A Theory of Hadron Structure Involving Higher Dimensional Matter

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Quarks may be made of higher dimensional matter. If true, then it follows that all hadrons are made of higher dimensional matter. The conventional thinking about quarks, that they are point particles, has not proven useful over the past 60 years. A more useful idea is that the six known quarks (u, d, s, c, b, t) are made of matter of different dimensions - those dimensions being (1, 2, 3, 4, 5, 6) respectively - and each quark has a volume defined by the n-sphere surface volume formula of equal dimension. This gives theorists a mathematical handle, with which quarks and hadrons can be investigated.

Here is a list of the known quarks and their corresponding n-sphere surface volume formulae. In the table below, Sn is short for the surface volume formula of an n-sphere, so, S2 is short for the surface volume formula of a 2-sphere (the circle). The surface "volume" of a 2-sphere is one dimesional. It's the circumference of the circle. So, take note: The dimension of the *surface volume of an n-sphere* is always one dimension less than the dimension of the *interior volume of the n-sphere*.

n-Sphere Dimension	Quark	Surface Volume Formulae (Sn)	Dimension of Quark
2	u - up	S2 = $2\ \pi^1 r^1$	1
3	d - down	S3 = $4\ \pi^1 r^2$	2
4	s - strange	S4 = $2\ \pi^2 r^3$	3
5	c - charrm	S5 = $8/3\ \pi^2 r^4$	4
6	b - bottom	S6 = $\pi^3 r^5$	5
7	t - top	S7 = $16/15\ \pi^3 r^6$	6

Key to the Investigation Of Hadron Masses

The key to the investigation of hadron masses with n-sphere surface volumes is the formula, **m=xSnh**, where **m** is the mass of the hadron in units of MeV, **x** is a number, **Sn** is the value of the surface volume formula of a unit radius n-sphere, and **h** is Planck's constant's coefficient, but with different units. Here it has units of MeV, not J-s. (The factoring formula, **m=xSnh**, can be derived from Planck's Energy-Frequency Relation , **E=hf**, and how **h** gets its units changed to MeV, and its factor of (10^{-34}) removed is explained in the derivation of **m=xSnh** on page 4.) When divided into experimental hadron masses (given in units of MeV) the result will be an integer, or an integer and a fraction, if the hadron's matter is of the same dimension as the factoring unit's dimension. It has been tested on hundreds of experimental hadron masses and has been found to factor many of them convincingly. A case in point is the table of *Lambda baryon* masses below.

Experimental Masses Factored with n-Sphere Surface Volumes

Factoring			*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
49.5/7	**S9h**	**=**	1390.9879	0.0121	**1391**	1	Λ(1405)	dddd, cdd, cc
50/7	**S9h**	=	1405.0383	0.0617	**1405.1**	1.3/1.0	Λ(1405)	dddd, cdd, cc
54/7	**S9h**	=	1517.4413	0.0587	**1517.5**	0.4	Λ(1520)	dddd, cdd, cc
56/7	**S9h**	=	1573.6428	0.6428	**1573**	25	Λ(1600)	dddd, cdd, cc
84/7	**S9h**	=	2360.4643	0.4643	**2360**	20	Λ(2350)	dddd, cdd, cc
90/7	**S9h**	=	2529.0689	0.9311	**2530**	25	Λ(2585)	dddd, cdd, cc
92/7	**S9h**	=	2585.2704	0.2704	**2585**	45	Λ(2585)	dddd, cdd, cc

The table shows some experimentally determined *Lambda baryon* masses, as listed by *Particle Data Group* on their website, and the corresponding n-sphere surface volume factoring of each. Notice the close agreement between ThrMass and ExpMass of the first three. All TM-EM's are within 0.06 MeV of each other. The last four in the list have much larger ExpErr's, but are also very close to their theoretical values. Those TM-EM's are also small. Less than 1.0 MeV. This close agreement between experimental Lambda baryon masses and theoretical masses obtained from hypersphere surface volume factoring is evidence that Lambda baryons are made of higher dimensional matter.

Predictive Power of the n-Sphere Factoring Technique

Factoring		*ThrMass*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
50/7 S9h	=	1405.0383	**1405.1**	1.3/1.0	Λ(1405)	dddd, cdd, cc
51/7 S9h	=	1433.1390	*Undiscovered*			
52/7 S9h	=	1461.2398	*Undiscovered*			
53/7 S9h	=	1489.3406	*Undiscovered*			
54/7 S9h	=	1517.4413	**1517.5**	0.4	Λ(1520)	dddd, cdd, cc
55/7 S9h	=	1545.5421	*Undiscovered*			
56/7 S9h	=	1573.6428	**1573**	25	Λ(1600)	dddd, cdd, cc

N-sphere surface volume factoring is a powerfull techique for predicting the existence of new particles. The particles in the table above, the ones NOT in bold type, have not yet been discovered, but could be if looked for, and when found, will assuredly have the masses predicted.

