spherical uniform distribution, total gravitational potential energy or gravitational binding energy ($-U_{gp}$) is

$$U_{gp} = -\frac{3}{5}\frac{G{M_{fr}}^2}{R} \quad (6)$$

$$U_{gp-Black-hole}(R = R_S) = -\frac{3}{5}\frac{G{M_{fr}}^2}{R} = -\frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{2GM_{fr}}{c^2})} = -0.3Mc^2 \quad (7)$$

Strictly speaking, the mass $M$ of a black hole is not the mass $M_{fr}$ in the free state, but the equivalent mass (or effective mass) including the binding energy. Here, $M_{fr}$ is used for simple estimation.

**2.4. In the case of black hole, the gravitational potential energy or gravitational binding energy must be taken into account**

In the general case, the value of gravitational potential energy is small enough to be negligible, compared to mass energy $Mc^2$. So generally, there was no need to consider gravitational potential energy. However the smaller $R$ becomes, the higher the absolute value of $U_{gp}$. For this reason, we can see that $U_{gp}$ is likely to offset the mass energy in a certain radius.

Thus, **looking for the size in which gravitational potential energy becomes equal to mass energy by comparing both,**

$$U_{gp} = |-\frac{3}{5}\frac{G{M_{fr}}^2}{R_{gp}}| = M_{fr}c^2 \quad (8)$$

$$R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} \quad (9)$$

This equation means that if mass $M_{fr}$ is uniformly distributed within the radius $R_{gp}$, negative gravitational potential energy for such an object equals positive mass energy in size. So, in case of such an object, positive mass energy and negative gravitational potential energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

**Comparing $R_{gp}$ with $R_S$, the radius of Schwarzschild black hole,**

In the rough estimate above, since the gravitational potential energy at the event horizon is $U_{gp} = -0.3M_{fr}c^2$, the mass energy of the black hole will be approximately $E_{BH} = 0.7M_{fr}c^2$.

$$R_S = \frac{2GM}{c^2} \approx \frac{2G(\frac{7}{10}M_{fr})}{c^2} = \frac{7}{5}\frac{GM_{fr}}{c^2} \quad (10)$$

$$R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} = \frac{3}{7}(\frac{7GM_{fr}}{5c^2}) \approx \frac{3}{7}R_S \approx 0.43R_S \tag{11}$$

This means that there exists the point where negative gravitational potential energy becomes equal to positive mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists approximately at the point $0.43R_S$.

Even if we apply the kinetic energy and virial theorem, the radius only decreases as negative energy cancels out positive energy, but the core claim that "there is a region that cannot be compressed any further due to negative gravitational potential energy" remains unchanged. Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem ($K = -U/2$),

$$R_{gp-vir} = \frac{1}{2}R_{gp} \tag{12}$$

### 2.5. There is no singularity at the center of a black hole, but rather a region of negative energy

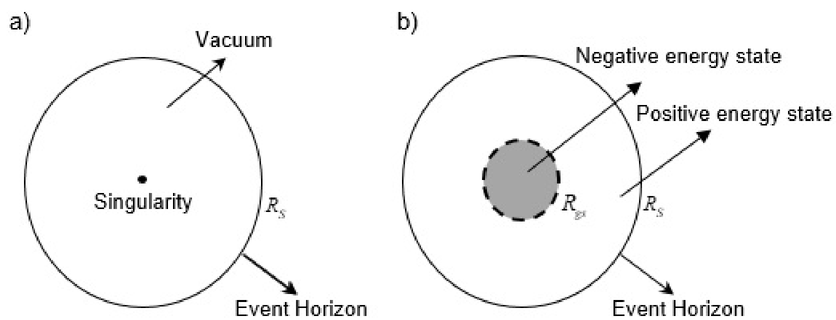


Figure 2: a) Existing Model. b) New Model. The area of within $R_{gp}(or R_{gp-vir})$ has gravitational potential energy (gravitational self-energy) of negative value, which is larger than mass energy of positive value. If $r$ is less than $R_{gp}(or R_{gp-vir})$, this area becomes negative energy(mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within $R_{gp}(or R_{gp-vir})$) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to $r = 0$. Therefore the distribution of mass (energy) can't be reduced to at least radius $R_{gp}(or R_{gp-vir})$.

