

A Theory of Hadron Structure Involving Higher Dimensional Matter

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Quarks may be made of higher dimensional matter. If true, then it follows that all hadrons are made of higher dimensional matter. The conventional thinking about quarks, that they are point particles, has not proven useful over the past 50 years. A more useful idea is that the six known quarks (u, d, s, c, b, t) are made of higher dimensional matter, the dimensions of which are respectively (1, 2, 3, 4, 5, 6), and each are equal to a volume of matter defined by the n-sphere surface volume formula of equal dimension. This gives theorists a mathematical handle, with which quarks and hadrons can be investigated.

Here is a list of the known quarks and their corresponding n-sphere surface volume formulae. In the table below, S_n is short for the surface volume formula of an n-sphere, so, S_2 is short for the surface volume formula of a 2-sphere (the circle). The surface "volume" of a 2-sphere is one dimensional. It's the circumference of the circle. So, take note: The dimension of the surface volume of an n-sphere is always one dimension less than the dimension of the interior volume of the n-sphere.

<u>n-Sphere Dimension</u>	<u>Quark</u>	<u>Surface Volume Formulae (S_n)</u>	<u>Dimension of Quark</u>
2	u - up	$S_2 = 2 \pi^1 r^1$	1
3	d - down	$S_3 = 4 \pi^1 r^2$	2
4	s - strange	$S_4 = 2 \pi^2 r^3$	3
5	c - charm	$S_5 = \frac{8}{3} \pi^2 r^4$	4
6	b - bottom	$S_6 = \pi^3 r^5$	5
7	t - top	$S_7 = \frac{16}{15} \pi^3 r^6$	6

Key to the Investigation Of Hadron Masses

The key to the investigation of hadron masses with n-sphere surface volumes is the formula ($xS_n h = \text{mass}$), where x is a number, S_n is the surface volume formula of a unit radius n-sphere, and h is Planck's constant's coefficient (6.62607015 J-s). When divided into experimental particle masses (given in units of MeV/c²) the result will be an integer, or an integer and a fraction (the denominator of which is a power of a prime number) if the hadron's matter is of the same dimension as the factoring unit's dimension. It has been tested on hundreds of experimentally determined particle masses and has been found to factor many of them convincingly. A case in point is the table of Lambda baryon masses below.

Experimental Masses Factored with n-Sphere Surface Volumes

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
49.5/7 S9h =	1390.9879	0.0121	1391	1	$\Lambda(1405)$	dddd, cdd, cc
50/7 S9h =	1405.0383	0.0617	1405.1	1.3/1.0	$\Lambda(1405)$	dddd, cdd, cc
54/7 S9h =	1517.4413	0.0587	1517.5	0.4	$\Lambda(1520)$	dddd, cdd, cc
56/7 S9h =	1573.6428	0.6428	1573	25	$\Lambda(1600)$	dddd, cdd, cc
84/7 S9h =	2360.4643	0.4643	2360	20	$\Lambda(2350)$	dddd, cdd, cc
90/7 S9h =	2529.0689	0.9311	2530	25	$\Lambda(2585)$	dddd, cdd, cc
92/7 S9h =	2585.2704	0.2704	2585	45	$\Lambda(2585)$	dddd, cdd, cc

The table shows some experimentally determined Lambda baryon masses, as listed by Particle Data Group on their website, and the corresponding n-sphere surface volume factoring of each. Notice the close agreement between ThrMass and ExpMass of the first three. All TM-EM's are within 0.06 MeV of each other. The last four in the list have much larger ExpErr's, but are also very close to their theoretical values. Those TM-EM's are also small. Less than 1.0 MeV. This close agreement between experimental Lambda baryon masses and theoretical masses obtained from hypersphere surface volume factoring is evidence that Lambda baryons are made of higher dimensional matter.

Predictive Power of the n-Sphere Factoring Technique

<u>Factoring</u>	<u>ThrMass</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
50/7 S9h =	1405.0383	1405.1	1.3/1.0	$\Lambda(1405)$	dddd, cdd, cc
51/7 S9h =	1433.1390	Undiscovered			
52/7 S9h =	1461.2398	Undiscovered			
53/7 S9h =	1489.3406	Undiscovered			
54/7 S9h =	1517.4413	1517.5	0.4	$\Lambda(1520)$	dddd, cdd, cc
55/7 S9h =	1545.5421	Undiscovered			
56/7 S9h =	1573.6428	1573	25	$\Lambda(1600)$	dddd, cdd, cc

N-sphere surface volume factoring is a powerful technique for predicting the existence of new particles. The particles in the table above, the ones NOT in bold type, have not yet been discovered, but could be if looked for, and when found, will assuredly have the masses predicted.