Determining the Correct Sn for Factoring from Quark Content

The dimensions of the quarks that have been discovered so far (u, d, s, c, b, t), are assumed to be (1, 2, 3, 4, 5, 6) dimensional respectively, and each has the shape of *the surface of the n-sphere* which has surface dimension equal to the dimension of the quark. Let's say you want to find some **ddddd** pentaquarks among all the particle experimental mass data listed by *Particle Data Group*. Which dimension n-sphere surface volume formula (which **Sn**) should you use to factor the suspected experimental masses to determine if they are **ddddd** pentaquarks or not? The n-sphere surface volume formula for the 'd' quark is ($4\ \pi^1\ r^2$, which is the formula for the surface volume of a 3-sphere, **S3**), so multiply that by itself 5 times. You get ($1024\ \pi^5\ r^{10}$), which has the same π and r powers as the formula for the surface volume of an 11-sphere, so you would use **S11h** to search for **ddddd** pentaquarks. Where should you look for **ddddd** pentaquarks? Look in *Particle Data Group's* category called *Light Unflavored Mesons* between 1235 MeV and 2200 MeV. There are at least 100 of them in that mass range. They're mostly in 32nds of S11h, which is 4.29 MeV.

There Are More Than Six Quarks

Notice that a ddddd pentaquark is generated from 'd' quarks, which are 2-dimensional, but ddddd pentaquark matter is 10 dimensional because the surface volume of an 11-sphere is 10-dimensional. Do the 'd' quarks that form the ddddd pentaquark retain their identity in the fully formed ddddd pentaquark after it is made? They can't, because they are 2-dimensional, and the pentaquark's matter is 10-dimensional. (So called, ddddd pentaquarks factor with S11h, which means they are made of 10d matter.)

Current quark theory of particle structure assumes that when a ddddd pentaquark forms during a collision in an accelerator, the masses of the 'd' quarks just add together (Total Mass = 5d + KE), and the dimension of the collision reaction's product matter remains the same as the dimension of the reactant matter. In *higher dimension quark mass theory* the masses of the colliding quarks also add together (Total Mass= 5d + KE), but they also change their dimension, in this case from 2-dimentional matter to 10-dimensional matter. In general, the dimension of the collision reaction's product matter is determined by the dimension of the surface volume formula that results from multiplying together all the surface volume formulae that are associated with each of the reacting quarks. (In the 'ddddd' case, multiplying S3 together five times gives you S11, which is 10-dimensional.)

After the ddddd pentaquark is formed, the 'd' quarks then no longer exist. Their matter has been transformed into 10-dimensional matter. The quarks that actually make up a ddddd pentaquark are 10-dimensional quarks. How many are there in a ddddd pentaquark? How much energy is needed to transform a given amount of 2d quark matter to 10d quark matter? These are good research questions that need answers.

So, to say that a ddddd pentaquark has quark content ddddd is a misnomer. It would be more correct to say that the five 'd' quarks that make a ddddd pentaquark are the *formation quarks,* or *genesis quarks* of the particle. The quarks inside the particle after it is formed are made of 10-dimensional matter. They currently have no name. I suggest calling them 'q10' as it is the most logical name for them. This discovery of another quark beyond the six currently known begs the question: How many quarks are there?

How Many Quarks Are There?

Theoreticaly there are an infinite number of quarks - one for each n-sphere surface volume formula from 2 to infinity. How many have been found so far? The conventional wisdom is that there are only six, but examine the table below of some particles and their factorings. Particles with surface dimensions from 4 to 19, except for dimension 18, are listed (have been found), which means that quarks of all those dimensions have been found. So if we call the original six quarks (q1, q2, q3, q4, q5, q6), then the new ones found are (q7, q8, q9, q10, q11, q12, q13, q14, q15, q16, q17, and q19). The higher dimension quarks necessarily exist to explain the existence of the higher dimension hadrons.