The total energy of the system, including the gravitational potential energy or binding energy, is

$$E_T(R) = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{13}$$

Let's gradually reduce $R$ from when $R$ is infinite.

This is assuming that it is stationary after the orbital transition. If there is kinetic energy due to rotation in the orbit, we can reflect only half of the negative gravitational potential energy term by using the virial theorem. $K = -\frac{1}{2}U$

$$E_T(R = \infty) = M_{fr}c^2 - \frac{3}{5}\frac{GM_{fr}{}^2}{R} = M_{fr}c^2 \tag{14}$$

$$E_T(R = R_S) = M_{fr}c^2 - \frac{3}{5}\frac{GM_{fr}{}^2}{R} \approx M_{fr}c^2 - \frac{3}{5}\frac{GM_{fr}{}^2}{(\frac{2GM_{fr}}{c^2})} = M_{fr}c^2 - \frac{3}{10}M_{fr}c^2 = 0.7M_{fr}c^2 \tag{15}$$

$$E_T(R = R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{5}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{16}$$

$$E_T(R = \frac{1}{10}R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{50}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - 10M_{fr}c^2 = -9M_{fr}c^2 \tag{17}$$

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point $R = 0$. From the point $R_{gp}$(or $R_{gp-vir}$), gravity is 0, and when it enters into the area of $R_{gp}$(or $R_{gp-vir}$), total energy within $R_{gp}$(or $R_{gp-vir}$) region corresponds to negative values enabling anti-gravity to exist. This $R_{gp}$(or $R_{gp-vir}$) region comes to exert repulsive effects of gravity on the particles outside of it, therefore it interrupting the formation of singularity at the near the area $R = 0$.

However, it still can perform the function as black hole because the emitted energy will exist in a region larger than $r > R_{gp}(or R_{gp-vir})$. Since the emitted energy cannot escape the black hole, it is distributed in the region $R_{gp}(or R_{gp-vir}) < r < R_S$. Since the total energy of the entire range $(0 \leq r < R_S)$ inside the black hole is positive, it functions as a black hole.

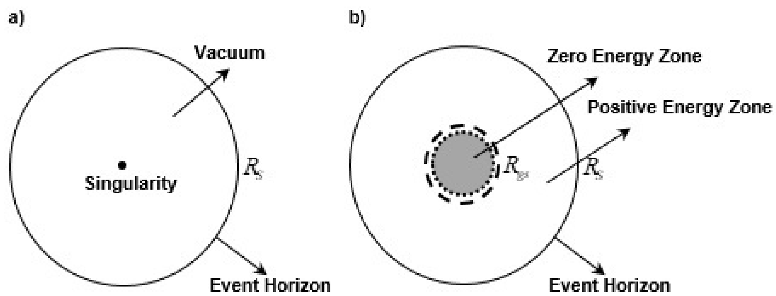


Figure 3: Internal structure of the black hole. a)Existing model b)New model. If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a zero (total) energy zone. Since there is a repulsive gravitational effect between negative energies (masses), the mass distribution expands, and when the mass distribution expands, the magnitude of the negative gravitational potential energy decreases, so it enters the positive energy state again. When the system (mass distribution) becomes a positive energy state, gravitational contraction will exist again. In this way, gravitational contraction and expansion will be repeated until the total energy of the system becomes 0, and finally it will stabilize at a state where the total energy is 0.

**If you have only the concept of positive energy, please refer to the following explanation.**
The total energy of the system, including the gravitational potential energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{18}$$

If, $R = R_{gp}$

$$E_T(R = R_{gp}) = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{R} = M_{fr}c^2 - \frac{3}{5}\frac{G{M_{fr}}^2}{(\frac{3}{5}\frac{GM_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{19}$$

From the point of view of mass defect, $r = R_{gp}(or R_{gp-vir})$ is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than $r = R_{gp}(or R_{gp-vir})$. So black hole doesn't have singularity.