Determining the Correct Sn for Factoring from Quark Content

The dimensions of the quarks that have been discovered so far (u, d, s, c, b, t), are assumed to be (1, 2, 3, 4, 5, 6) dimensional respectively, and each has the shape of the surface of the n-sphere which has surface dimension equal to the dimension of the quark. Let's say you want to find some **dddd** pentaquarks among all the particle experimental mass data listed by Particle Data Group. Which dimension n-sphere surface volume formula (which **Sn**) should you use to factor the suspected experimental masses to determine if they are **dddd** pentaquarks or not? The n-sphere surface volume formula for the 'd' quark is $(4 \pi^1 r^2)$, which is the formula for the surface volume of a 3-sphere, **S3**, so multiply that by itself 5 times. You get $(1024 \pi^5 r^{10})$, which has the same π and r powers as the formula for the surface volume of an 11-sphere, so you would use **S11h** to search for **dddd** pentaquarks. Where should you look for **dddd** pentaquarks? Look in Particle Data Group's category called Light Unflavored Mesons between 1235 MeV and 2200 MeV. There are at least 100 of them in that mass range. They're mostly in 32nds of S11h, which is 4.29 MeV.

There Are More Than Six Quarks

Notice that a **dddd** pentaquark is generated from 'd' quarks, which are 2-dimensional, but **dddd** pentaquark matter is 10-dimensional because the surface volume of an 11-sphere is 10-dimensional. Do the 'd' quarks that form the **dddd** pentaquark retain their identity in the fully formed **dddd** pentaquark after it is made? They can't, because they are 2-dimensional, and the pentaquark's matter is 10-dimensional. (So called, **dddd** pentaquarks factor with S11h, which means they are made of 10d matter.)

Current quark theory of particle structure assumes that when a **dddd** pentaquark forms during a collision in an accelerator, the masses of the 'd' quarks just add together (Total Mass = 5d + KE), and the dimension of the collision reaction's product matter remains the same as the dimension of the reactant matter. In higher dimensional quark mass theory the masses of the colliding quarks also add together (Total Mass= 5d + KE), but they also change their dimension, in this case from 2-dimensional matter to 10-dimensional matter. In general, the dimension of the collision reaction's product matter is determined by the dimension of the surface volume formula that results from multiplying together all the surface volume formulae that are associated with each of the reacting quarks. (In the 'dddd' case, multiplying S3 together five times gives you S11, which is 10-dimensional.)

After the **dddd** pentaquark is formed, the 'd' quarks then no longer exist. Their matter has been transformed into 10-dimensional matter. The quarks that actually make up a **dddd** pentaquark are 10-dimensional quarks. How many are there in a **dddd** pentaquark? How much energy is needed to transform a given amount of 2d quark matter to 10d quark matter? These are good research questions that need answers.

So, to say that a **dddd** pentaquark has quark content **dddd** is a misnomer. It would be more correct to say that the five 'd' quarks that make a **dddd** pentaquark are the formation quarks, or genesis quarks of the particle. The quarks inside the particle after it is formed are made of 10-dimensional matter. They currently have no name. I suggest calling them 'q10' as it is the most logical name for them. This discovery of another quark beyond the six currently known begs the question: How many quarks are there?

How Many Quarks Are There?

Theoretically there are an infinite number of quarks - one for each n-sphere surface volume formula from 2 to infinity. How many have been found so far? The conventional wisdom is that there are only six, but examine the table below of some particles and their factorings. Particles with surface dimensions from 4 to 19, except for dimension 18, are listed (have been found), which means that quarks of all those dimensions have been found. So if we call the original six quarks (q1, q2, q3, q4, q5, q6), then the new ones found are (q7, q8, q9, q10, q11, q12, q13, q14, q15, q16, q17, and q19). The higher dimension quarks necessarily exist to explain the existence of the higher dimension hadrons.