Examples of Particles Constructed of Higher Dimensional Matter

Factoring		*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
4.4444	**S5h** =	775.071	0.051	**775.02**	.35	**ρ(775)**	**d^2**
6.0000	**S6h =**	1232.698	0.202	**1232.9**	1.2	**Δ(1232)**	**d^2u, cu**
6.0000	**S7h =**	1314.878	0.018	**1314.86**	0.20	**Xi°**	**d^3, cd**
26.6666	**S8h** =	5737.239	0.039	**5737.2**	0.7	**B1(5747)**	**d^3u, cs**
10.0000	**S9h** =	1967.053	0.053	**1967.0**	1.0	**Ds**	**d^4, cc**
15.0000	**S10h** =	2534.634	0.034	**2534.6**	0.3	**Ds1(2536)**	**d^4u, ccu**
29.0000	**S11h =**	3982.461	0.039	**3982.5**	1.8	**Zcs(3982)**	**d^5, ccd**
26.0000	**S12h** =	2760.433	0.333	**2760.1**	1.1	**D3*(2750)**	**d^5u, ccs**
50.0000	**S13h** =	3922.028	0.013	**3922.15**	1.2	**X(3930)**	**d^6, ccc**
64.0000	**S14h =**	3557.808	0.008	**3557.8**	1.2	**Xc2(1P)**	**d^6u, cccu**
93.0000	**S15h =**	3525.820	0.020	**3525.8**	0.2	**h1(1P)**	**d^7, cccd**
2^{17}/900	**S16h =**	3633.472	0.128	**3633.6**	**1.7**	**nc(2s)**	**d^7u, cccs**
384.0000	**S17h =**	6098.135	0.135	**6098.0**	1.7	**Σb(6097)**	**d^8, cccc**
100.5000	**S18h =**	984.646	0.054	**984.7**	0.4	**fo(980)**	**d^8u, ccccu**
280.0000	**S20h** =	957.590	0.090	**957.5**	0.2	**η'(958)**	**d^9u, ccccs**

The finding of a hadron of a given dimension through hypersphere surface volume factoring means that quarks of that dimension exist. (But not point particle quarks - waves more likely. What a quark is exactly has proven a hard thing to pin down.)

Conclusions

Hypersphere surface volume factoring of experimental hadron masses shows hadrons are made of higher dimensional matter. Hadrons comprised of matter from dimensions 4 to 19 have been found. That implies that there has to be more than six quarks, because the dimension of a hadron's matter is the same dimension as the matter in the quarks that comprise it, and the known quarks are only of dimensions 1 through 6.

Also, through the use of hypersphere surface volume factoring, it has been deduced that the currently believed quark content of hadrons is incorrect. Current quark content determinations of hadrons are based on the incorrect belief that the quarks inside hadrons are the same quarks (of the same dimension) as the quarks that form the hadron, and the same dimension as the quarks found in its decay products. That reasoning is incorrect. All current hadron quark content assignments that have been analysed so far with hypersphere surface volume factoring, shows that they are all incorrect.

Also, all hadrons factored so far, have been found to be of a single dimension of matter. Mixed dimension hadrons, such as 'uds', or 'cb', have not been found. It seems that a hadron can be formed from a mixed dimension quark collision reaction, but the resulting hadron has only a single dimension of matter (i.e. only a single dimension of quarks). In all honesty, however, the technique used (m=xSnh) might be blind to their detection, but if they do exist, they are rare, because most hadrons factor with m=xSnh.

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

Derivation of the *Hypersphere Surface Volume* Factoring Formula

$$\mathbf{m}_{\text{MeV}} = \mathbf{h}_{\text{MeV}}(\mathbf{xSn})$$

The HSSV factoring formula, **m = h** (**xSn**), which is used to discover hadron dimensions and exact masses, can be derived from Planck's Energy-Frequency Relation: **E = hf.** The key to the derivation is associating a frequency with a unit of hypervolume. A main benefit of the derivation is that it explains how the 10^{-34} factor was removed from **h**, and its units changed from J-s to MeV.

If **m = h (xSn**) is correct, (and the factorings of hundreds of hadrons says it is) then a frequency of ($1.602176634 \times 10^{21}$ Hz) is associated with each unit of hypervolume of a hadron, no matter the dimension. In the example with **Ds** (See previous page), **Ds'**s hypervolume is **10.000 S9**, which equals 1967.053/**h** = 296.8657 hypervolume units. Multiplying 296.8657 by ($1.602176634 \times 10^{21}$ Hz/vol) - the frequency per unit hypervolume constant - will give you a frequency of $4.75631288 \times 10^{23}$ Hz as the frequency associated with the entire particle, which is correct. (Putting that frequency in Planck's energy-frequency law (**E=hf**) will give you the particle's mass in Joules.) So in terms of particle *hypervolume*, Planck's energy-frequency law can be rewritten as:

$$\mathbf{E}_{J} = \mathbf{h}_{J\text{-}s}\,(\mathbf{xSn}_{vol})\,(\mathbf{1.602176634 \times 10^{21}}\ \text{Hz/vol}) \qquad (\text{here } \mathbf{h} = 6.62607015 \times 10^{-34}\ \text{J-s})$$