**2.6. The gravitational singularity can be solved by gravity, not by quantum mechanics**

Think about a black hole with the size 10 billion times bigger than the solar mass. [7] [8] Schwarzschild radius of this black hole is $R_S = 3 \times 10^{10} km$ and $R_{gp}$ of this black hole $1.29 \times 10^{10} km$. Average density of this black hole is about $1.81 kg/m^3$. And average density of the Earth is about $5,200 kg/m^3$.

Is it a size that requires quantum mechanics? Black hole of this size is Newtonian mechanics' object and therefore, gravitational potential energy must be considered.

Let's reduce the mass of this black hole gradually and approach three times the solar mass, the smallest size of black hole where stars can be formed!

In case of the smallest black hole with three times the solar mass, [9] $R_S = 9km$. $R_{gp}$ of this object is as far as $3.87km$. In other words, **even in a black hole with smallest size that is made by the contraction of a star, the distribution of internal mass can't be reduced to at least radius 3.87km($R_{gs-vir} = 1.94km$).**

Before reaching quantum mechanical scales, the singularity problem is solved by gravity itself.

**2.7. The minimal size of existence**

[ **Existence = the sum of infinitesimal existences composing an existence** ]

A single mass $M$ for some object means that it can be expressed as $M = \sum dM$ and, for energy, $E = \sum dE$. The same goes for elementary particles, which can be considered a set of $dMs$, the infinitesimal mass.

$R_{gp}$ equation means that if masses are uniformly distributed within the radius $R_{gp}$, the size of negative binding energy becomes equal to that of mass energy. This can be the same that the rest mass, which used to be free for the mass defect effect caused by binding energy, has all disappeared. This means the total energy value representing "some existence" coming to 0 and "extinction of the existence". **Therefore, $R_{gp}$ is considered to act as "the minimal radius" or "a bottom line" of existence with some positive energy.**

Gravitational self-energy can provide the concept of minimal length or minimal ridius, one of the reasons for introducing string theory.

$$R_{\min} \geq R_{gp} = \frac{3}{5}\frac{GM}{c^2} \tag{20}$$

The important point here is that the minimum length or minimum radius is proportional to the mass $M$, i.e. energy $E$. In other words, there is a limit to compressing large energy into a small space.

## 3. Extension of general relativity and new solution [3]

**In all existing solutions, the mass term $M$ must be replaced by $(M_{fr} - M_{gp})$**

Let's think about the simplest case, the solution of the Schwarzschild black hole. When we find the Schwarzschild solution, we find the solution by making the Schwarzschild metric consistent with Newtonian mechanics in the Newton limit.

However, in this comparison with Newtonian mechanics, we use the following relationship:

$$\Phi_N = -\frac{GM}{r} \tag{21}$$

$$1 - \frac{C}{r} \approx 1 + \frac{2GM}{c^2 r} \tag{22}$$

$$C = 1 - \frac{2GM}{c^2} \tag{23}$$

[3] Chapter 3 is almost the same as the contents of the previous paper. [5] It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

Here, we should have asked one question. Is the mass $M$ the mass in the free state, $M_{free}$? Or is it the equivalent mass or the total mass $M^*$ because it is bound?

The mass of an object or the Earth in Newtonian mechanics is the equivalent mass or the total mass $M^*$. That is,

$M = M_{free} + M_{binding-energy}$

In a weak gravitational field, $M_{binding-energy}$ can be ignored because it is very small compared to $M_{free}$. For example, in the case of the Earth, the gravitational binding energy is about $4.17 \times 10^{-10}$ times the mass energy of the Earth. Therefore, in a weak gravitational field, $M \approx M_{free}$. However, in a strong gravitational field, the mass defect effect due to binding energy must be considered.

$M = M_{fr} - M_{gb} = M_{fr} - M_{gs} = M_{fr} - M_{gp}$

$M_{fr}$: Total mass when all components of the object are in a free state
$M_{gb}$: Equivalent mass of gravitational binding energy
$-M_{gs}$: Equivalent mass of gravitational self-energy
$-M_{gp}$: Equivalent mass of total gravitational potential energy of the object

In Newtonian mechanics, $-M_{gb}$, $-M_{gs}$, and $-M_{gp}$ have the same form and value in general situations. In addition, the energy of the gravitational field is also in the form of $U_{gf} = -k\frac{GM^2}{R}$, and depending on the integration interval, it can be the same as or different from the gravitational potential energy.