Examples of Particles Constructed of Higher Dimensional Matter

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
4.4444	S5h = 775.071	0.051	775.02	.35	ρ (775)	d²
6.0000	S6h = 1232.698	0.202	1232.9	1.2	Δ (1232)	d²u, cu
6.0000	S7h = 1314.878	0.018	1314.86	0.20	Xi^o	d³, cd
26.6666	S8h = 5737.239	0.039	5737.2	0.7	B1 (5747)	d³u, cs
10.0000	S9h = 1967.053	0.053	1967.0	1.0	Ds	d⁴, cc
15.0000	S10h = 2534.634	0.034	2534.6	0.3	Ds1 (2536)	d⁴u, ccu
29.0000	S11h = 3982.461	0.039	3982.5	1.8	Zcs (3982)	d⁵, ccd
26.0000	S12h = 2760.433	0.333	2760.1	1.1	D3* (2750)	d⁵u, ccs
50.0000	S13h = 3922.028	0.013	3922.15	1.2	X (3930)	d⁶, ccc
64.0000	S14h = 3557.808	0.008	3557.8	1.2	Xc2 (1P)	d⁶u, ccuu
93.0000	S15h = 3525.820	0.020	3525.8	0.2	h1 (1P)	d⁷, cccd
2 ¹⁷ /900	S16h = 3633.472	0.128	3633.6	1.7	nc (2s)	d⁷u, cccs
384.0000	S17h = 6098.135	0.135	6098.0	1.7	Σb (6097)	d⁸, cccc
100.5000	S18h = 984.646	0.054	984.7	0.4	fo (980)	d⁸u, ccccu
280.0000	S20h = 957.590	0.090	957.5	0.2	η' (958)	d⁹u, ccccs

The finding of a hadron of a given dimension through hypersphere surface volume factoring means that quarks of that dimension exist (maybe - the quark idea is still evolving).

Conclusions

Hypersphere surface volume factoring of experimental hadron masses shows hadrons are made of higher dimensional matter. Hadrons comprised of matter from dimensions 4 to 19 have been found. That implies that there has to be more than six quarks, because the dimension of a hadron's matter is the same dimension as the matter in the quarks that comprise it, and the known quarks are only of dimensions 1 through 6.

Also, through the use of hypersphere surface volume factoring, it has been deduced that the currently believed quark content of hadrons is incorrect. Current quark content determinations of hadrons are based on the incorrect belief that the quarks inside hadrons are the same quarks (of the same dimension) as the quarks that form the hadron, and the same dimension as the quarks found in its decay products. That reasoning is incorrect. All current hadron quark content assignments that have been analysed so far with hypersphere surface volume factoring, shows that the currently believed quark content of all hadrons is incorrect.

Also, all hadrons factored so far, have been found to be of a single dimension of matter. Mixed dimension hadrons, such as 'uds', or 'cb', have not been found. It seems that a hadron can be formed from a mixed quark collision reaction, but the resulting hadron has only a single dimension of matter (i.e. only a single dimension of quarks).

More Proof That Hadrons Are Made of Higher Dimensional Matter

More examples of higher dimensional hadrons follows, from dimension 4 to 18. Also, there are four appendices of useful information.

S5h Factoring

4D Matter

(5-spheres have a 4D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
4.44444 S5h =	775.071	0.051	775.02	.35	p(775)	dd

S6h Factoring

5D Matter

(6-spheres have a 5D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
6.00000 S6h =	1232.698	0.202	1232.9	1.2	Δ(1232)	ddu, sd, cu
12.00000 S6h =	2465.397	0.003	2465.4	0.2	D2(2460)+	ddu, sd, cu
12.55555 S6h =	2579.535	0.035	2579.5	3.4	D(2550)0	ddu, sd, cu

S7h Factoring

6D Matter

(7-spheres have a 6D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
2.50000 S7h =	547.866	0.001	547.865	0.031	η	ddd, cd
25/7 S7h =	782.665	0.015	782.65	0.12	ω	ddd, cd
6.00000 S7h =	1314.878	0.018	1314.86	0.20	Xi⁰	ddd, cd
6.03125 S7h =	1321.726	0.016	1321.71	0.07	Xi⁻	ddd, cd
7.00000 S7h =	1534.024	0.376	1534.4	1.1	Xi(1530)⁻	ddd, cd
7680/900 S7h =	1870.049	0.049	1870.0	1.0	D+	ddd, cd
8256/900 S7h =	2010.303	0.043	2010.26	0.05	D*(2010)+	ddd, cd