Which says a frequency (and therefore energy) is associated with a volume. To convert **h** to units of MeV divide the right hand side by $1.602176634 \times 10^{-13}$ Joules/MeV (the Joules to MeV conversion factor). The result is **h** in units of MeV and a factor of (1×10^{34}) times **h(xSn)** on the right . (**E** on the left hand side of the equation then has units of MeV by default.) When that factor, (1×10^{34}), is multiplied by Planck's constant, ($6.62607015 \times 10^{-34}$ MeV), you are left with just Planck's constant's coefficient (6.62607015 MeV) for **h**. The result is:

$$\mathbf{m}_{\text{MeV}} = \mathbf{h}_{\text{MeV}}\,(\mathbf{xSn}) \qquad (\text{So, here } \mathbf{h} = 6.62607015\ \text{MeV, } \mathbf{not}\ 6.62607015 \times 10^{-34}\ \text{J-s.})$$

Where **m** is in units of MeV, **h** = 6.62607015 MeV, and **Sn** is the hypervolume calculated from the surface volume formula for an n-sphere using a radius of one (a unit radius). (**Snh** values are given in an appendix for all **n** from dimensions 2 to 21.) That formula seems to work on any dimension of hadron, *which implies that the mass density of the hypervolume of hadrons remains the same over all dimensions*. What is the density of the hypervolume of any hadron? It is 6.62607015 MeV per unit hypervolume. That's what the formula says if it is rearranged.

$$\mathbf{h}_{\text{MeV}} = \mathbf{m}_{\text{MeV}} / (\mathbf{xSn})$$

So, if m=h(xSn) is valid, it means that if a correct factoring can be found for a hadron then, a dimension and a precise mass can be assigned to it.

More Proof That Hadrons Are Made of Higher Dimensional Matter

More examples of higher dimensional hadrons follows, from dimension 4 to 18. Also, there are four appendices of useful information.

S5h Factoring
4/5 D Matter
(5-spheres have a 4D surface)

Factoring	ThrMass	TM-EM	ExpMass	ExpErr	Particle	Formation Quarks
4.44444 S5h =	775.071	0.051	**775.02**	.35	**ρ(775)**	**dd**

S6h Factoring
5/6 D Matter
(6-spheres have a 5D surface)

Factoring	ThrMass	TM-EM	ExpMass	ExpErr	Particle	Formation Quarks
8/3 S6h =	547.866	0.001	**547.865**	0.031	**η**	**ddu, sd, cu**
6.00000 S6h =	1232.698	0.202	**1232.9**	1.2	**Δ(1232)**	**ddu, sd, cu**
7.00000 S6h =	1438.148	0.852	**1439**	19	**N(1440)**	**ddu, sd, cu**
8.00000 S6h =	1643.598	0.598	**1643**	6	**Δ(1700)**	**ddu, sd, cu**
9.00000 S6h =	1879.047	0.953	**1848**	9	**Δ(1930)**	**ddu, sd, cu**
8192/900 S6h =	1870.049	0.049	**1870.0**	1.0	**D+**	**ddu, sd, cu**
12.00000 S6h =	2465.397	0.003	**2465.4**	0.2	**D2(2460)+**	**ddu, sd, cu**
12.55555 S6h =	2579.535	0.035	**2579.5**	3.4	**D(2550)o**	**ddu, sd, cu**

S7h Factoring
6/7 D Matter
(7-spheres have a 6D surface)

Factoring	ThrMass	TM-EM	ExpMass	ExpErr	Particle	Formation Quarks
2.50000 S7h =	547.866	0.001	**547.865**	0.031	**η**	**ddd, cd**
25/7 S7h =	782.665	0.015	**782.65**	0.12	**ω**	**ddd, cd**
6.00000 S7h =	1314.878	0.018	**1314.86**	0.20	**Xi°**	**ddd, cd**
6.03125 S7h =	1321.726	0.016	**1321.71**	0.07	**Xi^{-}**	**ddd, cd**
7.00000 S7h =	1534.024	0.376	**1534.4**	1.1	**$Xi(1530)^{-}$**	**ddd, cd**
7680/900 S7h =	1870.049	0.049	**1870.0**	1.0	**D+**	**ddd, cd**
8256/900 S7h =	2010.303	0.043	**2010.26**	0.05	**D*(2010)+**	**ddd, cd**

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

S8h Factoring

7/8 D Matter

(8-spheres have a 7D surface)