We can solve the problem of singularity by separating the term($-M_{gp}$) of gravitational potential energy (gravitational self-energy) from mass and including it in the solutions of field equation.

$M \rightarrow (M_{fr}) + (-M_{gp})$, In all existing solutions(Schwarzschild, Kerr, Reissner-Nordstr$\ddot{o}$m, ... ), the mass term $M$ must be replaced by $(M_{fr} - M_{gp})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{24}$$

**Schwarzschild-Choi solution** is

$$ds^2 = -(1 - \frac{2G(M_{fr} - M_{gp})}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M_{fr} - M_{gp})}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{25}$$

In the case of a spherical uniform distribution,

$$-M_{gp} = -\frac{3}{5}\frac{G{M_{fr}}^2}{Rc^2} \tag{26}$$

1) If $M_{fr} \gg |-M_{gp}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.
2) If $M_{fr} = |-M_{gp}|$

$$ds^2 = -c^2dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{27}$$

It has a flat space-time.
3) If $M_{fr} \ll |-M_{gp}|$, in other words if $0 \leq r \ll R_{gp}$,

$$ds^2 \simeq -(1 + \frac{2GM_{gp}}{c^2 r})c^2dt^2 + \frac{1}{(1 + \frac{2GM_{gp}}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{28}$$

In the domain of $0 \leq r \ll R_{gp}$,

The area of within $R_{gp}$ has gravitational potential energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. [10] So, we can assume that $-M_{gp}$ is almost evenly distributed. Therefore $-\rho_{gp}$ is constant. And we must consider the Shell Theorem.

$$-M_{gp} = \frac{4\pi r^3}{3}(-\rho_{gp}) \tag{29}$$

$$(1+\frac{2GM_{gp}}{c^2r})=1+\frac{2G(\frac{4\pi}{3}r^3\rho_{gp})}{c^2r}=1+\frac{8\pi G\rho_{gp}r^2}{3c^2} \tag{30}$$

$$ds^2\simeq-(1+\frac{8\pi G\rho_{gp}r^2}{3c^2})c^2dt^2+\frac{1}{(1+\frac{8\pi G\rho_{gp}r^2}{3c^2})}dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2 \tag{31}$$

If $r\to 0$,

$$ds^2\simeq -c^2dt^2+dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2 \tag{32}$$

There is no singularity.

# 4. Effective renormalization of gravity

### 4.1. Asymptotic Safety Method

Since Newton's constant $G_N$ has a negative mass dimension ($[G_N]=-2$ in 4 dimensions), it is difficult to renormalize because high-order infinities appear during perturbation expansion. However, the Asymptotic Safety method is the idea that even in theories such as quantum gravity, which are difficult to renormalize using traditional perturbation methods, a theory that can be predicted at UV (ultra-high energy) can be constructed using a nonperturbative method. [2] [3] [4]

Generally, the RG (Renormalization Group) flow for coupling $g_i$ is expressed as follows:
Beta function equation

$$\beta_i(g)=\frac{dg_i}{d\ln k} \tag{33}$$

The conventional beta function form of $G(k)$ in nonperturbative RG flow

$$\frac{dG(k)}{d\ln k}=\beta(G)=(d-2)G-cG^2 \tag{34}$$

d: spacetime dimension (usually d = 4 is assumed)
c: quantum correction factor, which varies depending on the details of the theory.
When solving the RG flow equation, the general solution of $G(k)$ is expressed as follows.

$$G(k)=\frac{G_0}{1+cG_0\ln(k/k_0)} \tag{35}$$

$G_0$: Initial Newton constant (value at low energy, usually known as $G_N$)
$k_0$ : Initial energy scale

### 4.2. Find $G(k)$ or $M_{eff}(k)$

Usually, when applying RG flow, $G(k)$ is used as follows.

$$F=-\frac{G(k)Mm}{r^2} \tag{36}$$

$G(k)$ is a function that varies with distance or energy, and k basically means energy scale (or momentum scale). $k\sim p\sim\frac{E}{c}$