S8h Factoring 7D Matter

(8-spheres have a 7D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
8.00	S8h = 1721.171	0.171	1721	13	a2 (1700)	ddd <u>u</u> , cs , bd
64/7	S8h = 1967.053	0.053	1967.0	1.0	Ds	ddd <u>u</u> , cs , bd
80/7	S8h = 2458.817	0.083	2458.9	1.5	Ds (2460)	ddd <u>u</u> , cs , bd
50255/2048	S8h = 5279.388	0.008	5279.38	0.11	B+	ddd <u>u</u> , cs , bd
50257/2048	S8h = 5279.598	0.018	5279.58	0.15	Bo	ddd <u>u</u> , cs , bd
2560/96	S8h = 5737.239	0.039	5737.2	0.7	B2 (5747) +	ddd <u>u</u> , cs , bd
2561/96	S8h = 5739.480	0.020	5739.5	0.7	B2 (5747) o	ddd <u>u</u> , cs , bd

S9h Factoring 8D Matter

(9-spheres have an 8D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
10.00000	S9h = 1967.053	0.053	1967.0	1.0	Ds	ddd <u>u</u> , cc
13.66666	S9h = 2688.306	0.306	2688	4	Ds (2700)	ddd <u>u</u> , cc
13.77777	S9h = 2710.162	0.162	2710	2	Ds (2700)	ddd <u>u</u> , cc
29.00000	S9h = 5704.455	0.455	5704	4	Bj (5732)	ddd <u>u</u> , cc

S10h Factoring 9D Matter

(10-spheres have a 9D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
12.5000	S10h = 2112.195	0.005	2112.2	0.4	Ds*	ddd <u>u</u> , ccu
12.4666	S10h = 2106.563	0.037	2106.6	2.1	Ds*	ddd <u>u</u> , ccu
15.0000	S10h = 2534.634	0.034	2534.6	0.3	Ds1 (2536)	ddd <u>u</u> , ccu
15.2222	S10h = 2572.185	0.015	2572.2	0.3	Ds2 (2573)	ddd <u>u</u> , ccu
15.3333	S10h = 2590.960	0.040	2591	6	Dso (2590)	ddd <u>u</u> , ccu
25.6666	S10h = 4337.041	0.041	4337	7	Pc (4337)	ddd <u>u</u> , ccu
26.3333	S10h = 4449.692	0.108	4449.8	1.7	Pc (4450)	ddd <u>u</u> , ccu
26.6666	S10h = 4506.017	0.017	4506	11	Xco (4500)	ddd <u>u</u> , ccu

S11h Factoring

10D Matter

(11-spheres have a 10D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
9-1/128	S11h = 1234.863	0.137	1235	15	b1 (1235)	dddd, ccd
9.00000	S11h = 1235.936	0.064	1236	16	b1 (1235)	dddd, ccd
9+1/128	S11h = 1237.009	0.009	1237	7	b1 (1235)	dddd, ccd
15.875	S11h = 2180.054	0.054	2180	8	Xc0 (2170)	dddd, ccd
15.90625	S11h = 2184.345					
15.9375	S11h = 2188.637	0.637	2188	10	Xc0 (2170)	dddd, ccd
15.96875	S11h = 2192.928	0.072	2193	2	Xc0 (2193)	dddd, ccd
16.	S11h = 2197.219	0.181	2197.4	4.4	Xc0 (1P)	dddd, ccd
16.03125	S11h = 2201.511	0.511	2201	19	Xc0 (1P)	dddd, ccd
16.0625	S11h = 2205.802	0.198	2206	12	Xc0 (1P)	dddd, ccd
16.09375	S11h = 2210.094					
16.125	S11h = 2214.384	0.384	2214	20	Xc0 (1P)	dddd, ccd
16.3125	S11h = 2240.134	0.934	2239.2	7.1	X (2240)	dddd, ccd
17.875	S11h = 2454.706	0.294	2455	3	D2* (2460)⁰	dddd, ccd
17.90625	S11h = 2458.998	0.002	2459	3	D2* (2460)⁰	dddd, ccd
17.9375	S11h = 2463.289	0.011	2463.3	0.6	D2* (2460)⁰	dddd, ccd
29.000	S11h = 3982.461	0.039	3982.5	1.8	Zcs (3982)	dddd, ccd
29.375	S11h = 4033.958	0.042	4034	6	X (4040)	dddd, ccd
29.500	S11h = 4051.124	0.124	4051	14	X (4050)	dddd, ccd
31.125	S11h = 4274.279	0.121	4274.4	8.4		
32.125	S11h = 4411.605	0.605	4411	7	Ψ (4415)	dddd, ccd
32.250	S11h = 4428.771	0.229	4429	9	Ψ (4415)	dddd, ccd
32.33333	S11h = 4440.215	0.085	4440.3	1.3	Pc (4440)	dddd, ccd
34.000	S11h = 4669.092	0.229	4669	21	Ψ (4660)	dddd, ccd