Factoring		*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
56/7	**S8h =**	1721.171	0.171	**1721**	13	**a2(1700)**	**dddu, cs, bd**
60/7	**S8h =**	1844.112	0.112	**1844**	9	**X(1835)**	**dddu, cs, bd**
64/7	**S8h** =	1967.053	0.053	**1967.0**	1.0	**Ds**	**dddu, cs, bd**
68/7	**S8h =**	2089.994	0.006	**2090**	10	**fo(2100)**	**dddu, cs, bd**
72/7	**S8h =**	2212.935	1.065	**2214**	20	**η(2225)**	**dddu, cs, bd**
76/7	**S8h =**	2335.876	1.124	**2337**	14	**fo(2230)**	**dddu, cs, bd**
80/7	**S8h** =	2458.817	0.083	**2458.9**	1.5	**Ds(2460)**	**dddu, cs, bd**
50255/2048	**S8h** =	5279.388	0.008	**5279.38**	0.11	**B+**	**dddu, cs, bd**
50257/2048	**S8h =**	5279.598	0.018	**5279.58**	0.15	**Bo**	**dddu, cs, bd**
2560/96	**S8h** =	5737.239	0.039	**5737.2**	0.7	**B2(5747)+**	**dddu, cs, bd**
2561/96	**S8h =**	5739.480	0.020	**5739.5**	0.7	**B2(5747)o**	**dddu, cs, bd**

S9h Factoring

8/9 D Matter

(9-spheres have an 8D surface)

Factoring		*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
10.00000	**S9h** =	1967.053	0.053	**1967.0**	1.0	**Ds**	**dddd, cc**
12.50000	**S9h =**	2458.817	0.083	**2458.9**	1.5	**Ds(2460)**	**dddd, cc**
13.66666	**S9h** =	2688.306	0.306	**2688**	4	**Ds(2700)**	**dddd, cc**
13.77777	**S9h** =	2710.162	0.162	**2710**	2	**Ds(2700)**	**dddd, cc**
29.00000	**S9h** =	5704.455	0.455	**5704**	4	**Bj(5732)**	**dddd, cc**

S10h Factoring

9/10 D Matter

(10-spheres have a 9D surface)

Factoring		*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
12.5000	**S10h** =	2112.195	0.005	**2112.2**	0.4	**Ds***	**ddddu, ccu**
12.4666	**S10h** =	2106.563	0.037	**2106.6**	2.1	**Ds***	**ddddu, ccu**
15.0000	**S10h** =	2534.634	0.034	**2534.6**	0.3	**Ds1(2536)**	**ddddu, ccu**
15.2222	**S10h** =	2572.185	0.015	**2572.2**	0.3	**Ds2(2573)**	**ddddu, ccu**
15.3333	**S10h** =	2590.960	0.040	**2591**	6	**Dso(2590)**	**ddddu, ccu**
25.6666	**S10h =**	4337.041	0.041	**4337**	7	**Pc(4337)**	**ddddu, ccu**
26.3333	**S10h** =	4449.692	0.108	**4449.8**	1.7	**Pc(4450)**	**ddddu, ccu**
26.6666	**S10h** =	4506.017	0.017	**4506**	11	**Xco(4500)**	**ddddu, ccu**

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

S11h Factoring

10/11 D Matter

(11-spheres have a 10D surface)

Factoring		*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
9-1/128	**S11h =**	1234.863	0.137	**1235**	15	**b1(1235)**	**ddddd, ccd**
9.00000	**S11h =**	1235.936	0.064	**1236**	16	**b1(1235)**	**ddddd, ccd**
9+1/128	**S11h =**	1237.009	0.009	**1237**	7	**b1(1235)**	**ddddd, ccd**
15.875	**S11h** =	2180.054	0.054	**2180**	8	**Xc0(2170)**	**ddddd, ccd**
15.90625	**S11h** =	2184.345					
15.9375	**S11h =**	2188.637	0.637	**2188**	10	**Xc0(2170)**	**ddddd, ccd**
15.96875	**S11h =**	2192.928	0.072	**2193**	2	**Xc0(2193)**	**ddddd, ccd**
16.	**S11h** =	2197.219	0.181	**2197.4**	4.4	**Xc0(1P)**	**ddddd, ccd**
16.03125	**S11h =**	2201.511	0.511	**2201**	19	**Xc0(1P)**	**ddddd, ccd**
16.0625	**S11h =**	2205.802	0.198	**2206**	12	**Xc0(1P)**	**ddddd, ccd**
16.09375	**S11h =**	2210.094					
16.125	**S11h =**	2214.384	0.384	**2214**	20	**Xc0(1P)**	**ddddd, ccd**
16.3125	**S11h =**	2240.134	0.934	**2239.2**	7.1	X(2240)	**ddddd, ccd**
17.875	**S11h =**	2454.706	0.294	**2455**	3	**D2*(2460)^0**	**ddddd, ccd**
17.90625	**S11h =**	2458.998	0.002	**2459**	3	**D2*(2460)^0**	**ddddd, ccd**
17.9375	**S11h =**	2463.289	0.011	**2463.3**	0.6	**D2*(2460)^0**	**ddddd, ccd**
29.000	**S11h =**	3982.461	0.039	**3982.5**	1.8	**Zcs(3982)**	**ddddd, ccd**
29.375	**S11h** =	4033.958	0.042	**4034**	6	**X(4040)**	**ddddd, ccd**
29.500	**S11h** =	4051.124	0.124	**4051**	14	**X(4050)**	**ddddd, ccd**
31.125	**S11h** =	4274.279	0.121	**4274.4**	8.4		
32.125	**S11h** =	4411.605	0.605	**4411**	7	**Ψ(4415)**	**ddddd, ccd**
32.250	**S11h** =	4428.771	0.229	**4429**	9	**Ψ(4415)**	**ddddd, ccd**
32.33333	**S11h** =	4440.215	0.085	**4440.3**	1.3	**Pc(4440)**	**ddddd, ccd**
34.000	**S11h** =	4669.092	0.229	**4669**	21	**Ψ(4660)**	**ddddd, ccd**
4096/7	**S11h =**	**80,355.473**	1.473	**80,354**	23	**W boson**	**[2]**
4100/7	**S11h =**	**80,433.945**	0.445	**80,433.5**	9.4	**W boson**	**[2]**

