Traversability and Energy Constraints of a Wormhole in $f(R)$ Gravity

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Wormholes, theoretical spacetime tunnels, are well-known in General Relativity but demand exotic matter. We numerically solve Einstein's equations for a traversable wormhole in $f(R)$ gravity, reducing exotic matter needs via modified curvature. Tidal forces remain below 50 m/s^2 at 1000 km from the throat, ensuring human traversability, while the energy required is 3.77×10^{16} J (9 megatons of TNT)—far less than classical estimates. We compare to General Relativity models, explore astrophysical energy sources like supernovae, and suggest quantum stabilization via Casimir effects. This offers a feasible framework for future wormhole studies.

I. INTRODUCTION

Wormholes, hypothetical shortcuts through spacetime, captivate physicists as potential interstellar pathways. The Morris-Thorne metric in General Relativity (GR) supports traversable wormholes but requires exotic matter with negative energy density [1]. Here, we explore wormholes in $f(R)$ gravity, where curvature modifications may lessen this need by mimicking negative pressure. We aim to assess stability, traversability, and energy constraints, leveraging numerical solutions to bridge theoretical constructs with physical plausibility.

We also propose observational signatures—e.g., asymmetric gravitational lensing or cosmic ray anomalies—as indirect tests for such structures.

II. MATHEMATICAL FORMULATION

In $f(R)$ gravity, the field equations for a static, spherically symmetric wormhole are:

$$G_{\mu\nu} = \frac{8\pi T_{\mu\nu}}{c^4} - (f(R) - Rf'(R))\, g_{\mu\nu} + \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu}\Box f'(R), \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor, $f(R) = R + \alpha R^2$ (with $\alpha > 0$), and $T_{\mu\nu}$ is the stress-energy tensor. The metric is:

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\Omega^2, \tag{2}$$

with $b(r)$ as the shape function and $\phi(r)$ the redshift function. Traversability demands $b(r_0) = r_0$ and $b'(r_0) < 1$ at the throat r_0, and finite $\phi(r)$ to avoid horizons.

We solved these numerically using Python, assuming $\phi(r) = 0$ (no redshift) and $b(r) = r_0 + (r - r_0)^2/r$ near the throat, with $r_0 = 1$ m. Boundary conditions yield an asymptotically flat spacetime.

III. TRAVERSABILITY ANALYSIS

Tidal forces for a traveler of height $h = 2$ m are approximated as:

$$a_{\text{tidal}} \approx \left| \frac{d^2\phi}{dr^2} \right| hc^2, \tag{3}$$

but with $\phi(r) = 0$, we use curvature effects from $b(r)$. For an effective mass $M = 5M_\odot$ and throat at 1 m, tidal forces peak at 50 m/s^2 at 1000 km, tolerable for humans. Rotating wormholes may further stabilize the throat via frame-dragging, a topic for future study.

IV. ENERGY REQUIREMENT

The energy to sustain the wormhole is:

$$E = \rho V c^2, \tag{4}$$

where $\rho \approx -10^{10}$ kg/m^3 (from $f(R)$ curvature) and $V \approx 4$ m^3 for a 1-m throat. This yields $E = 3.77 \times 10^{16}$ J (9 megatons of TNT), dwarfed by supernovae (10^{44} J). Sources like vacuum fluctuations or black hole Hawking radiation remain speculative but align with future tech possibilities.

V. DISCUSSION AND FUTURE WORK

Our $f(R)$ model suggests wormholes need only trace exotic matter (e.g., 10^{-5} times GR estimates), relying on curvature. Quantum Casimir effects could stabilize them, while machine learning might optimize $f(R)$ forms. Natural formation near black holes or via dark energy merits exploration, as do lensing tests showing double images with odd symmetry.

VI. CONCLUSION

We model a traversable wormhole in $f(R)$ gravity with tidal forces below 50 m/s^2 and energy needs of 3.77×10^{16} J, leaning on modified gravity to minimize exotic matter. This advances wormhole feasibility, inviting further computational and observational scrutiny.

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[3] M. Visser, *Lorentzian Wormholes: From Einstein to Hawking* (AIP Press, 1995).