S12h Factoring

11D Matter

(12-spheres have an 11D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
26.00000 S12h =	2760.433	0.333	2760.1	1.1	D3* (2750)	d⁵u, ccs
27.00000 S12h =	2866.605	0.005	2866.6	AVG	Ds3 (2860)⁺	d⁵u, ccs
28.00000 S12h =	2972.775	0.975	2971.8	8.7	D (3000)⁰	d⁵u, ccs
28.33333 S12h =	3008.165	0.065	3008.1	4.0	D (3000)⁰	d⁵u, ccs
28.66666 S12h =	3043.555	0.444	3044	8	Dsj (3040)⁰	d⁵u, ccs
30.06666 S12h =	3510.705	0.005	3510.71	0.04	Xc1 (1P)	d⁵u, ccs
35.55555 S12h =	3774.952	0.548	3775.5	2.4	Ψ (3770)	d⁵u, ccs
36.00000 S12h =	3822.139	0.061	3822.2	1.2	Ψ2 (3823)	d⁵u, ccs

Note: 9 x **35.55555** = 320 = 256 + 64
 9 x **36.00000** = 324 = 256 + 64 + 4

S13h Factoring

12D Matter

(13-spheres have a 12D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
16.0000 S13h =	1255.049	0.049	1255	7	a1 (1260)	d⁶, ccc
49-8/90 S13h =	3915.056	0.056	3915	3	X (3930)	d⁶, ccc
50.0000 S13h =	3922.028	0.013	3922.15	1.2	X (3930)	d⁶, ccc
50+8/90 S13h =	3929.001	0.001	3929	5	X (3930)	d⁶, ccc

S14h Factoring

13D Matter

(14-spheres have a 13D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
40.00000 S14h =	2223.630	0.270	2223.9	2.5	fj (2220)	d⁶u, cccu
41.50000 S14h =	2307.016	0.016	2307	6	ρ5 (2350)	d⁶u, cccu
61.44000 S14h =	3415.496	0.004	3415.5	0.4	Xc0 (1P)	d⁶u, cccu
64.00000 S14h =	3557.808	0.008	3557.8	1.2	Xc2 (1P)	d⁶u, cccu

Note: **6144** = 4096 + 2048
6400 = 4096 + 2048 + 256

S15h Factoring

14D Matter

(15-spheres have a 14D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
48.0000	S15h = 1819.778	0.378	1819.4	3.1	Xi (1820)	d⁷, cccd
93.0000	S15h = 3525.820	0.020	3525.8	0.2	h1 (1P)	d⁷, cccd
113.0000	S15h = 4284.061	0.061	4284	17	Y (4260)	d⁷, cccd

S16h Factoring

15D Matter

(16-spheres have a 15D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
2 ¹⁷ /900	S16h = 3633.472	0.128	3633.6	1.7	nc (2s)	d⁷u, cccs
2 ¹⁷ +128 /900	S16h = 3637.020	0.020	3637.0	5.7	nc (2s)	d⁷u, cccs
2 ¹⁷ +256 /900	S16h = 3640.569	0.069	3640.5	3.2	nc (2s)	d⁷u, cccs

S17h Factoring

16D Matter

(17-spheres have a 16D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
222.0000	S17h = 3525.484	0.084	3525.40	0.13	hc (1P)	d⁸, cccc
384.0000	S17h = 6098.135	0.135	6098.0	1.7	Σb (6097)	d⁸, cccc
668.0000	S17h = 10608.215	0.115	10608.1	1.2	Zb (10610)	d⁸, cccc

S18h Factoring

17D Matter

(18-spheres have a 17D surface)