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

S12h Factoring

11/12 D Matter

(12-spheres have an 11D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	Formation *Quarks*
26.00000 **S12h** =	2760.433	0.333	**2760.1**	1.1	**D3*(2750)**	**d^{5}u, ccs**
27.00000 **S12h** =	2866.605	0.005	**2866.6**	AVG	**Ds3(2860)$^+$**	**d^{5}u, ccs**
28.00000 **S12h** =	2972.775	0.975	**2971.8**	8.7	**D(3000)^0**	**d^{5}u, ccs**
28.33333 **S12h** =	3008.165	0.065	**3008.1**	4.0	**D(3000)^0**	**d^{5}u, ccs**
28.66666 **S12h** =	3043.555	0.444	**3044**	8	**Dsj(3040)^0**	**d^{5}u, ccs**
30.06666 **S12h** =	3510.705	0.005	**3510.71**	0.04	**Xc1(1P)**	**d^{5}u, ccs**
35.55555 **S12h** =	3774.952	0.548	**3775.5**	2.4	**Ψ(3770)**	**d^{5}u, ccs**
36.00000 **S12h** =	3822.139	0.061	**3822.2**	1.2	**Ψ2(3823)**	**d^{5}u, ccs**

S13h Factoring

12/13 D Matter

(13-spheres have a 12D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	Formation *Quarks*
16.0000 **S13h =**	1255.049	0.049	**1255**	7	**a1(1260)**	**d^6, ccc**
50-8/90 **S13h** =	3915.056	0.056	**3915**	3	**X(3930)**	**d^6, ccc**
50.0000 **S13h** =	3922.028	0.013	**3922.15**	1.2	**X(3930)**	**d^6, ccc**
50+8/90 **S13h** =	3929.001	0.001	**3929**	5	**X(3930)**	**d^6, ccc**

S14h Factoring

13/14 D Matter

(14-spheres have a 13D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
40.00000 **S14h** =	2223.630	0.270	**2223.9**	2.5	**fj(2220)**	**d^{6}u, cccu**
41.50000 **S14h =**	2307.016	0.016	**2307**	6	**ρ5(2350)**	**d^{6}u, cccu**
61.44000 **S14h** =	3415.496	0.004	**3415.5**	0.4	**Xc0(1P)**	**d^{6}u, cccu**
64.00000 **S14h =**	3557.808	0.008	**3557.8**	1.2	**Xc2(1P)**	**d^{6}u, cccu**

Note: 6144 = 4096 + 2048
6400 = 4096 + 2048 + 256

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

S15h Factoring

14/15 D Matter

(15-spheres have a 14D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
48.0000 **S15h** = 1819.778		0.378	**1819.4**	3.1	**Xi(1820)**	**d^7, cccd**
93.0000 **S15h =** 3525.820		0.020	**3525.8**	0.2	**h1(1P)**	**d^7, cccd**
113.0000 **S15h =** 4284.061		0.061	**4284**	17	**Y(4260)**	**d^7, cccd**

S16h Factoring

15/16 D Matter

(16-spheres have a 15D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
2^{17} /900 S16h =	3633.472	0.128	**3633.6**	1.7	**nc(2s)**	**d^7u, cccs**
2^{17}+128 /900 S16h =	3637.020	0.020	**3637.0**	5.7	**nc(2s)**	**d^7u, cccs**
2^{17}+256 /900 S16h =	3640.569	0.069	**3640.5**	3.2	**nc(2s)**	**d^7u, cccs**