Existing researchers are having difficulties while focusing on $G(k)$, but let's think a little differently,

$$F=-\frac{G(k)Mm}{r^2}=-\frac{G_N(\frac{G(k)}{G_N}M)m}{r^2}=-\frac{G_NM_{eff}m}{r^2} \tag{37}$$

In other words, **instead of changing the gravitational coupling constant $G(k)$, we can change it to changing the mass $M$.** $G_N$ is Newton's gravitational constant

$M\to M_{eff}=\frac{G(k)}{G_N}M$

Previously, when solving the singularity problem of black holes, we were able to know that the mass $M$ changes by including binding energy or gravitational potential energy. This is a method that utilizes that.

For a simple calculation, assuming a spherical uniform distribution,

$$M_{eff} = M_{fr} - M_{gp} = M_{fr} - \frac{3}{5}\frac{G_N {M_{fr}}^2}{Rc^2} \tag{38}$$

$$M_{eff}(k) = (1 - \frac{3}{5}\frac{G_N M_{fr}}{Rc^2})M_{fr} = (1 - \frac{3}{5}\frac{G_N \frac{E}{c^2}}{Rc^2})M_{fr} = (1 - \frac{3G_N}{5Rc^3}k)M_{fr} \tag{39}$$

**This can be reorganized and expressed in the form of $G(k)$.** [4]

$$F = -\frac{G(k)Mm}{r^2} = -\frac{G_N M_{eff} m}{r^2} = -\frac{G_N(1 - \frac{3G_N}{5Rc^3}k)M_{fr}m}{r^2} = -(1 - \frac{3G_N}{5Rc^3}k)\frac{G_N M_{fr} m}{r^2} \tag{40}$$

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N \tag{41}$$

If $B \equiv \frac{3G_N}{5Rc^3}$ is defined,

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N = (1 - Bk)G_N \tag{42}$$

If, $k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$

$G(k^*) = 0$ means that at that particular energy scale (for example, in the UV regime) the effective gravitational coupling vanishes. In other words, rather than diverging to infinity at high energies, the gravitational interaction actually disappears at that scale.

We want $\lim_{r\to 0} \frac{M_{eff}}{r^2} = 0$ to eliminate divergence. That is, $M_{eff}$ must decrease faster than $r^2$.

In the previous analysis, $R_{gp} = \frac{3}{5}\frac{GM_{fr}}{c^2} \approx \frac{3}{7}R_S$

At $R_{gs} = \frac{3}{5}\frac{GM_{fr}}{c^2}$ before r reaches 0, $M_{eff}$ goes to 0. Therefore, we can solve the gravitational divergence problem.

Also, in the low energy limit, $G(k) \to G_N$. And, in the $M_{eff}$ equation, when $r \gg R_S$, we can see that it is consistent with the Newton equation.

### 4.3. New beta function

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N = (1 - Bk)G_N \tag{43}$$

Differentiating both sides with respect to $\ln k$:

$$\frac{dG}{d\ln k} = \frac{d}{d\ln k}(1 - Bk)G_N = G_N(-B)\frac{dk}{d\ln k} \tag{44}$$

$$\frac{dk}{d\ln k} = k \tag{45}$$

$$\beta(G) = -BG_N k \tag{46}$$

At a specific $k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$, the value of the beta function is

$$\beta(G)|_{k=\frac{1}{B}} = -BG_N k = -G_N \tag{47}$$

[4] Here I am using the gravitational potential energy value from Newtonian mechanics. This may not be a completely accurate value. However, we use approximations in many fields. If you can find a better binding energy function or gravitational potential energy function, you can use that.

Therefore, the new $\beta(G)$ is, if we adjust the existing equation,

$$\beta(G) = (d-2)G - cG^2(1 - \frac{G}{G_N}) - G_N \tag{48}$$

Looking at this equation, if $k = k^* = \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R = R_{gp}$, $G(k) = 0$, and we get $\beta(0) = -G_N$, which is consistent with the previous result.