<u>Factoring</u>	<u>ThrMass</u>	<u>TM-EM</u>	<u>ExpMass</u>	<u>ExpErr</u>	<u>Particle</u>	<u>Formation Quarks</u>
99.000	S18h = 969.950	0.150	969.8	4.5	fo (980)	d⁸u, ccccu
99.750	S18h = 977.296	0.004	977.3	0.9	fo (980)	d⁸u, ccccu
100.250	S18h = 982.197	0.003	982.2	0.6	fo (980)	d⁸u, ccccu
100.500	S18h = 984.646	0.054	984.7	0.4	fo (980)	d⁸u, ccccu
101.250	S18h = 991.994	0.006	992.0	8.5	fo (980)	d⁸u, ccccu
101.375	S18h = 993.219	0.019	993.2	6.5	fo (980)	d⁸u, ccccu

APPENDIX A

Quark Assignments
to
n-Sphere Surface Volume Formulae

<u>Sphere Dimension</u>	<u>Quark Names</u>			<u>Corresponding n-Sphere Surface Formula</u>
	<u>Old</u>	<u>New</u>		
2	u	q1	=	$2 \pi^1 r^1$
3	d	q2	=	$4 \pi^1 r^2$
4	s	q3	=	$2 \pi^2 r^3$
5	c	q4	=	$8/3 \pi^2 r^4$
6	b	q5	=	$\pi^3 r^5$
7	t	q6	=	$16/15 \pi^3 r^6$
8	-----	q7	=	$1/3 \pi^4 r^7$
9	-----	q8	=	$32/105 \pi^4 r^8$
10	-----	q9	=	$1/12 \pi^5 r^9$
11	-----	q10	=	$64 / 945 \pi^5 r^{10}$
12	-----	q11	=	$1 / 60 \pi^6 r^{11}$
13	-----	q12	=	$128 / 10395 \pi^6 r^{12}$
14	-----	q13	=	$1 / 360 \pi^7 r^{13}$
15	-----	q14	=	$256 / 135135 \pi^7 r^{14}$
16	-----	q15	=	$1 / 2520 \pi^8 r^{15}$
17	-----	q16	=	$512 / 2027025 \pi^8 r^{16}$
18	-----	q17	=	$1 / 20160 \pi^9 r^{17}$
19	-----	q18	=	$1024 / 34459425 \pi^9 r^{18}$
20	-----	q19	=	$1 / 181440 \pi^{10} r^{19}$
21	-----	q20	=	$2048 / 654729075 \pi^{10} r^{20}$

APPENDIX B

n-Sphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

<u>Sphere</u> <u>Dimension</u>	<u>S_n</u>	<u>Surface</u> <u>Volume Formula</u>	<u>(π, r)</u> <u>Powers</u>
2	S2 =	2 $\pi^1 r^1$	(1, 1)
3	S3 =	4 $\pi^1 r^2$	(1, 2)
4	S4 =	2 $\pi^2 r^3$	(2, 3)
5	S5 =	8/3 $\pi^2 r^4$	(2, 4)
6	S6 =	$\pi^3 r^5$	(3, 5)
7	S7 =	16/15 $\pi^3 r^6$	(3, 6)
8	S8 =	1/3 $\pi^4 r^7$	(4, 7)
9	S9 =	32/105 $\pi^4 r^8$	(4, 8)
10	S10 =	1/12 $\pi^5 r^9$	(5, 9)
11	S11 =	64 / 945 $\pi^5 r^{10}$	(5, 10)
12	S12 =	1 / 60 $\pi^6 r^{11}$	(6, 11)
13	S13 =	128 / 10395 $\pi^6 r^{12}$	(6, 12)
14	S14 =	1 / 360 $\pi^7 r^{13}$	(7, 13)
15	S15 =	256 / 135135 $\pi^7 r^{14}$	(7, 14)
16	S16 =	1 / 2520 $\pi^8 r^{15}$	(8, 15)
17	S17 =	512 / 2027025 $\pi^8 r^{16}$	(8, 16)
18	S18 =	1 / 20160 $\pi^9 r^{17}$	(9, 17)
19	S19 =	1024 / 34459425 $\pi^9 r^{18}$	(9, 18)
20	S20 =	1 / 181440 $\pi^{10} r^{19}$	(10, 19)
21	S21 =	2048 / 654729075 $\pi^{10} r^{20}$	(10, 20)