S17h Factoring

16/17 D Matter

(17-spheres have a 16D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
222.0000 **S17h =**	**3525.484**	0.084	**3525.40**	0.13	**hc(1P)**	**d^8, cccc**
384.0000 **S17h =**	**6098.135**	0.135	**6098.0**	1.7	**Σb(6097)**	**d^8, cccc**
668.0000 **S17h =**	**10608.215**	0.115	**10608.1**	1.2	**Zb(10610)**	**d^8, cccc**

S18h Factoring

17/18 D Matter

(18-spheres have a 17D surface)

Factoring	*ThrMass*	*TM-EM*	*ExpMass*	*ExpErr*	*Particle*	*Formation Quarks*
99.000 **S18h** =	969.950	0.150	**969.8**	4.5	**fo(980)**	**d^8u, ccccu**
99.750 S18h =	977.296	0.004	**977.3**	0.9	**fo(980)**	**d^8u, ccccu**
100.250 S18h =	982.197	0.003	**982.2**	0.6	**fo(980)**	**d^8u, ccccu**
100.500 S18h =	984.646	0.054	**984.7**	0.4	**fo(980)**	**d^8u, ccccu**
101.250 S18h =	991.994	0.006	**992.0**	8.5	**fo(980)**	**d^8u, ccccu**
101.375 S18h =	993.219	0.019	**993.2**	6.5	**fo(980)**	**d^8u, ccccu**

Source of Mass Data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX A

Quark Assignments
to
n-Sphere Surface Volume Formulae

Sphere Dimension	Quark Names Old	Quark Names New		Corresponding n-Sphere Surface Formula
2	**u**	q1	=	$2\ \pi^1 r^1$
3	**d**	q2	=	$4\ \pi^1 r^2$
4	**s**	q3	=	$2\ \pi^2 r^3$
5	**c**	q4	=	$8/3\ \pi^2 r^4$
6	**b**	q5	=	$\pi^3 r^5$
7	**t**	q6	=	$16/15\ \pi^3 r^6$
8	------	q7	=	$1/3\ \pi^4 r^7$
9	------	q8	=	$32/105\ \pi^4 r^8$
10	------	q9	=	$1/12\ \pi^5 r^9$
11	------	q10	=	$64 / 945\ \pi^5 r^{10}$
12	------	q11	=	$1 / 60\ \pi^6 r^{11}$
13	------	q12	=	$128 / 10395\ \pi^6 r^{12}$
14	------	q13	=	$1 / 360\ \pi^7 r^{13}$
15	------	q14	=	$256 / 135135\ \pi^7 r^{14}$
16	------	q15	=	$1 / 2520\ \pi^8 r^{15}$
17	------	q16	=	$512 / 2027025\ \pi^8 r^{16}$
18	------	q17	=	$1 / 20160\ \pi^9 r^{17}$
19	------	q18	=	$1024 / 34459425\ \pi^9 r^{18}$
20	------	q19	=	$1 / 181440\ \pi^{10} r^{19}$
21	------	q20	=	$2048 / 654729075\ \pi^{10} r^{20}$

APPENDIX B

n-Sphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

Sphere Dimension	Sn		Surface Volume Formula	(π, r) Powers
2	**S2**	=	$2 \ \pi^1 r^1$	(1, 1)
3	**S3**	=	$4 \ \pi^1 r^2$	(1, 2)
4	**S4**	=	$2 \ \pi^2 r^3$	(2, 3)
5	**S5**	=	$8/3 \ \pi^2 r^4$	(2, 4)
6	**S6**	=	$\pi^3 r^5$	(3, 5)
7	**S7**	=	$16/15 \ \pi^3 r^6$	(3, 6)
8	**S8**	=	$1/3 \ \pi^4 r^7$	(4, 7)
9	**S9**	=	$32/105 \ \pi^4 r^8$	(4, 8)
10	**S10**	=	$1/12 \ \pi^5 r^9$	(5, 9)
11	**S11**	=	$64 / 945 \ \pi^5 r^{10}$	(5, 10)
12	**S12**	=	$1 / 60 \ \pi^6 r^{11}$	(6, 11)
13	**S13**	=	$128 / 10395 \ \pi^6 r^{12}$	(6, 12)
14	**S14**	=	$1 / 360 \ \pi^7 r^{13}$	(7, 13)
15	**S15**	=	$256 / 135135 \ \pi^7 r^{14}$	(7, 14)
16	**S16**	=	$1 / 2520 \ \pi^8 r^{15}$	(8, 15)
17	**S17**	=	$512 / 2027025 \ \pi^8 r^{16}$	(8, 16)
18	**S18**	=	$1 / 20160 \ \pi^9 r^{17}$	(9, 17)
19	**S19**	=	$1024 / 34459425 \ \pi^9 r^{18}$	(9, 18)
20	**S20**	=	$1 / 181440 \ \pi^{10} r^{19}$	(10, 19)
21	**S21**	=	$2048 / 654729075 \ \pi^{10} r^{20}$	(10, 20)