In the Asymptotic Safety method, when the energy goes to infinity ($k \to \infty$), we find a Non-Gaussian Fixed Point (NGFP) where the coupling constants have a specific finite value. However, in this model, G(k) does not simply converge to a finite value, but there is a point where $G(k) \to 0$ at a specific scale $R = R_{gp}$. This solves the divergence problem of gravity in a new way.

Also, when $k > \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R < R_{gp}$, we get $G(k) < 0$, a repulsive force occurs. This repulsive force prevents gravitational collapse, so that a singularity is not formed at the center of the black hole.

And, in the existing model, a quantum correction term was added to produce the Non-Gaussian Fixed Point (NGFP) and repulsive effects. However, in this model, if $k > \frac{1}{B}$, antigravity is generated, solving the singularity problem. Therefore, there is no need to introduce a quantum correction term.

Therefore, If the quantum correction term is deleted, the beta function becomes a simpler form.

$$\beta(G) = (d-2)G - G_N \tag{49}$$

$$G(k) = (1 - \frac{3G_N}{5Rc^3}k)G_N = (1 - \frac{3G_N M_{fr}}{5Rc^2})G_N = (1 - \frac{R_{gp}}{R})G_N \tag{50}$$

$R$ is the radius of the mass or energy distribution.

If $R > R_{gp}$, $G(k) > 0$, and attractive force acts.
If $R = R_{gp}$, $G(k) = 0$, and since the total energy is zero, gravity is also zero.
If $R < R_{gp}$, $G(k) < 0$, and repulsive force or antigravity acts.
This repulsive force prevents gravitational collapse and prevents the formation of a singularity at the center of the black hole.

Einstein-Hilbert action is

$$S = \frac{1}{16\pi G(k)} \int d^4x \sqrt{-g} R \tag{51}$$

$$S = \frac{1}{16\pi(1 - \frac{R_{gp}}{R})G_N} \int d^4x \sqrt{-g} R \tag{52}$$

$R_{gp}$: The radius where the gravitational potential energy (or binding energy) with a negative value is equal to the positive energy. In the case of a spherical uniform distribution, $R_{gs} = \frac{3}{5}\frac{GM_{fr}}{c^2} \approx \frac{3}{7}R_S$. The point where $G(k) = 0$ suggests an inflection point where the force changes from attractive to repulsive.

$R$: The radius of the mass distribution or energy distribution

# 5. Conclusion

In the case of a combined object, we must consider the binding energy. Thus, it is possible that the mass $M$ of the combined object varies with the binding energy. By using the fact that the mass $M$ changes to $M_{eff}$, which reflects the binding energy, we can obtain the RG flow and gravitational coupling constant $G(k)$.

When $k^* = \frac{5Rc^3}{3G_N}$, $G(k^*) = 0$, which means that gravity is zero. Therefore, we can solve the problem of gravitational divergence at high energy. In addition, when $R < R_{gp}$, $G(k) < 0$, so antigravity occurs, and it prevents substances from gravitational collapse and forming a singularity. Therefore, the singularity problem is also solved.

[ **Problems in physics and cosmology related to gravity** ]
1) Black hole singularity problem
2) Dark energy problem
3) Inflation cause or mechanism problem
4) Gravitational divergence problem and gravity renormalization problem

The mainstream recognizes all four problems as different problems, and therefore presents as-hoc hypotheses for each of them. But these four problems may actually be different aspects of one problem. That is, problems that can be explained by the existence of repulsion or antigravity in the gravitational problem.

The singularity problem, inflation problem, divergence problem, and dark energy seem to be on different scales, right? So it seems like multiple sources are needed?

The only thing we need is a mechanism that creates repulsion or anti-gravity in the problem of gravity.

Let's look at the gravitational potential energy or gravitational self-energy term.

$$U_{gp} = -\frac{3}{5}\frac{GM^2}{R} \tag{53}$$

The total energy, including the gravitational potential energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{54}$$

The negative gravitational potential energy term can be larger than the positive mass energy when the energy distribution radius $R$ is very small. It applies to the singularity problem, inflation problem, and divergence problem. [6] [11] [12]

The negative gravitational potential energy term can be larger than the positive mass energy when $M$ is very large. It applies to the dark energy problem, which accelerates the expansion of the universe. [13]

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