APPENDIX C

Values of n-Sphere Surface Volume
Units of Factorization

(Dimension 2 - Dimension 21)

<u>Sphere Dimension</u>	<u>Unit of Factorization</u>	<u>Formula</u>	<u>Value (MeV/c²)</u>
2	S2h =	$2 \pi^1 r^1 h =$	41.63282661
3	S3h =	$4 \pi^1 r^2 h =$	83.26565322
4	S4h =	$2 \pi^2 r^3 h =$	130.7933822
5	S5h =	$8/3 \pi^2 r^4 h =$	174.3911763
6	S6h =	$\pi^3 r^5 h =$	205.4497644
7	S7h =	$16/15 \pi^3 r^6 h =$	219.1464153
8	S8h =	$1/3 \pi^4 r^7 h =$	215.1464901
9	S9h =	$32/105 \pi^4 r^8 h =$	196.7053624
10	S10h =	$1/12 \pi^5 r^9 h =$	168.9756582
11	S11h =	$64 / 945 \pi^5 r^{10} h =$	137.3262492
12	S12h =	$1 / 60 \pi^6 r^{11} h =$	106.1705373
13	S13h =	$128 / 10395 \pi^6 r^{12} h =$	78.44057013
14	S14h =	$1 / 360 \pi^7 r^{13} h =$	55.59076334
15	S15h =	$256 / 135135 \pi^7 r^{14} h =$	37.91204905
16	S16h =	$1 / 2520 \pi^8 r^{15} h =$	24.94907624
17	S17h =	$512 / 2027025 \pi^8 r^{16} h =$	15.88056197
18	S18h =	$1 / 20160 \pi^9 r^{17} h =$	9.797479330
19	S19h =	$1024 / 34459425 \pi^9 r^{18} h =$	5.869441980
20	S20h =	$1 / 181440 \pi^{10} r^{19} h =$	3.419965454
21	S21h =	$2048 / 654729075 \pi^{10} r^{20} h =$	1.940989032

APPENDIX D

Smallest Formation Quarks per n-Sphere

(Dimension 2 - Dimension 21)

<u>Sphere Dimension</u>	<u>S_n</u>	<u>Surface Volume Formula</u>	<u>(π, r) Powers</u>	<u>Formation Quarks</u>	
2	S2 =	2 π ¹ r ¹	(1, 1)	u	
3	S3 =	4 π ¹ r ²	(1, 2)	d	
4	S4 =	2 π ² r ³	(2, 3)	du	di-quarks
5	S5 =	8/3 π ² r ⁴	(2, 4)	dd	
6	S6 =	π ³ r ⁵	(3, 5)	ddu	tri-quarks
7	S7 =	16/15 π ³ r ⁶	(3, 6)	ddd	
8	S8 =	1/3 π ⁴ r ⁷	(4, 7)	dddu	tetra-quarks
9	S9 =	32/105 π ⁴ r ⁸	(4, 8)	dddd	
10	S10 =	1/12 π ⁵ r ⁹	(5, 9)	ddddu	penta-quarks
11	S11 =	64 / 945 π ⁵ r ¹⁰	(5, 10)	ddddd	
12	S12 =	1 / 60 π ⁶ r ¹¹	(6, 11)	dddddu	hexa-quarks
13	S13 =	128 / 10395 π ⁶ r ¹²	(6, 12)	dddddd	
14	S14 =	1 / 360 π ⁷ r ¹³	(7, 13)	ddddddu	hepta-quarks
15	S15 =	256 / 135135 π ⁷ r ¹⁴	(7, 14)	ddddddd	
16	S16 =	1 / 2520 π ⁸ r ¹⁵	(8, 15)	dddddddu	octa-quarks
17	S17 =	512 / 2027025 π ⁸ r ¹⁶	(8, 16)	dddddddd	
18	S18 =	1 / 20160 π ⁹ r ¹⁷	(9, 17)	ddddddduu	nona-quarks
19	S19 =	1024 / 34459425 π ⁹ r ¹⁸	(9, 18)	ddddddddu	
20	S20 =	1 / 181440 π ¹⁰ r ¹⁹	(10, 19)	ddddddduuu	deca-quarks
21	S21 =	2048 / 654729075 π ¹⁰ r ²⁰	(10, 20)	ddddddduuuu	

References

[1] P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update