APPENDIX C

Values of n-Sphere Surface Volume Units of Factorization

(Here **h** = 6.62607015 MeV, **not** 6.62607015 x 10-34 J-s)

(Dimension 2 - Dimension 21)

Sphere Dimension	Unit of Factorization		Formula		Value (MeV)
2	**S2h**	=	2 $\pi^1 r^1 h$	=	41.63282661
3	**S3h**	=	4 $\pi^1 r^2 h$	=	83.26565322
4	**S4h**	=	2 $\pi^2 r^3 h$	=	130.7933822
5	**S5h**	=	8/3 $\pi^2 r^4 h$	=	174.3911763
6	**S6h**	=	$\pi^3 r^5 h$	=	205.4497644
7	**S7h**	=	16/15 $\pi^3 r^6 h$	=	219.1464153
8	**S8h**	=	1/3 $\pi^4 r^7 h$	=	215.1464901
9	**S9h**	=	32/105 $\pi^4 r^8 h$	=	196.7053624
10	**S10h**	=	1/12 $\pi^5 r^9 h$	=	168.9756582
11	**S11h**	=	64 / 945 $\pi^5 r^{10} h$	=	137.3262492
12	**S12h**	=	1 / 60 $\pi^6 r^{11} h$	=	106.1705373
13	**S13h**	=	128 / 10395 $\pi^6 r^{12} h$	=	78.44057013
14	**S14h**	=	1 / 360 $\pi^7 r^{13} h$	=	55.59076334
15	**S15h**	=	256 / 135135 $\pi^7 r^{14} h$	=	37.91204905
16	**S16h**	=	1 / 2520 $\pi^8 r^{15} h$	=	24.94907624
17	**S17h**	=	512 / 2027025 $\pi^8 r^{16} h$	=	15.88056197
18	**S18h**	=	1 / 20160 $\pi^9 r^{17} h$	=	9.797479330
19	**S19h**	=	1024 / 34459425 $\pi^9 r^{18} h$	=	5.869441980
20	**S20h**	=	1 / 181440 $\pi^{10} r^{19} h$	=	3.419965454
21	**S21h**	=	2048 / 654729075 $\pi^{10} r^{20} h$	=	1.940989032

APPENDIX D

Smallest Formation Quarks per n-Sphere

(Dimension 2 - Dimension 21)

Sphere Dimension	Sn	Surface Volume Formula	(π, r) Powers	Formation Quarks	
2	**S2** =	$2\ \pi^1 r^1$	(1, 1)	u	
3	**S3** =	$4\ \pi^1 r^2$	(1, 2)	d	
4	**S4** =	$2\ \pi^2 r^3$	(2, 3)	du	di-quarks
5	**S5** =	$8/3\ \pi^2 r^4$	(2, 4)	dd	
6	**S6** =	$\pi^3 r^5$	(3, 5)	ddu	tri-quarks
7	**S7** =	$16/15\ \pi^3 r^6$	(3, 6)	ddd	
8	**S8** =	$1/3\ \pi^4 r^7$	(4, 7)	dddu	tetra-quarks
9	**S9** =	$32/105\ \pi^4 r^8$	(4, 8)	dddd	
10	**S10** =	$1/12\ \pi^5 r^9$	(5, 9)	ddddu	penta-quarks
11	**S11** =	$64 / 945\ \pi^5 r^{10}$	(5, 10)	ddddd	
12	**S12** =	$1 / 60\ \pi^6 r^{11}$	(6, 11)	dddddu	hexa-quarks
13	**S13** =	$128 / 10395\ \pi^6 r^{12}$	(6, 12)	dddddd	
14	**S14** =	$1 / 360\ \pi^7 r^{13}$	(7, 13)	ddddddu	hepta-quarks
15	**S15** =	$256 / 135135\ \pi^7 r^{14}$	(7, 14)	ddddddd	
16	**S16** =	$1 / 2520\ \pi^8 r^{15}$	(8, 15)	dddddddu	octa-quarks
17	**S17** =	$512 / 2027025\ \pi^8 r^{16}$	(8, 16)	dddddddd	
18	**S18** =	$1 / 20160\ \pi^9 r^{17}$	(9, 17)	ddddddddu	nona-quarks
19	**S19** =	$1024 / 34459425\ \pi^9 r^{18}$	(9, 18)	ddddddddd	
20	**S20** =	$1 / 181440\ \pi^{10} r^{19}$	(10, 19)	dddddddddu	deca-quarks
21	**S21** =	$2048 / 654729075\ \pi^{10} r^{20}$	(10, 20)	dddddddddd	

References

1. P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update
2. S. Navaset al.(Particle Data Group), Phys. Rev. D110, 030001 (2024)